Non-perturbative studies of QFT in Minkowski space

(3 credits – April-May 2020 – G. Salmè)

The non-perturbative regime of QFTs has its paradigmatic example in the existence of bound states, that appear as poles of the proper n-point Green's function. As is well-known, the standard and very successful approach for facing with the paramount challenge of a detailed description of QFTs, in whole dynamical range, is to shift from the Minkowski space to the Euclidean one, and *to discretize* the space on a 4D lattice, in order to evaluate the needed correlation functions, as dictated by the path-integral approach.

Nonetheless, it could be interesting, for a possible cross-fertilization, to develop *continuous* non-perturbative approaches, as the ones based on the Dyson-Schwinger equations (DSEs) [1,2], that amount to an infinite tower of coupled equations, to be fulfilled by the Green's functions of the theory under scrutiny, possibly in Minkowski space, i.e. the space where the physical processes take place. It is important to emphasize that such approaches necessarily contain a truncation of DSE hierarchy, and the main efforts are focused on the self-consistency of the final scheme. In the last decades, successful efforts have been undertaken to establish a proper framework, where one could investigate the continuous QFT (see, e.g. [3]), by using a suitable truncation of the DSEs and taking into account also the Bethe-Salpeter equation (BSE) [4], that leads to determine the amplitude of a bound state within a genuine QFT framework. The developed approaches are still played in the Euclidean space, quite advantageous for the analytic properties one can exploit and the correspondence with the spacelike-region of the Minkowski space. Therefore, attempts to extend the continuous-QFT framework to the Minkowski space seem to be quite worthwhile [5]. A possible Minkowski space framework can be constructed by relying on the so-called Nakanishi integral representation [6] of the n-legs transition amplitudes (e.g. 2-legs → self-energy, 3-legs → vertex function), that can be applied to the study of both the BSE for bound states and the DSEs [7,8].

Aim of the proposed 3-credit course is a self-consistent presentation of i) the Dyson-Schwinger equations governing the n-point functions (Green's functions); ii) the Bethe-Salpeter equation for bound states, with and without spin degrees of freedom; iii) the Nakanishi integral representations to be used for solving both the Bethe-Salpeter equation and Dyson-Schwinger equations within a self-consistent truncation scheme, in order to obtain dynamical observables, that cannot be evaluated within a purely Euclidean approach to QFT, like the momentum distributions and the probability of the state with the lowest number of constituents (aka valence state), belonging to the Fock expansion of a given bound state.

Since the subject, crossing several fields, is highly *interdisciplinary*, the presentation of the items will not assume any prerequisite, except for the basic knowledge of QFT (Reference Textbook: C. Itzykson and J. B. Zuber, *Quantum Field Theory,* McGraw-Hill, New York, 1980).

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[8]V. Sauli, *Minkowski solution of Dyson-Schwinger equations in momentum subtraction scheme*, JHEP 02 (2003) 001; C. Mezrag and G. Salmè, *Solving the Fermion and Photon gap-equations in Minkowski space*, in preparation.

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