Cattedra Enrico Fermi 2015-20116

La teoria delle stringhe: l'ultima rivoluzione in fisica?

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Lezione # 3.2: 19.11.2015

Poli di Regge e andamento a grandi energie

Sketch of Regge's theory of complex J

Consider non-relativistic potential scattering.
Expand the scattering amplitude in partial waves:

$$A(E,\theta) = \sum_{J=0}^{\infty} A_J(E) P_J(cos\theta)$$

In 1959 Tullio Regge had the bold idea of looking at $A_J(E)$ as an analytic function of complex J. He found that, quite generically, there were poles in J at an energy-dependent position i.e. at $J = \alpha(E)$:

$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

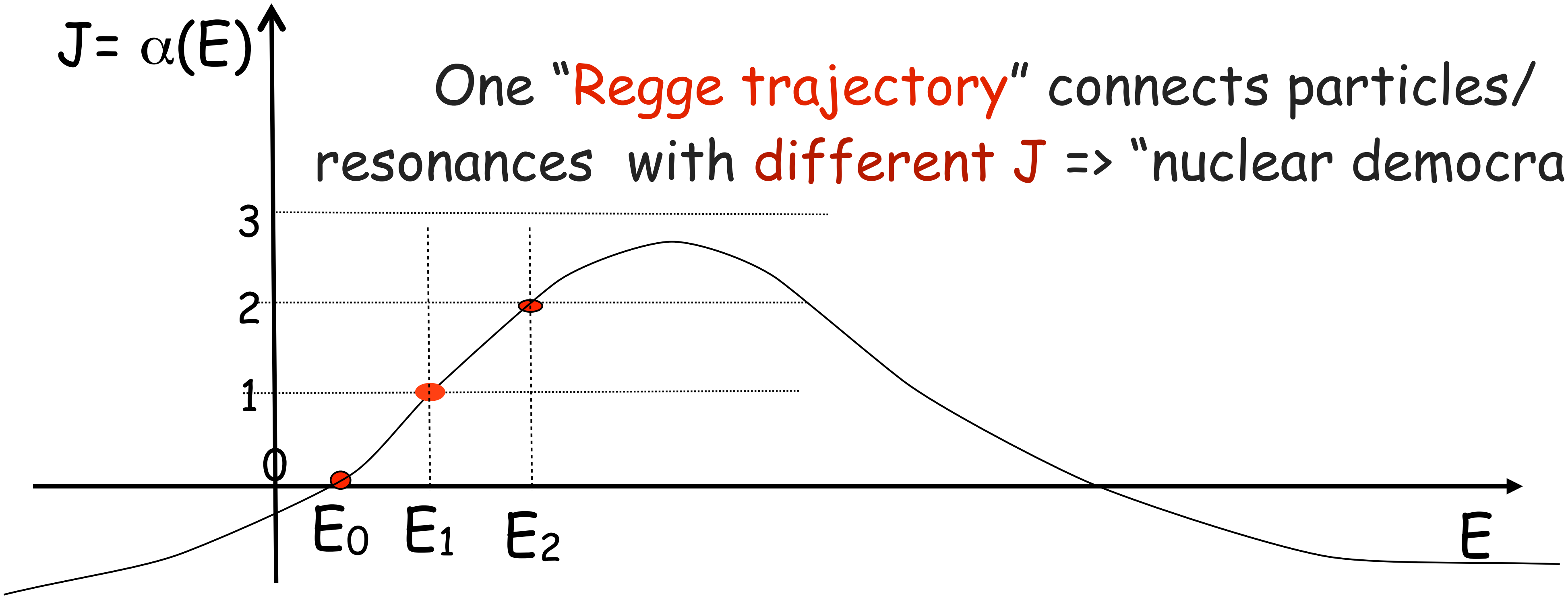
$\alpha(E)$ is called a Regge trajectory

$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

Assume $\alpha(E)$ to go through a positive integer n at $E = E_n$.

$$\alpha(E_n) = n \Rightarrow A(E,\theta) \sim \frac{\beta(E_n)}{n - \alpha(E)} P_n(cos\theta) \sim -\frac{\beta(E_n)}{\alpha'(E - E_n)} P_n(cos\theta)$$

i.e. just the contribution of a single resonance of energy E_n.

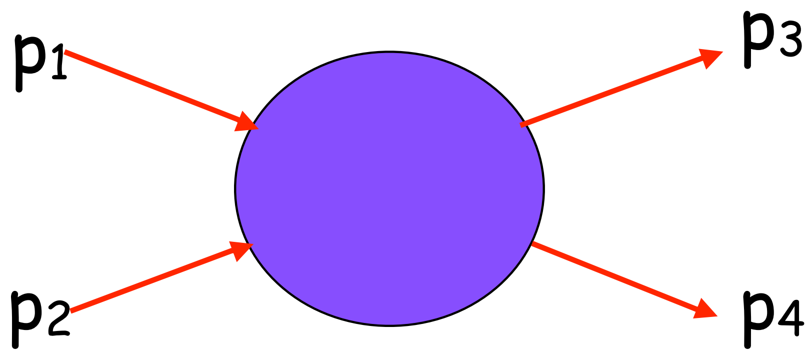


One "Regge trajectory" connects particles/ resonances with different J => "nuclear democracy".

Chew-Mandelstam's application of Regge theory in relativistic scattering

In s-channel region expand A(s,t) in t-channel partial waves:

$$\begin{aligned} s = -(p_1+p_2)^2 &= -(p_3+p_4)^2 \\ t = -(p_1-p_3)^2 &= -(p_2-p_4)^2 \\ u = -(p_1-p_4)^2 &= -(p_2-p_3)^2 \\ s+t+u &= \sum m_i^2 \end{aligned}$$



$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) \quad ; \quad cos\theta_t = 1 + 2s/t$$

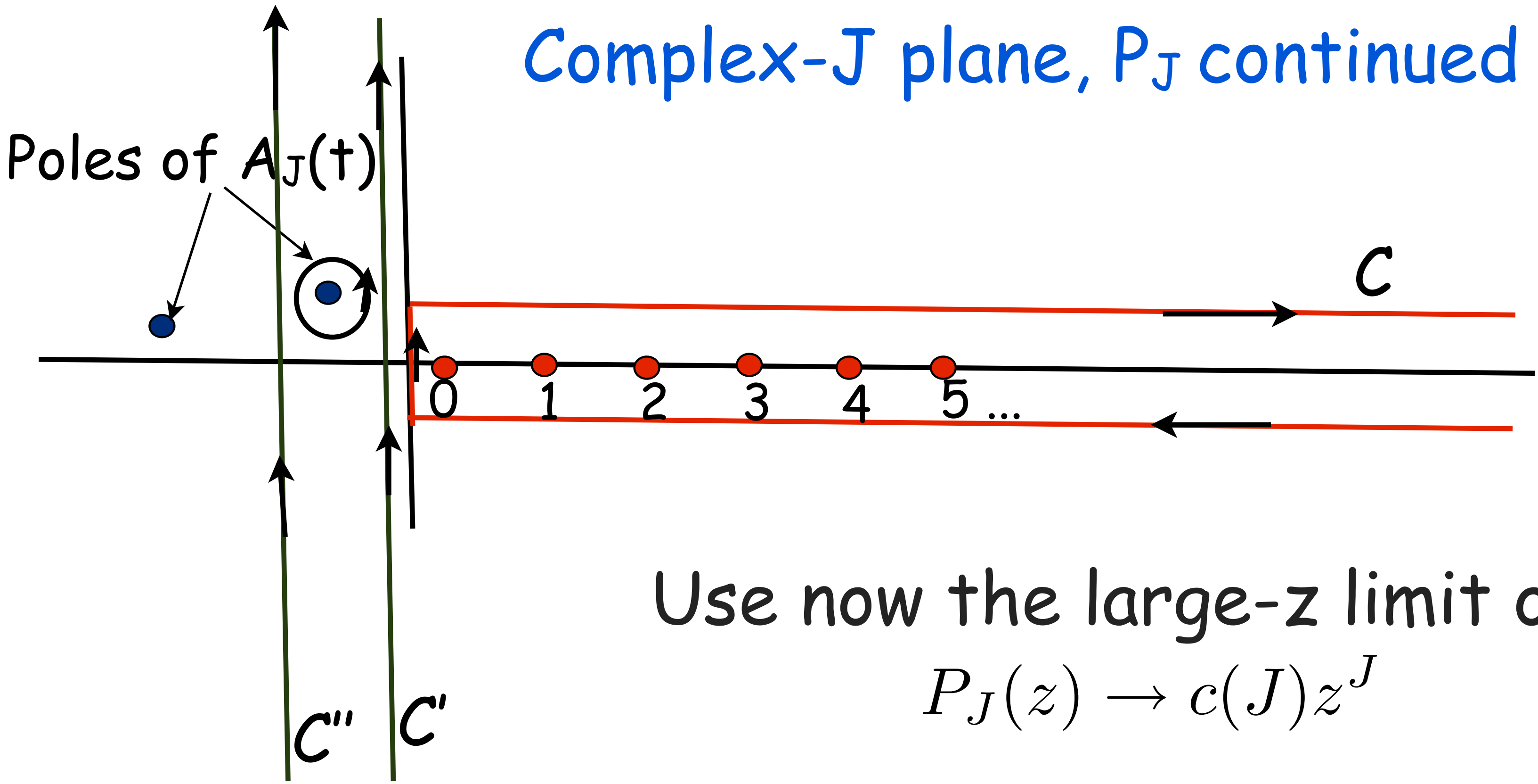
At large s > 0 & fixed t < 0, the contribution of the J^{th} term grows like s^J. The series is badly divergent but, under some assumptions, can be extended analytically

The sum diverges but can be analytically continued using a trick due to Froissart & Gribov

$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) = \frac{1}{2i}\int_C dJ \frac{e^{i\pi J}}{sin(\pi J)} A_J(t) P_J(cos\theta_t)$$

where the contour C is as in the figure.

Complex-J plane, P_J continued as well

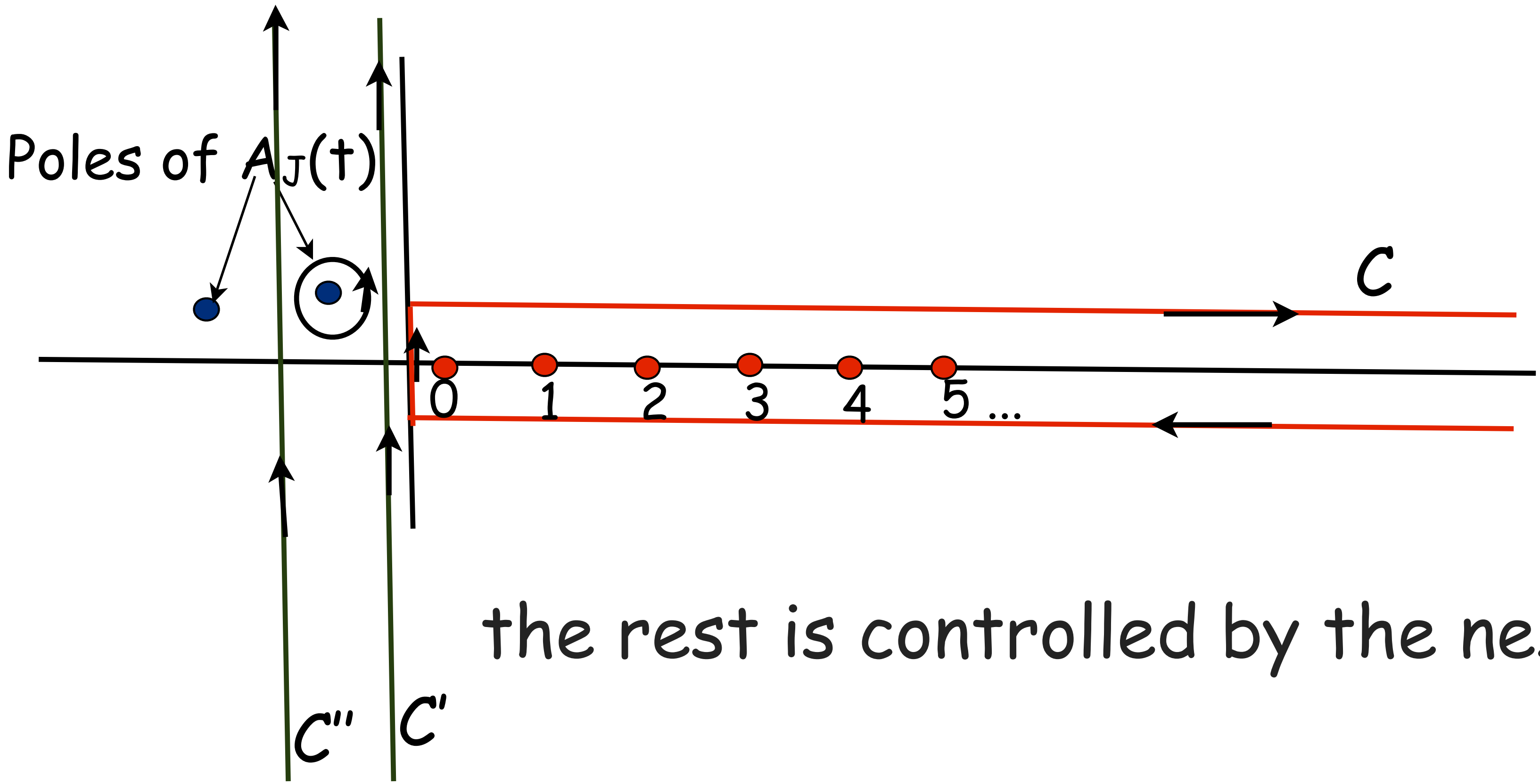


Use now the large-z limit of $P_J(z)$:

$$P_J(z) \to c(J) z^J$$

Deforming the contour from C to C' to C'' (which includes the little circle around the rightmost Regge pole) we get, from the latter:

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$



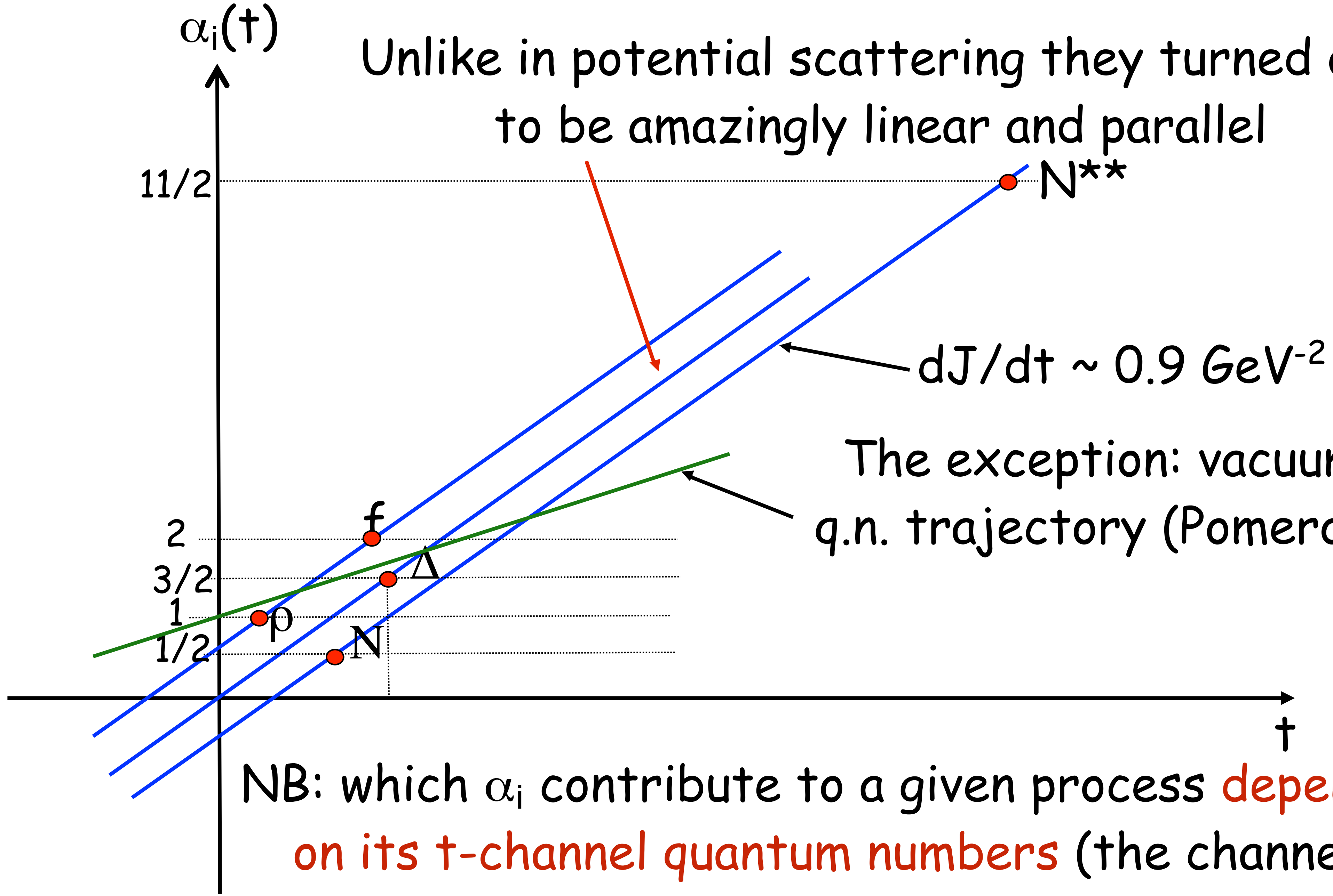
the rest is controlled by the next pole..

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$

Note that Regge theory gives, in general a complex scattering amplitude while usual single particle exchanges do not. Interpreting this imaginary part turned out to be crucial.

A very interesting experimental discovery of the sixties was the shape of the Regge trajectories:

Unlike in potential scattering they turned out to be amazingly linear and parallel



NB: which α_i contribute to a given process depends on its t-channel quantum numbers (the channel whose Mandelstam variable is kept fixed)