

Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Una soluzione esatta del bootstrap
e sue principali proprietà

Punti principali dell'ultima lezione

- I poli di Regge con le loro traiettorie $\alpha_i(t)$ associano, nella regione $t > 0$, particelle di spin diverso.
- Nella regione $t < 0$ le stesse traiettorie di Regge determinano l'andamento dell'ampiezza $A(s,t)$ a grande energia ($s \gg s_0$) e t fisso.
- Assumendo andamento alla Regge e analicità si ottengono regole di somma che vanno saturate con stati intermedi nel canale s .

- La dualita' di DHS ci dice che tali stati intermedi si possono identificare **con le sole risonanze** che compaiono nel canale s .
- Queste devono dunque riprodurre, in media, la parte immaginaria dell'ampiezza di Regge.
- In questo consiste il bootstrap "economico".

Alcune nozioni da chiarire?

1. Unità di misura in fisica delle particelle
2. Significato di sezione d'urto e di luminosità
3. Relazione fra $A(s,t)$ e sezioni d'urto

Alcune nozioni matematiche

Le funzioni Gamma e Beta di Eulero e loro proprietà:

$$\Gamma(a) = \int_0^{\infty} dx x^{a-1} e^{-x}$$

$$\Gamma(1) = 1 \quad ; \quad \Gamma(a + 1) = a \Gamma(a)$$

La Γ estende a valori complessi il fattoriale,

$\Gamma(N+1) = N!$ Inoltre $\Gamma(z)$ ha poli a $z = 0, -1, -2, \dots$

$$B(a, b) = \int_0^1 dx x^{a-1} (1-x)^{b-1} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Il suo inverso è un coefficiente binomiale...

riprendendo dalla settimana scorsa...

πN scattering looked too complicated
We* decided to consider a simpler case:

$\pi \pi \rightarrow \pi \omega$ (very symmetric & very selective in QN's)
($\rho, \rho^* ..$)

Between the fall of 1967 and the summer of 1968 we made much progress in finding accurate solutions to this "Cheap Bootstrap".

*) Ademollo, Rubinstein, Virasoro, GV (+Bishari & Schwimmer)
with much advice and encouragement by Sergio Fubini

The ARVV ansatz that worked amazingly well for the DHS bootstrap in $\pi\pi \rightarrow \pi\omega$ was simply*:

$$\text{Im } A(s, t) = \frac{\beta(t)}{\Gamma(\alpha(t))} (\alpha' s)^{\alpha(t)-1} (1 + O(1/s))$$

with: $\beta(t) \sim \text{const.}, \alpha(t) = \alpha_0 + \alpha' t$

i.e. a **linear** leading Regge **trajectory** accompanied by parallel "daughter" trajectories. The latter, if suitably tuned, were improving the agreement in an increasingly large range of t

Which was the road that led from the above ansatz to an "exact solution"? **Three** main **ingredients** were used:

*The extra $1/s$ is due to the spin 1 of ω

1. Look at **A** rather than $\text{Im } A$ ($A = \text{analytic function}$)
2. Impose exact **crossing** symmetry : $A(s,t) = A(t,s)$
3. Emphasize **resonances** over Regge (only poles)

1. Easy to show that

$$\text{Im } A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))} (\alpha' s)^{\alpha(t)-1} (1 + O(1/s))$$

corresponds to (up to a trivial factor π):

$$A(s,t) = \beta(t) \Gamma(1 - \alpha(t)) (-\alpha' s)^{\alpha(t)-1} (1 + O(1/s))$$

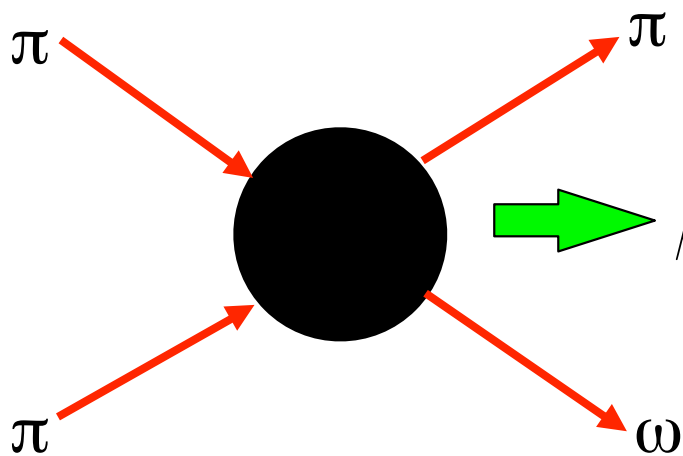
after using a well known formula: $\Gamma(x)\Gamma(1 - x) = \frac{\pi}{\sin \pi x}$

3. $A(s,t)$ already exhibits resonances (poles) in the t -channel but still only a smooth Regge behaviour in s : However, using

$$\frac{\Gamma(1 - \alpha(s))}{\Gamma(2 - \alpha(s) - \alpha(t))} \rightarrow (-\alpha' s)^{\alpha(t) - 1} (1 + O(1/s))$$

we can satisfy both 2. and 3. by simply writing:

$$A(s,t) = \beta \frac{\Gamma(1 - \alpha(s)) \Gamma(1 - \alpha(t))}{\Gamma(2 - \alpha(s) - \alpha(t))} = \beta B(1 - \alpha(s), 1 - \alpha(t))$$



Full x -ing symmetry then implies:

$$\begin{aligned} \beta^{-1} A(s,t) &= B(1 - \alpha(s), 1 - \alpha(t)) \\ &+ B(1 - \alpha(u), 1 - \alpha(t)) \\ &+ B(1 - \alpha(s), 1 - \alpha(u)) \end{aligned}$$

Exact DHS duality is implied by:

1. **Analyticity** (dispersion relations);
2. All singularities are **poles** corresponding to resonances
3. **Regge asymptotics!**

=> duality between two infinite sets of resonances in "dual" channels!

Properties best analyzed by using:

$$B(1 - \alpha(s), 1 - \alpha(t)) = \int_0^1 dx x^{-\alpha(s)} (1 - x)^{-\alpha(t)}$$

Analytic cont. needed: only converges for suff. negative s, t .

$$\beta(t) \sim \text{const.}, \alpha(t) = \alpha_0 + \alpha' t$$

$$B(1 - \alpha(s), 1 - \alpha(t)) = \int_0^1 dx x^{-\alpha(s)} (1 - x)^{-\alpha(t)}$$

1. Crossing symmetry: $x \rightarrow (1-x)$

2. All singularities are poles: expanding integrand in powers of x (or of $(1-x)$) gives s - t duality in a nicer form:

$$\sum_{n=0}^{\infty} \frac{C_n(t)}{s - m_n^2} = \sum_{n=0}^{\infty} \frac{C_n(s)}{t - m_n^2}$$

3. Good (Regge) asymptotics: as s becomes very large (with t fixed) integral is dominated by $x \sim 1$ region. Implies duality!

$$\begin{aligned} \int_0^1 dx (1-x)^{-\alpha_s} x^{-\alpha_t} &\sim \int_0^{\dots} dx e^{x\alpha_s} x^{-\alpha_t} \\ &\sim (-\alpha_s)^{\alpha_t-1} \int_0^{\infty} dz e^{-z} z^{-\alpha_t} = (-\alpha_s)^{\alpha_t-1} \Gamma(1 - \alpha_t) \end{aligned}$$

$$B(1 - \alpha(s), 1 - \alpha(t)) = \int_0^1 dx x^{-\alpha(s)} (1 - x)^{-\alpha(t)}$$

4. High energy, fixed angle ($t/s = - (1 - \cos\theta)/2 = z$)

$$\int_0^1 dx x^{-\alpha_s} (1 - x)^{-\alpha_t} = \int_0^1 dx e^{-\alpha' s [\log x + z \log(1 - x)]} \sim e^{-\alpha' s [\log x^* + z \log(1 - x^*)]}$$

where x^* denotes the saddle point of the integrand:

$$\frac{1}{x^*} - \frac{z}{(1 - x^*)} = 0 \Rightarrow x^* = \frac{1}{1 + z} \quad \text{giving (for } t, s < 0)$$

$$\begin{aligned} A(s, t) &\rightarrow e^{\alpha' s \log(1 + \frac{t}{s}) + \alpha' t \log(1 + \frac{s}{t})} && \text{i.e. mysteriously} \\ &= e^{-\alpha' s \log(s) - \alpha' t \log(t) - \alpha' u \log(u)} && \text{symmetric in } s, t, u. \end{aligned}$$

NB: such a "soft" behavior had never been seen before...

Paragone con dati?

A prima vista il processo $\pi\pi \rightarrow \pi\omega$ sembra di interesse puramente accademico. Ma non è così perché la simmetria di crossing ci dice che, in questo caso, c'è una quarta regione fisica nel diagramma di Maldestam: quella in cui s, t, u sono tutte e 3 positive, maggiori di $4m_\pi^2$ e con

$$s+t+u = m_\omega^2 + 3m_\pi^2$$

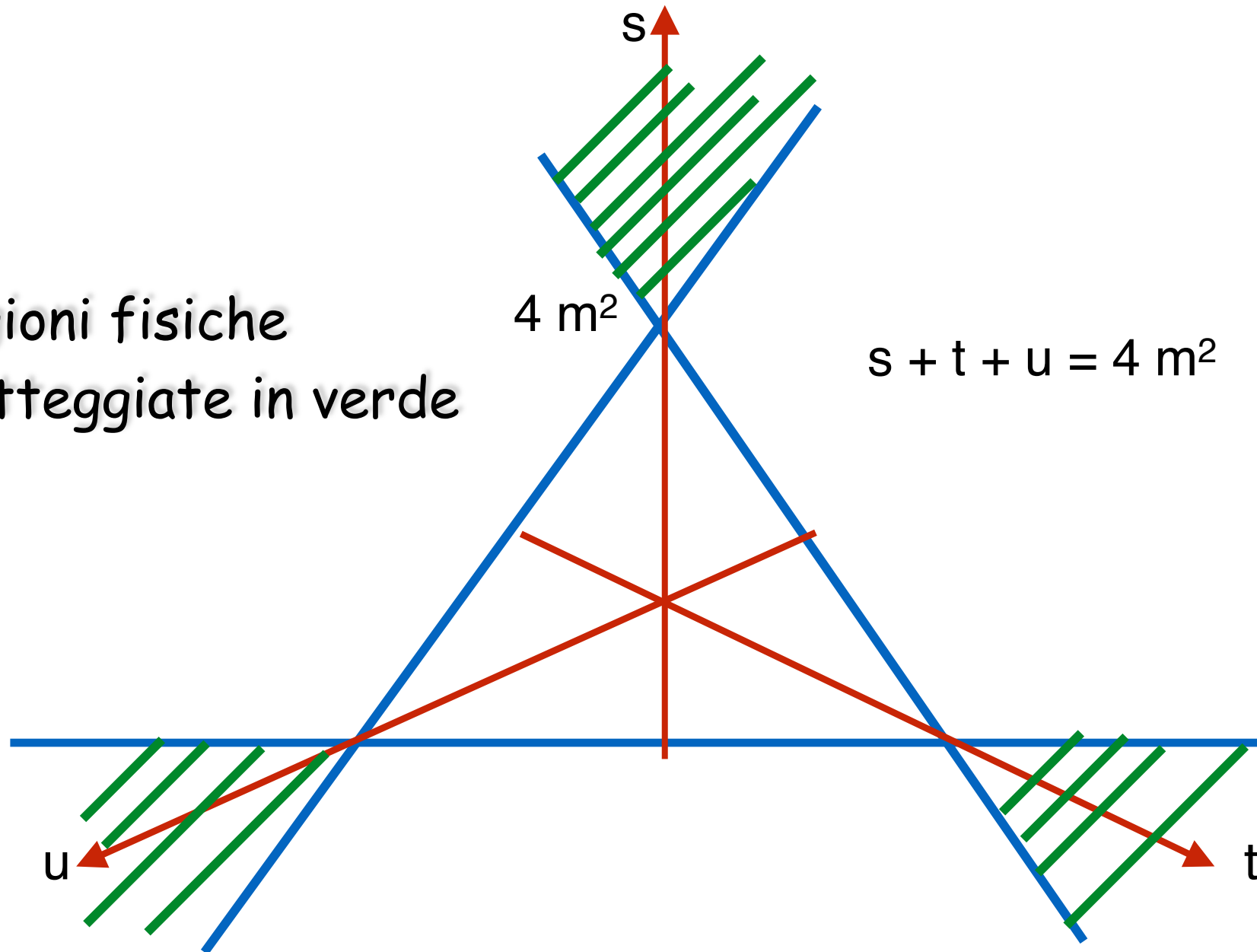
In questa regione la $A(s, t)$ descrive il decadimento

$$\omega \rightarrow \pi\pi\pi$$

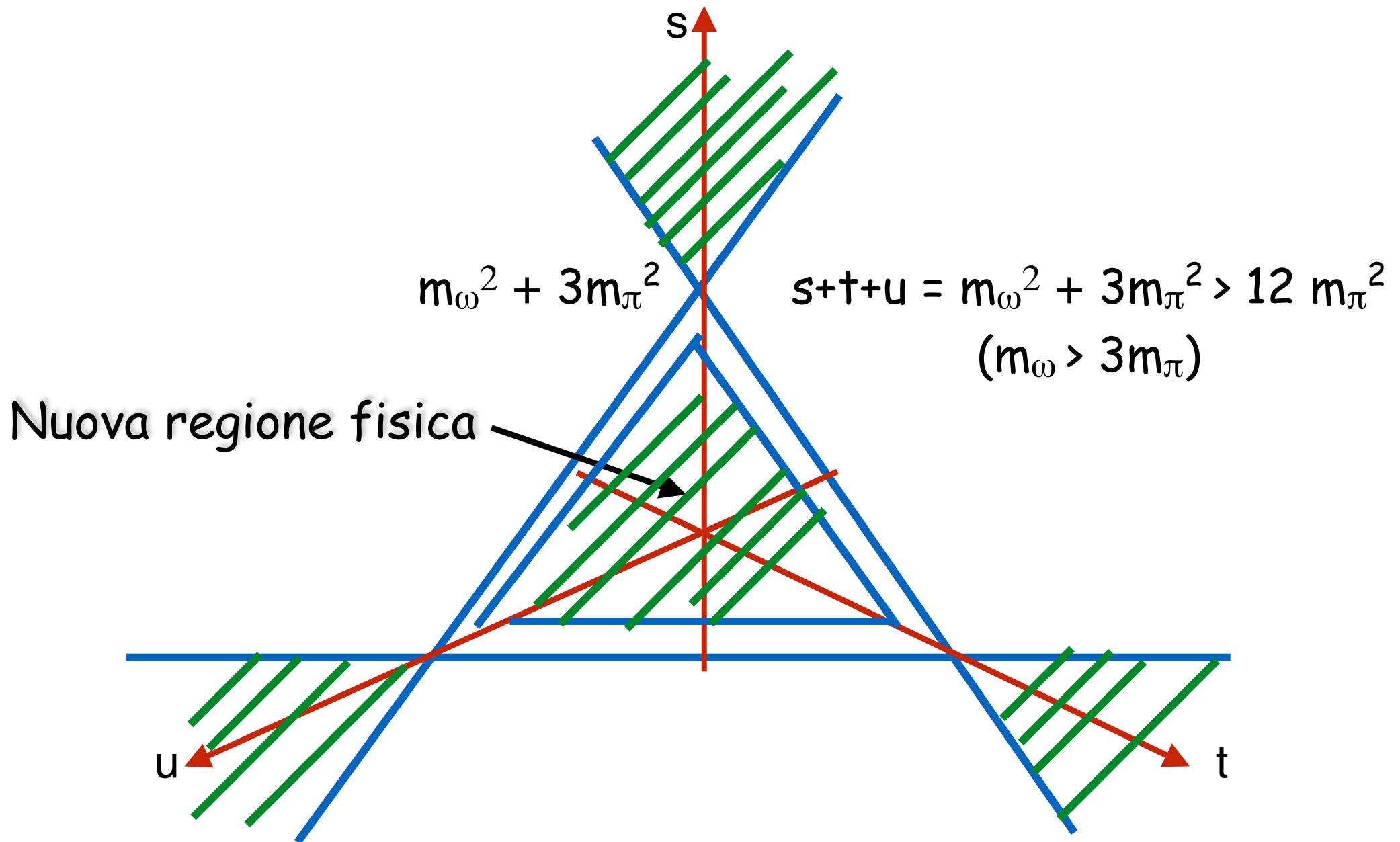
con notevole successo (cf. zeri di Odorico nel plot di Dalitz)

Mandelstam plane (equal masses)

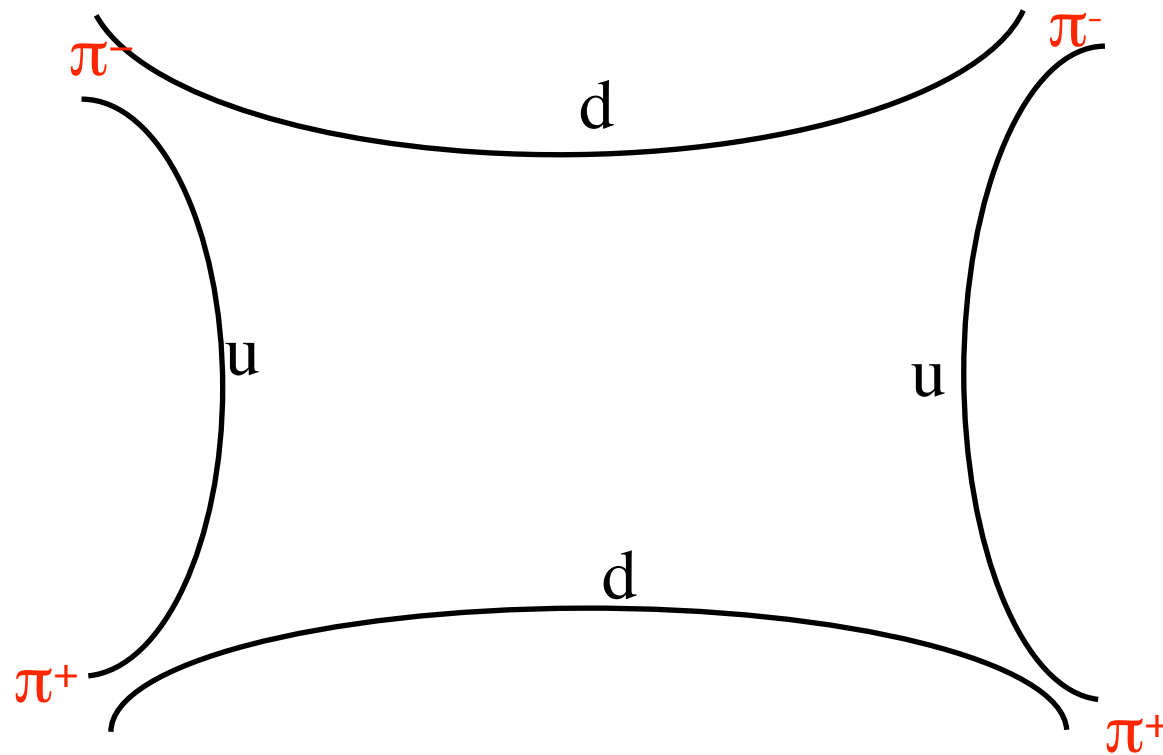
Regioni fisiche
tratteggiate in verde



Mandelstam plane ($\omega \rightarrow \pi \pi \pi$)



An even more successful model was proposed by Lovelace for $\pi\pi$ scattering. For instance:



$$A(\pi^+ \pi^- \rightarrow \pi^+ \pi^-) = g^2 \frac{\Gamma(1 - \alpha(s)) \Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))}$$

$$\alpha(t) = \alpha_\rho(t) \sim 0.5 + 0.9t \text{ GeV}^{-2}$$

These successes raised high hopes that a new theory of strong interactions (with a new fundamental scale α') was soon to be found...

It took a few years to realize that the DRM was a theory of strings. Till about 1972 it looked like a very strange theory, mysteriously different from anything that had been seen before, like QFT or General Relativity.

As such it polarized the community with the opponents (particularly within the establishment) outnumbering the (mostly young, but with exceptions) enthusiasts.

Even modern string theory remembers its very unconventional origins and uses concepts and methods that are very different from those people are accustomed to, e.g. in QFT.

"A piece of physics from the 21st century
that fell, by accident, in the 20th"
(S. Fubini ~1969)

Questo conclude la "preistoria"

il seguito:

- Estensione a processi con più di 4 particelle
- Identificazione dello spettro
- Formulazione in termini di un numero infinito di oscillatori armonici quantizzati con una frequenza fondamentale e le sue armoniche superiori
- Suggesti dell'esistenza di una stringa sottostante al DRM
- Formulazione ab-inizio della stringa relativistica, sua quantizzazione e recupero del DRM

Questo sarebbe il menu delle 3
prossime lezioni
lasciando al prossimo semestre:

- Problemi fenomenologici alla luce di nuovi dati
- Competizione con QCD
- Il limite di grandi N in QCD e la sua misteriosa stringa (ancora da scoprire)
- Abbandono della stringa adronica
- Proposta di reinterpretare la stringa come una teoria piu' fondamentale, che include la quantizzazione della gravità.