Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Condizioni per "esorcizzare" il DRM
Una stringa ben nascosta?
La stringa classica di Nambu-Goto

Punti principali dell'ultima lezione

- Dopo un breve richiamo dell'oscillatore armonico e sua quantizzazione abbiamo introdotto il DRM come estensione della funzione Beta a processi che coinvolgono un numero arbitrario di particelle.
- Dalle proprietà di queste si trova un'insieme sufficiente di stati per descrivere lo spettro della teoria.
- Caratteristiche principali dello spettro: degenerazione esponenziale (nella massa dello stato) e possibile presenza di stati a norma (prob. di produzione) negativa.

Piano della lezione

- Condizione di Virasoro e vincolo sul D
- Indicazioni a favore di una stringa sottostante
- La stringa classica di Nambu-Goto

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- Fermioni, GSO, supersimmetria
- Problemi fenomenologici e sopravvento di QCD

A sufficient set of states consists of the eigenstates of momentum and of the occupation numbers of an infinite set of harmonic oscillators:

$$[q_\mu \ , \ p_\nu] = i\eta_{\mu\nu} \ , \quad [a_{n,\mu} \ , \ a^\dagger_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

$$(n = 1, 2, \ldots \ ; \ \mu = 0, 1, 2, \ldots D-1)$$

$$|N_{n,\mu}, k\rangle \sim \prod_{n,\mu} \left(a^\dagger_{n,\mu}\right)^{N_{n,\mu}} e^{iqk}|0\rangle \quad ; \quad a_{n,\mu}|0\rangle = p_\mu|0\rangle = 0$$

$$-\alpha' k^2 = \alpha' M^2 = -1 + \sum_{n,\mu} n \ a^\dagger_{n,\mu} \ a^\mu_n$$

(relativistic analog of E of h.o.)

Because of the "wrong" sign of the time-like c.r., states created by an odd number of time-like operators are ghosts. One hope remained: all those states were sufficient but perhaps only a (ghost-free?) subset was necessary.

In the FV69 paper the following (so-called "spurious") states were found to be unnecessary (for any value of $\alpha(0)$):

$$L_{-1}|X\rangle \equiv \left(p\cdot a_1^\dagger + \sum_n \sqrt{n(n+1)}\, a_{n+1}^\dagger \cdot a_n\right)|X\rangle$$

(with |X> any state)

This was hopefully sufficient to eliminate the ghosts created by the time component of a^+_1. But what about all others?

The situation looked almost desperate... until Virasoro (1969) made a crucial discovery. Iff $\alpha(0) = 1$ one could enlarge enormously the space of "spurious" states to:

$$L_{-m}|X\rangle \equiv \left(p\cdot a_m^\dagger + \sum_n \sqrt{n(n+m)}\, a_{n+m}^\dagger \cdot a_n\right)|X\rangle$$

(with m=1,2,..)

=> for $\alpha(0) =1$, there was a chance to eliminate all the ghosts!

These, plus their h.c. L_{+m}, are the Virasoro operators!

The “ghost hunting” project was a “tour de force” that culminated in the proof of a “no-ghost theorem” by R. Brower and, independently, by P. Goddard & Ch.Thorn.

At the basis of the theorem was the discovery (Fubini-Veneziano-Weis, 1970) of the infinite-dimensional Virasoro algebra (of which the previous O(2,1) symmetry is a subgroup generated by L_0, L_1 , L_{-1}) and the explicit construction of an infinite set of positive-norm physical (Di Vecchia-Del Giudice-Fubini or DDF) states (using only D-2 components of the operators)

There were a couple of heavy prices to pay for the absence of ghosts: the Regge intercept, α_0, had to be exactly 1 (implying a massless spin one particle and a spin zero "tachyon") and D had to be less than or equal to 26 (this was only checked later, after Lovelace's observation, see below).

At exactly D=26 the physical Hilbert space would be completely spanned by the DDF states corresponding to oscillators in (D-2)=24 dimensions. At D<26 one needed additional (so-called Brower) states.

Meanwhile, C. Lovelace had shown that loops were consistent with unitarity only if D=26! Had he gone crazy?

For D=26 and α_0=1 the model looked consistent except for the presence of a tachyon ($M^2 = -1/\alpha'$).

It took a while before it was realized that the DRM was a theory of strings. Till about 1972 it looked like a very strange kind of object, mysteriously different from anything that had been seen before, like QFT or GR.

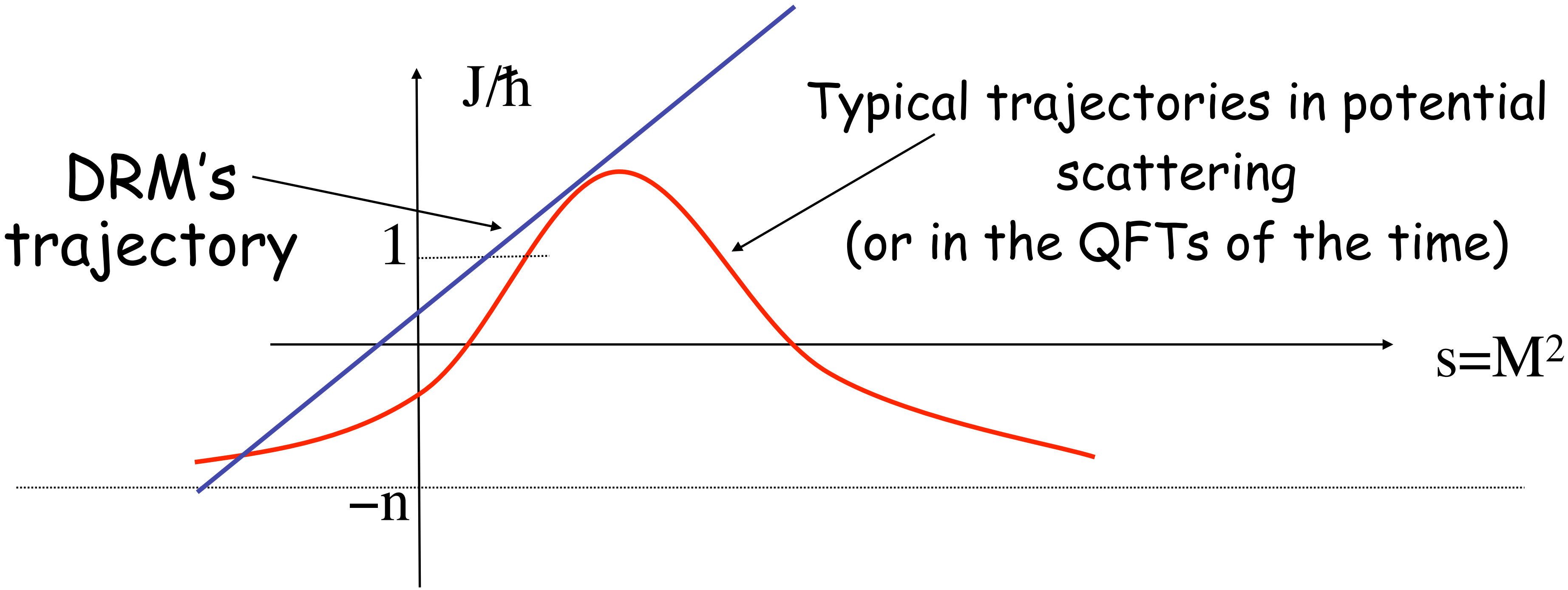
As such it polarized the community with the opponents (particularly within the establishment) outnumbering the (mostly young but with exceptions) enthusiasts.

(For earlier attitudes towards DRM, read Louis Clavelli: http://bama.ua.edu/~lclavell/Weston/)

Missed hints of an underlying string?

1. From linear Regge trajectories
2. From duality diagrams
3. From the harmonic oscillators
4. From the DDF «transverse» states

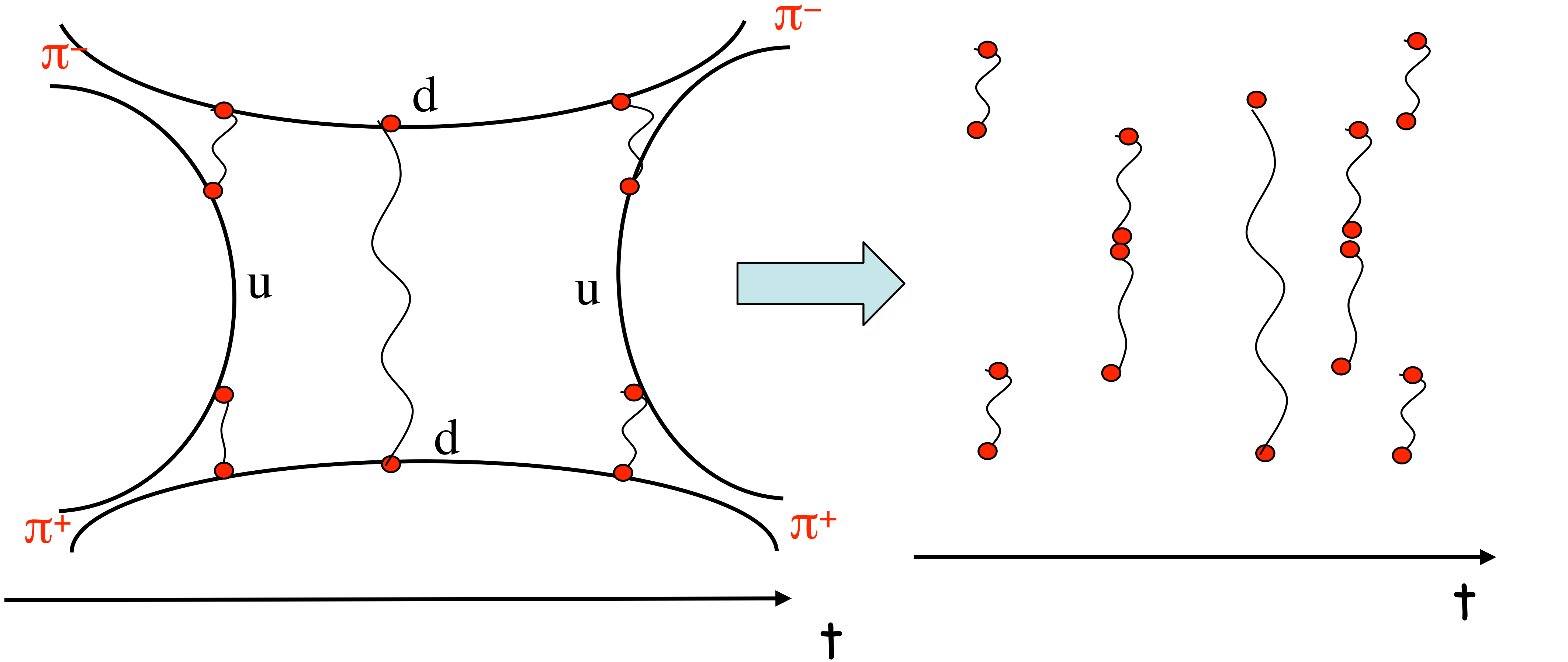
From linear Regge trajectories



$\alpha' = dJ/dM^2 \sim 10^{-13}$ cm/GeV ~ constant.

Its inverse, 10^{13} GeV/cm, has dimensions of a string tension (NB, c=1 but no $\hbar$ needed)!

Strings from duality diagrams



From the harmonic oscillators

This was a very clear hint since a string is a collection of harmonic oscillators of integer frequencies (wrt a fundamental one).

From DDF «transverse» states

The physical vibrations of a string are orthogonal to the string itself: the number of physical dof should therefore be proportional to (D-2), like for the DDF states.

The Nambu-Goto action

After some incomplete attempts to formulate a string theory that would reproduce the DRM (Nielsen, Susskind, Nambu), a decisive step forward was made in 1970-'71 by Nambu and, independently, by Goto.

They wrote a geometric action for the classical relativistic string in strict analogy with the well-known action of the relativistic particle.

This was the true birth of String Theory although, for the moment, just at the classical level.

The Nambu-Goto action

The Nambu-Goto action for the relativistic string is the straightforward generalization of the well-known action of a relativistic particle.

The latter can be written in a general (but given, fixed) spacetime metric $g_{\mu\nu}(x)$ and for any D as:

$$S_p = -mc \int ds = -mc \int d\tau \sqrt{-\frac{dx^\mu(\tau)}{d\tau}\frac{dx^\nu(\tau)}{d\tau} g_{\mu\nu}(x(\tau))}$$

The action of a point-particle is thus proportional (with mc as the proportionality constant) to the proper length of the "world-line" described by the particle's motion and parametrized by $x^{\mu}(\tau)$. The classical motion is the one minimizing that length (a *geodesic* in the given metric).

In complete analogy, for a relativistic string NG wrote a geometric action proportional to the area of the surface ("world-sheet" in analogy with "world-line") swept by the string. T, the string tension, is the proportionality constant.

The string motion is parametrized by $X^{\mu}(\sigma, \tau)$ where:

$\mu = 0, 1, \ldots D-1$; $0 < \sigma < \pi$ (by convention), τ unconstrained.

NG did this in Minkowski spacetime ($g_{\mu\nu}(x) = \eta_{\mu\nu}(x)$) but, like for point particles, the construction can be easily generalized to an arbitrary metric $G_{\mu\nu}(x)$ and to any D.

$$S_{NG} = -T \int d(Area) = -T \int d^2\xi \sqrt{-det\gamma_{\alpha\beta}} \equiv -T \int d\xi^0 \int_0^\pi d\xi^1 \sqrt{-det\gamma_{\alpha\beta}}$$

where

$$\gamma_{\alpha\beta} \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^\alpha} \frac{\partial X^\nu(\xi)}{\partial \xi^\beta} G_{\mu\nu}(X(\xi)) \ , \quad \alpha, \beta = 0, 1 \quad , \quad \xi^0 = \tau, \ \xi^1 = \sigma$$

Again by analogy with the point-particle, the classical motion of the string is obtained by varying the action and corresponds to minimizing the area of the surface swept.

But, unlike in the case of point-particles, the problem is already non trivial even in Minkowski space-time (at the quantum level).

Another major difference: for point particles we can add in the action different particles of different mass thus introducing many free parameters.

Also, interactions have to be added by hand (and it is not so simple!) and are quite arbitrary.

This is not the case for the string: there is just one T and interactions are automatically included in a "geometric" way!

Postponing constraints, boundary conditions…

A convenient choice of coordinates

There is a lot of freedom in the choice of the two WS coordinates (σ, τ). It can be used to impose two useful conditions (defining the orthonormal gauges):

$$\begin{aligned} \dot{X}^2 + X'^2 &\equiv -(\dot{X}_0)^2 + (\dot{X}_i)^2 - (X_0')^2 + (X_i')^2 = 0 \\ \dot{X} \cdot X' &= 0 \quad , \quad \text{i.e.} \ \ (\dot{\mathrm{X}} \pm \mathrm{X}')^2 = 0 \end{aligned}$$

The equations of motion become simply: $\ddot{X}_\mu = X_\mu''$

w/ solution: $X_\mu(\sigma, \tau) = F_\mu(\tau - \sigma) + G_\mu(\tau + \sigma)$

General solution of $\ddot{X}_\mu = X''_\mu$

Open (Neumann) strings ($X'_\mu(\sigma=0, \pi) = 0$). Def. $2\pi\alpha' = 1/T$

$$X_\mu(\sigma,\tau) = q_\mu + 2\alpha' p_\mu \tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,\mu}}{\sqrt{n}}e^{-in\tau} - \frac{a^*_{n,\mu}}{\sqrt{n}}e^{in\tau}\right] cos(n\sigma)$$

Closed strings $X_\mu(\sigma=0) = X_\mu(\sigma=\pi)$

$$\begin{aligned} X_\mu(\sigma,\tau) &= q_\mu + 2\alpha' p_\mu \tau + \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,\mu}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^*_{n,\mu}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right] \\ &+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{\tilde{a}_{n,\mu}}{\sqrt{n}}e^{-2in(\tau+\sigma)} - \frac{\tilde{a}^*_{n,\mu}}{\sqrt{n}}e^{2in(\tau+\sigma)}\right] \end{aligned}$$

to be added

$$\begin{aligned} \dot{X}\cdot X' &= 0 \\ \dot{X}^2 + X'^2 &\equiv -(\dot{X}_0)^2 + (\dot{X}_i)^2 - (X'_0)^2 + (X'_i)^2 = 0 \end{aligned}$$

Simplest classical solution: open string, the rotating rod

The equations to be solved are $\ddot{X}_\mu = X''_\mu$

subject to the constraints

$$\begin{aligned} \dot{X} \cdot X' &= 0 \\ \dot{X}^2 + X'^2 &\equiv -(\dot{X}_0)^2 + (\dot{X}_i)^2 - (X'_0)^2 + (X'_i)^2 = 0 \end{aligned}$$

and to the b.c. $X'_\mu(\sigma = 0, \pi) = 0 \Rightarrow \sum_i \left(\frac{dX_i}{dX_0}\right)^2 (\sigma = 0, \pi) = 1$

Ends move with speed of light. A simple solution is:

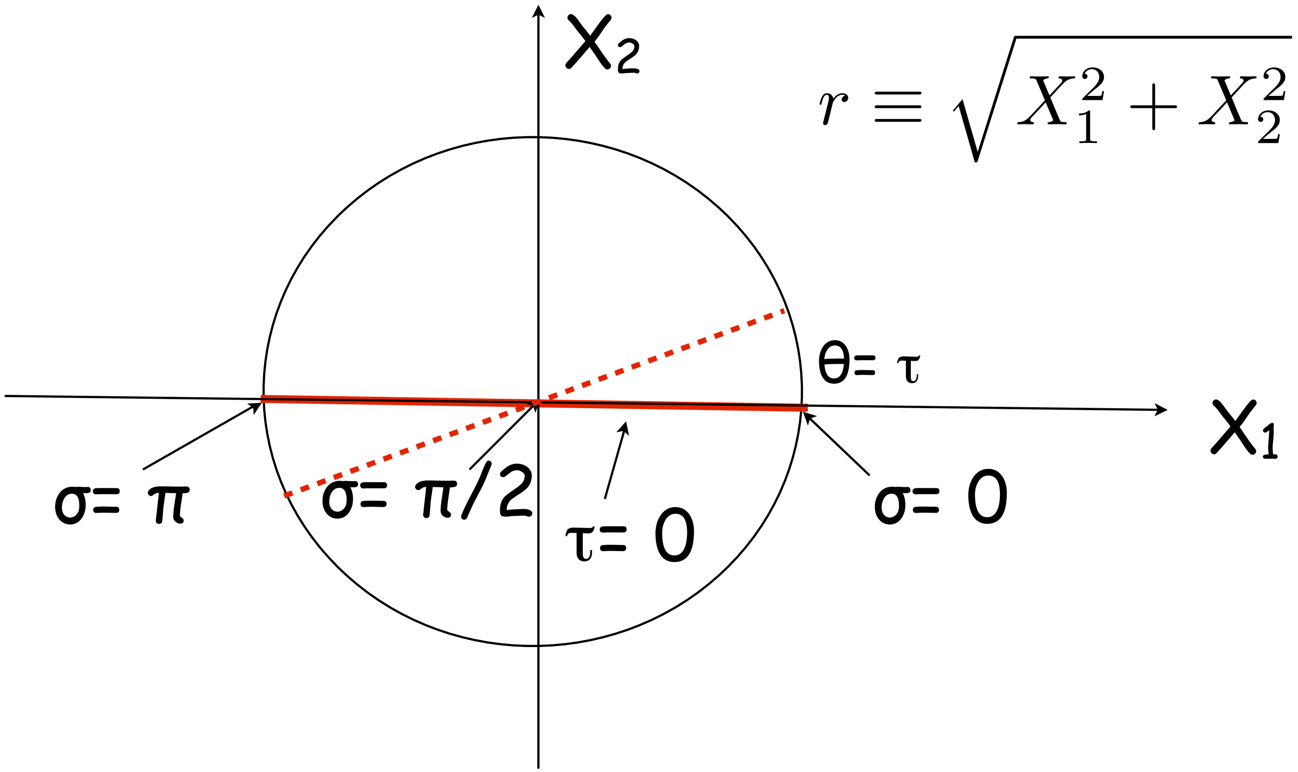
$$X_0 = A\tau \quad ; \quad X_1 = A\, cos\tau\ cos\sigma\ ,\ X_2 = A\, sin\tau\ cos\sigma$$

$$X_i = 0 \ , \quad (i = 3, 4, \ldots D - 1)$$

e.o.m., constraints and b.c. easily checked!

$$X_0 = A\tau \quad ; \quad X_1 = A\ cos\tau\ cos\sigma\ ,\ X_2 = A\ sin\tau\ cos\sigma$$

$$X_i \quad = \quad 0 \quad , \quad (i = 3, 4, \ldots D-1)$$



$$r \equiv \sqrt{X_1^2 + X_2^2} = A|cos\sigma|$$

A rigid, rotating rod whose ends move with the speed of light since $dl/dX_0 = r\ d\theta/Ad\,\tau = r/A = |cos\sigma|$.

Let us now compute the energy (mass) and angular momentum of this classical string.

$$X_0 = A\tau \quad ; \quad X_1 = A\ cos\tau\ cos\sigma\ ,\ X_2 = A\ sin\tau\ cos\sigma$$

$$X_i \ = \ 0 \ , \ (i = 3, 4, \ldots D-1)$$

$$p_i = \int_0^\pi d\sigma P_i(\sigma) = T \int_0^\pi d\sigma \dot{X}_i = 0$$

$$E = M = \int_0^\pi d\sigma P_0(\sigma) = T \int_0^\pi d\sigma \dot{X}_0 = \pi T A$$

$$\begin{aligned} J_{12} &= \int_0^\pi d\sigma (X_1 P_2 - X_2 P_1)(\sigma) \\ &= TA^2 \int_0^\pi d\sigma (sin^2\tau cos^2\sigma + cos^2\tau cos^2\sigma) = \frac{\pi}{2} TA^2 \end{aligned}$$

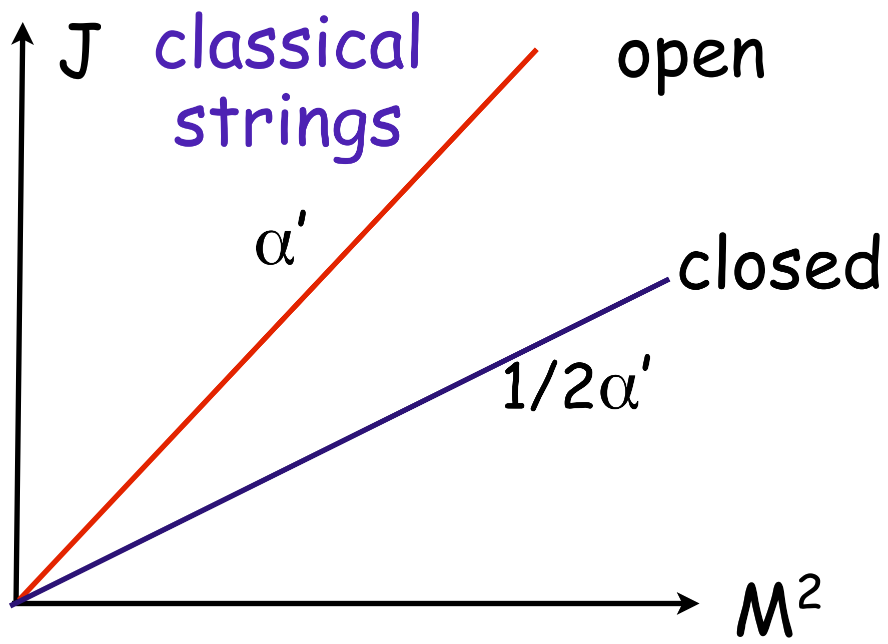
thus $$J = \frac{M^2}{2\pi T} = \alpha' M^2\ ,\ \alpha' \equiv \frac{1}{2\pi T}$$

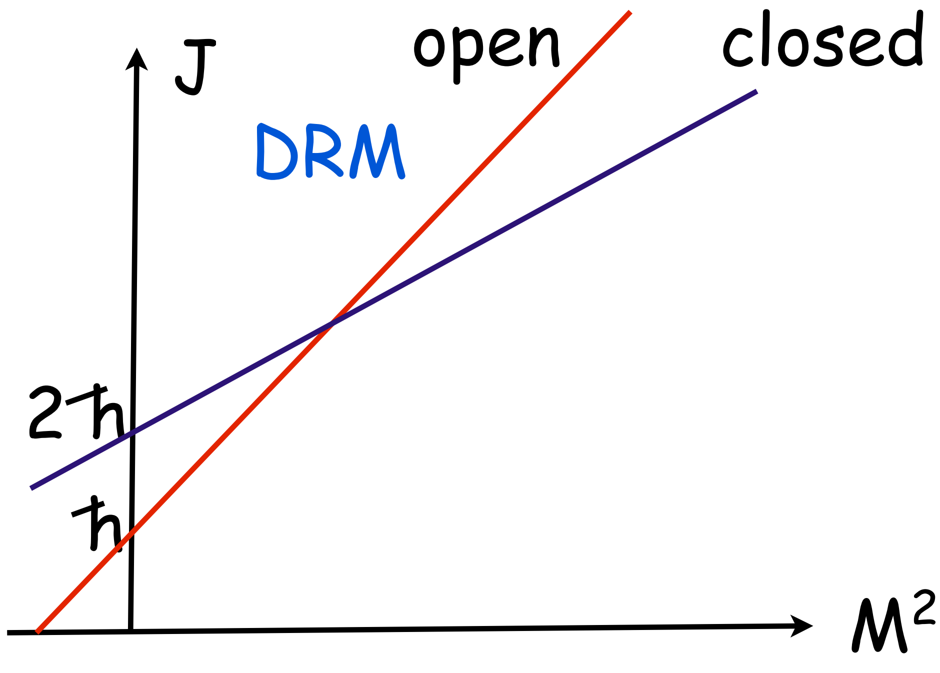
It is quite obvious that this solution maximizes the ratio J/M^2. The relation is very similar to the one given by the linear Regge trajectory we have been discussing in DRM.

For closed strings one finds the same relation between J and M^2 except for T --> 2 T (simple interpretation: for the same total length the closed string is half as big since it has to "come back on itself"). Hence α' -> 1/2 α'

Yet classical strings and DRM differ in crucial way.

In the classical theory, J and M^2 can take any real value with $J < \alpha' M^2$ => classical strings cannot have J without having mass! But in the DRM there were such states:





- The discrepancy between strings and the DRM disappears completely once we move from classical to quantum strings.
- This is where points and strings start to differ in a fundamental, qualitative way.
- Point particles can be quantized in an arbitrary background metric. This turns out not to be true for strings!
- *As we shall see, even flat spacetime is in general forbidden... unless D takes a (so-called) critical value!*
- String quantization is not trivial and can be done in many different ways. But the end result is always the same.
- Lovelace had not gone crazy…

Pausa ?

Alcuni dettagli che saranno riprese nella seconda parte

The classical constraints

S_p is invariant under reparametrization of the world-line, $\tau \to \tau'(\tau)$. This leads to 1 constraint (easy to check):

$$p_\mu(\tau) \equiv \frac{\delta S_p}{\delta \dot{x}^\mu(\tau)} \Rightarrow p_\mu(\tau) p_\nu(\tau) g^{\mu\nu}(x(\tau)) = -m^2$$

Similarly, S_{NG} is invariant under reparametrization of the world-sheet by an arbitrary redefinition $\xi^a \to \xi'^a(\xi^a)$

This leads now to 2 constraints (easy again to check):

$$\begin{aligned} P_\mu(\xi) &\equiv \frac{\delta S_{NG}}{\delta \dot{X}^\mu(\xi)} \quad \Rightarrow P_\mu(\xi) X'^\mu(\xi) = 0 \\ & \quad P_\mu(\xi) P_\nu(\xi) G^{\mu\nu}(X(\xi)) + T^2 X'^\mu(\xi) X'^\nu(\xi) G_{\mu\nu}(X(\xi)) = 0 \\ \dot{X}^\mu(\xi) &\equiv \frac{\partial X^\mu(\xi)}{\partial \xi^0}, \quad X'^\mu(\xi) \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^1} \end{aligned}$$

Strings in Minkowski spacetime: action and equations of motion

Since classical string motion (and even more quantization) is already non-trivial in Minkowski spacetime let us consider that case (also needed for connection with DRM) but let's keep the dimensionality of spacetime D arbitrary.

$$S_{NG} = -T \int d^2\xi \sqrt{-det\gamma_{\alpha\beta}}$$

$$\gamma_{\alpha\beta} \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^\alpha} \frac{\partial X^\nu(\xi)}{\partial \xi^\beta} G_{\mu\nu}(X(\xi)) \ , \quad \alpha, \beta = 0, 1 \quad , \quad \xi^0 = \tau, \ \xi^1 = \sigma$$

becomes: $$S_{NG} = -T \int d^2\xi \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^\mu(\xi) \equiv \frac{\partial X^\mu(\xi)}{\partial \tau} \ , \quad X'^\mu(\xi) \equiv \frac{\partial X^\mu(\xi)}{\partial \sigma}$$

Lorentz product understood

The equations of motion for the point-particle are trivial while for the string they look quite frightening:

$$\delta S_{NG} \propto \int d\tau \int_0^\pi d\sigma \left[\left(\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{X}^\mu} + \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial X'^\mu} \right) \delta X^\mu - \frac{\partial}{\partial \sigma} \left(\frac{\partial L}{\partial X'^\mu} \delta X^\mu \right) \right]$$

$$\begin{aligned}
\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{X}^\mu} \quad &+ \quad \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial X'^\mu} = 0 \\
\frac{\partial L}{\partial \dot{X}^\mu} \quad &= \quad T \frac{\dot{X}_\mu X'^2 - X'_\mu (\dot{X} \cdot X')}{\sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}} \\
\frac{\partial L}{\partial X'^\mu} \quad &= \quad T \frac{X'_\mu \dot{X}^2 - \dot{X}_\mu (\dot{X} \cdot X')}{\sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}}
\end{aligned}$$

Boundary conditions

Boundary conditions are very important and they differ in a crucial way for open and closed strings. We need, at all τ,

$$\left(\frac{\partial L}{\partial X'^{\mu}}\delta X^{\mu}\right)(\sigma = 0) = \left(\frac{\partial L}{\partial X'^{\mu}}\delta X^{\mu}\right)(\sigma = \pi) \quad ; \quad (\text{no sum over } \mu)$$

For closed strings the points $\sigma = 0$ and $\sigma = \pi$ are physically the same point. If spacetime is topologically trivial this implies $X^{\mu}(0, \tau) = X^{\mu}(\pi, \tau)$ and the b.c. is satisfied.

For open strings we have two options:

Neumann b.c. $$\frac{\partial L}{\partial X'^{\mu}} = 0 \quad , \qquad \sigma = 0, \pi$$

Dirichlet b.c. $$\delta X^{\mu} = 0 \quad , \qquad \sigma = 0, \pi$$

For the moment we will consider N. b.c. for open strings