Cattedra Enrico Fermi 2015-20116

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Poli di Regge e loro duplice significato
Un bootstrap caro ed uno economico

Punti principali dell'ultima lezione

- L'ampiezza di scattering 2->2, A(s,t), è funzione analitica dei suoi argomenti.
- Le singolarità di questa funzione (poli, punti di diramazione, tagli) in una variabile di Mandelstam sono legate ai possibili stati intermedi nel corrispondente canale.
- Una singola particella scambiata da un polo, uno stato a molte particelle da un taglio (tutti sull'asse reale)
- Una risonanza da un polo vicino all'asse reale (in effetti sul secondo foglietto di Riemann)

Organizing the hadronic zoo

A) Group theory:

- $SU(2)_I$, $SU(3)_F$, same-J particles (e.g. p <-> n)

B) Regge theory of complex J

- For associating different-J particles (Regge)
- For describing high-energy scattering (Chew-Mandelstam)
- Most important for the birth of string theory

Sketch of Regge theory of complex J

Consider non-relativistic scattering from a spherically symmetric potential.

Expand the scattering amplitude in partial waves:

$$A(E,\theta) = \sum_{J=0}^{\infty} A_J(E) P_J(cos\theta)$$

In 1959 Tullio Regge had the bold idea of looking at $A_J(E)$ as an analytic function of complex J. He found that, quite generically, there are poles in J at an energy-dependent position $J = \alpha(E)$:

$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

$\alpha(E)$ is called a Regge trajectory

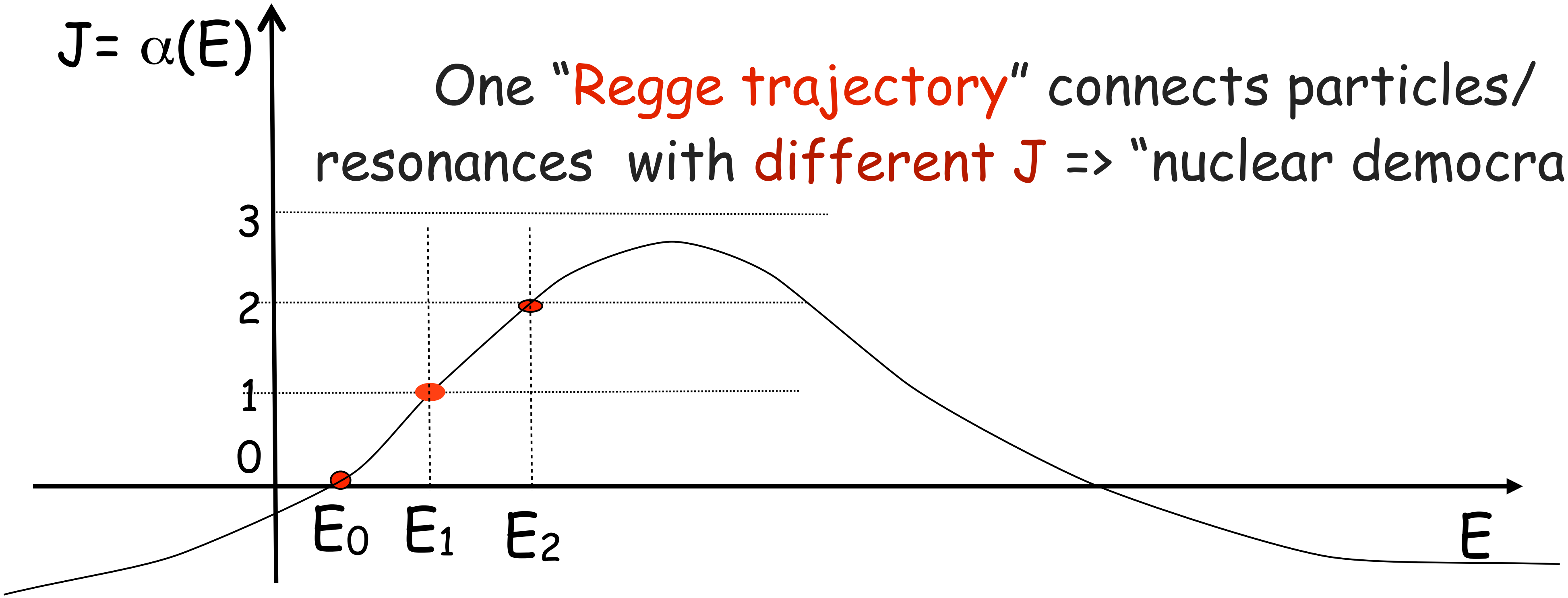
$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

Assume α(E) to go through a positive integer n at E = E_n.

$$\alpha(E_n) = n \Rightarrow A(E,\theta) \sim \frac{\beta(E_n)}{n - \alpha(E)} P_n(cos\theta) \sim -\frac{\beta(E_n)}{\alpha'(E - E_n)} P_n(cos\theta)$$

i.e. just the contribution of a single state of energy E_n.

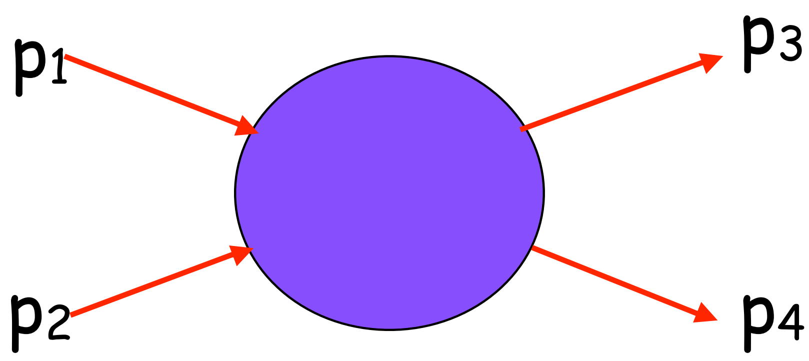
One "Regge trajectory" connects particles/ resonances with different J => "nuclear democracy".



Chew-Mandelstam's use of Regge theory for high-energy behavior (in rel. scattering)

In s-channel region ($s > 0$, $t,u < 0$) expand $A(s,t)$ in t-channel partial waves:

$$\begin{aligned} s = -(p_1+p_2)^2 &= -(p_3+p_4)^2 \\ t = -(p_1-p_3)^2 &= -(p_2-p_4)^2 \\ u = -(p_1-p_4)^2 &= -(p_2-p_3)^2 \\ s+t+u &= \sum m_i^2 \end{aligned}$$



$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) \quad ; \quad cos\theta_t = 1 + 2s/t$$

$$cos\theta_s = 1 + 2t/s \to 1$$

At large $s > 0$ & fixed $t < 0$, the contribution of the J^{th} term grows like s^J. The series is badly divergent but, under some assumptions, can be extended analytically.

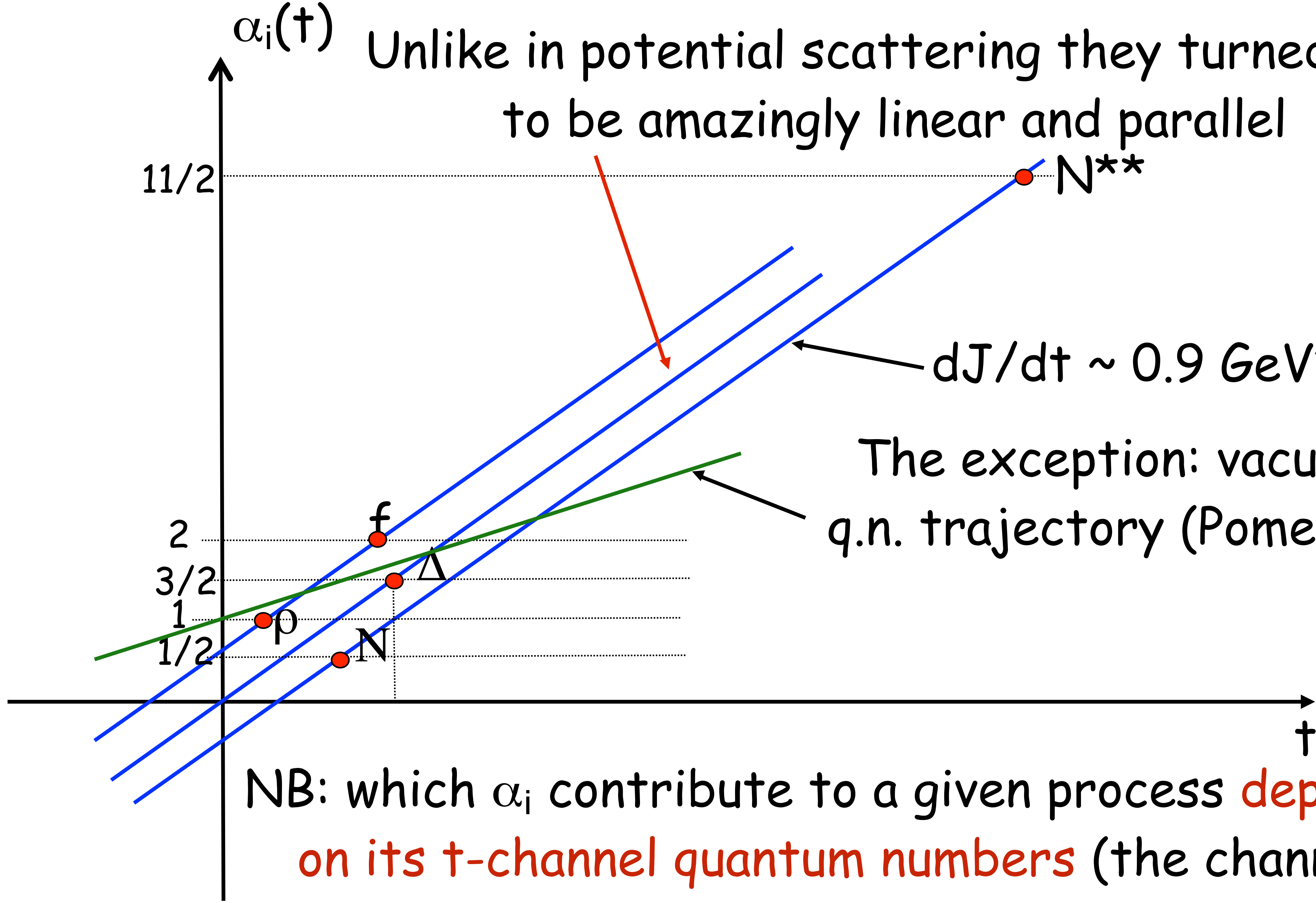
Leaving the bloody details to the next hour here is the result:

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{-i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$

At large s and fixed t < 0 this behavior is much much better than the one of each partial wave or single particle exchange!

Also note that Regge theory gives a complex scattering amplitude while usual single particle exchanges do not. Interpreting correctly Im A(s,t) turned out to be crucial.

A very interesting experimental discovery of the sixties was the unexpected shape of the Regge trajectories:



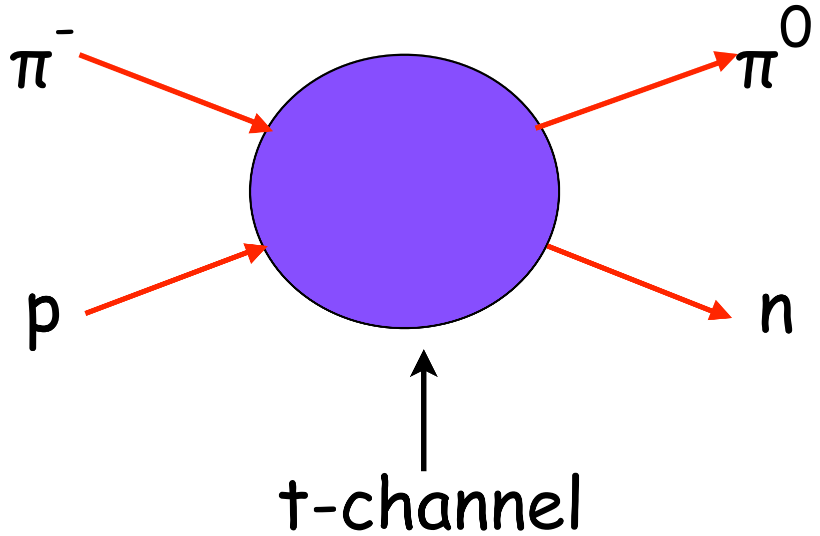
Unlike in potential scattering they turned out to be amazingly linear and parallel

The exception: vacuum q.n. trajectory (Pomeron)

NB: which α_i contribute to a given process depends on its t-channel quantum numbers (the channel whose Mandelstam variable is kept fixed)

Examples

1. pion-nucleon charge exchange

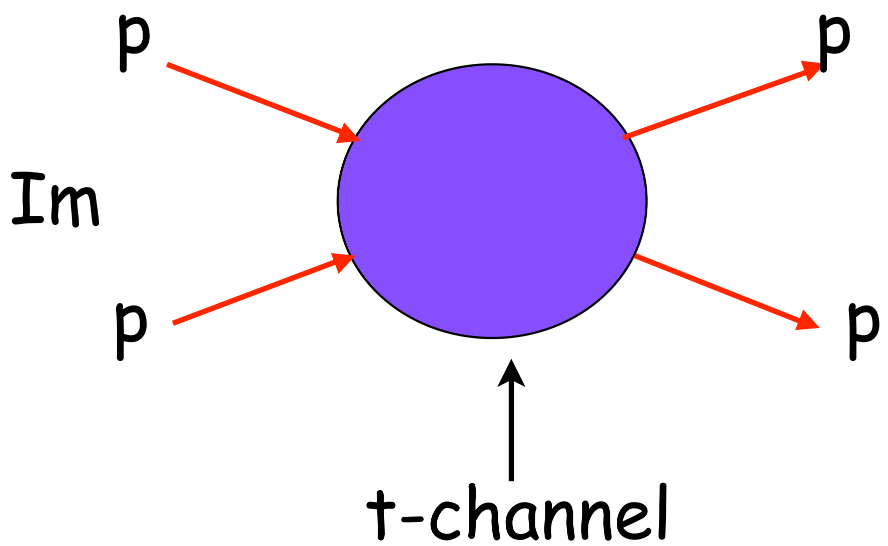


I=1 trajectories of both signatures can contribute

$$A(s,t) \sim \frac{\beta_\rho(t)[e^{-i\pi\alpha_\rho} - 1]}{sin(\pi\alpha_\rho(t))} s^{\alpha_\rho(t)} + \frac{\beta_{A2}(t)[e^{-i\pi\alpha_{A2}} + 1]}{sin(\pi\alpha_{A2}(t))} s^{\alpha_{A2}(t)}$$

Fitting data gives $\alpha_\rho(0) \sim \alpha_{A2}(0) \sim 0.57$ explaining quite well the scattering data above a few GeV. Distinctive prediction: shrinkage of forward peak

2. proton-proton total cross section (LHC)



I=0, 1 trajectories of both signatures can contribute

Highest one has vacuum quantum numbers

$$\sigma_T = \frac{1}{s} Im A(s,0) \sim \frac{1}{s} Im \frac{\beta_{\mathcal{P}}(0)[e^{i\pi\alpha_{\mathcal{P}}}+1]}{sin(\pi\alpha_{\mathcal{P}}(0))} s^{\alpha_{\mathcal{P}}(0)} + \dots = \beta_{\mathcal{P}(0)} s^{\alpha_{\mathcal{P}}(0)-1} + \dots$$

Fitting data gives $\alpha_P(0) \sim 1.07$ violating a famous (Froissart) bound ($\log^2 s$): the story must be more complicated!

Chew's "expensive" bootstrap…

Add to the general constraints of symmetry, causality, unitarity that of Nuclear Democracy

"All hadrons lie on Regge trajectories @ $M^2>0$;

All asymptotics fixed by same trajectories @ $M^2<0$"

Will this give a unique S-matrix?

A posteriori Chew's program was too ambitious. We now believe the answer to the question to be negative.

String theory is a perfect example of Nuclear Democracy and satisfies the other constraints as well…but adds to them a crucial new dynamical input: strings!

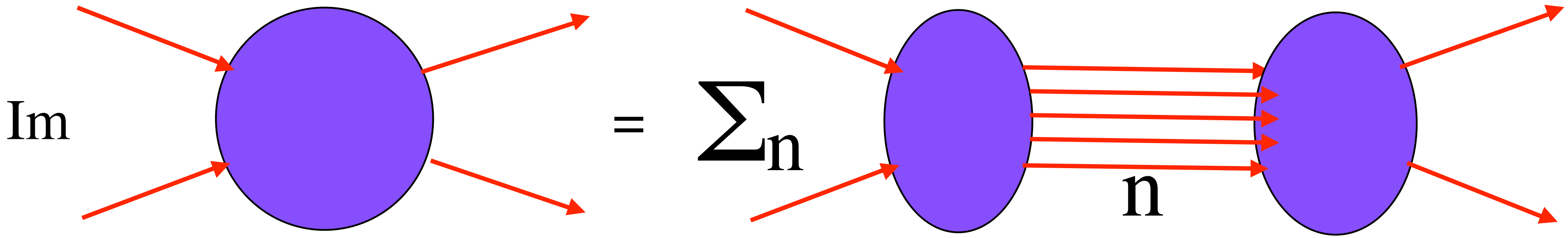
Also, many variations of QCD should satisfy ND.

In any case people derived some general consequences from Regge+Chew+Maldestam's theory adding:

1. Analyticity in the form of dispersion relations (proof?):

$$A(s,t) = \frac{1}{\pi}\int ds' \frac{ImA(s',t)}{s'-s-i\epsilon}$$

2. Unitarity which relates Im A(s,t) to a sum over all the physical intermediate states that can appear in the s-channel with total energy $s^{1/2}$:



They can be combined to get some interesting "sum rules"

Superconvergence (S. Fubini ~ 1966)

Superconvergence applies to the case in which the t-channel quantum numbers are such that A(s,t) decreases at infinity faster than 1/s (corresponding α < -1). Writing a fixed-t (unsubtracted) dispersion relation for *A*,

$$A(s,t) = \frac{1}{\pi} \int ds' \frac{Im A(s',t)}{s' - s - i\epsilon}$$

and imposing that *sA*—> 0 at large s we must have:

$$\int ds \; Im A(s,t) = 0$$

Inserting low-energy "data" met with very reasonable success

Finite-energy sum rules (FESR)

In this case we use our theoretical (Regge) model at high energy and write a superconvergence relation for a subtracted amplitude: $A^{(sub)} = A(s,t) - A^{(R)}(s,t)$ so that s $A^{(sub)}$ goes to zero at large s. Limiting the integral to a finite value s_0 we get:

$$\int_0^{s_0} ds\ ImA(s,t) \sim \int_0^{s_0} ds\ ImA^{(R)}(s,t)$$

s_0 has to be taken judiciously. Using two such reasonable s_0

$$\int_{s_1}^{s_2} ds\ ImA(s,t) \sim \int_{s_1}^{s_2} ds\ ImA^{(R)}(s,t) = \sum \beta_i(t) \frac{s_2^{\alpha_i(t)+1} - s_1^{\alpha_i(t)+1}}{\alpha_i(t)+1}$$

Unitarity relates Im A to s-channel intermediate states hence we get a relation between s and t-channel quantities

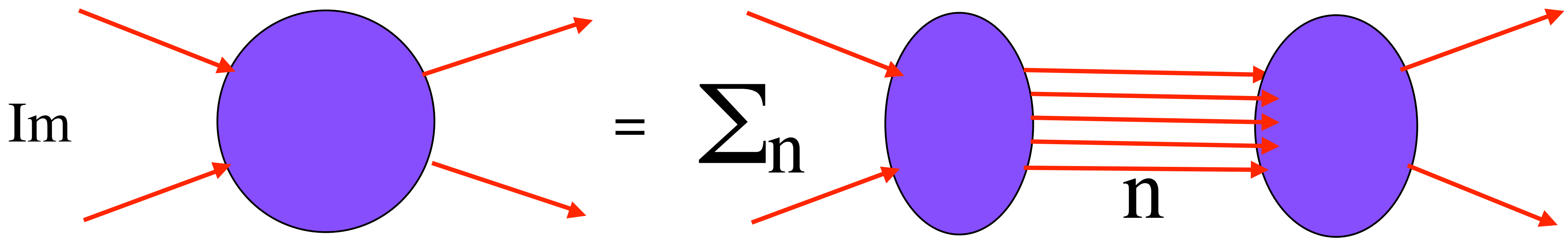
Thanks to Regge-Chew-Mandelstam we think we know what to put on the t-channel (r.h.) side of the FESR.

The question is: what should we put on the s-channel (l.h.) side of the FESR?

Giving the correct answer to this question turned out to be one of the crucial steps towards the ultimate discovery of string theory...

In very special cases one can use actual data. But in most cases one is forced to use some theoretical model.

What's on the l.h.s. of FESR?

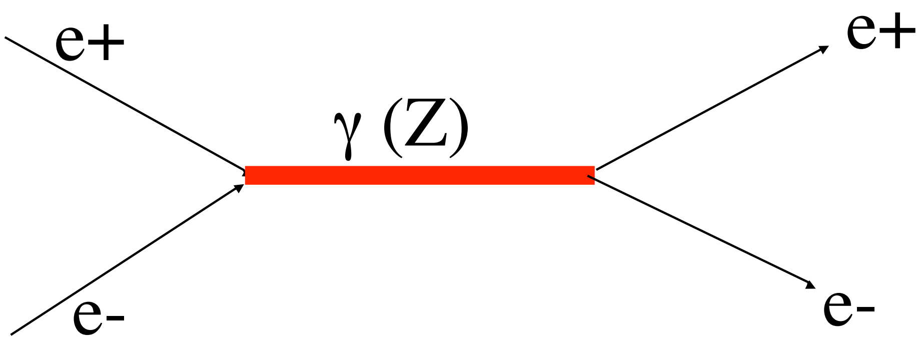


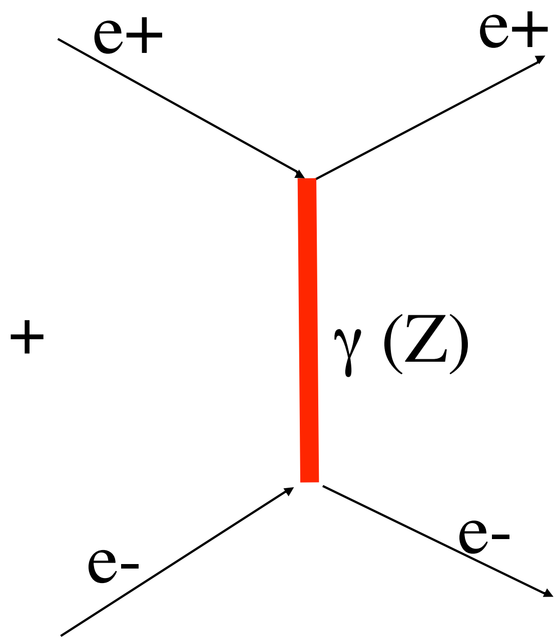
One obvious contribution was the one due to the resonances that could be produced in the s-channel (supposedly lying on that channel's Regge trajectories)

But what about other contributions that were making up the imaginary part of the t-channel Regge pole contribution at high energy?

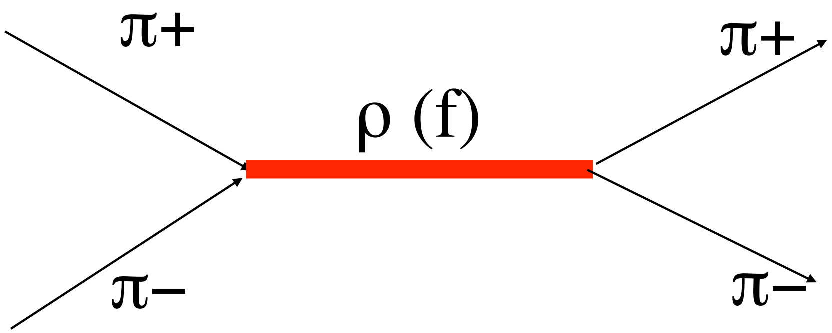
The prevailing belief at the time was that those two contributions had nothing to do with each other and that, therefore, should be added.

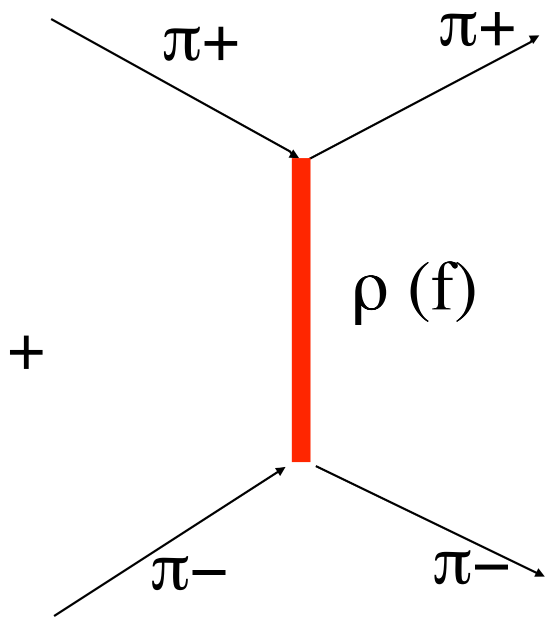
This was supported by QED calculations and also by QFT models for Regge poles.

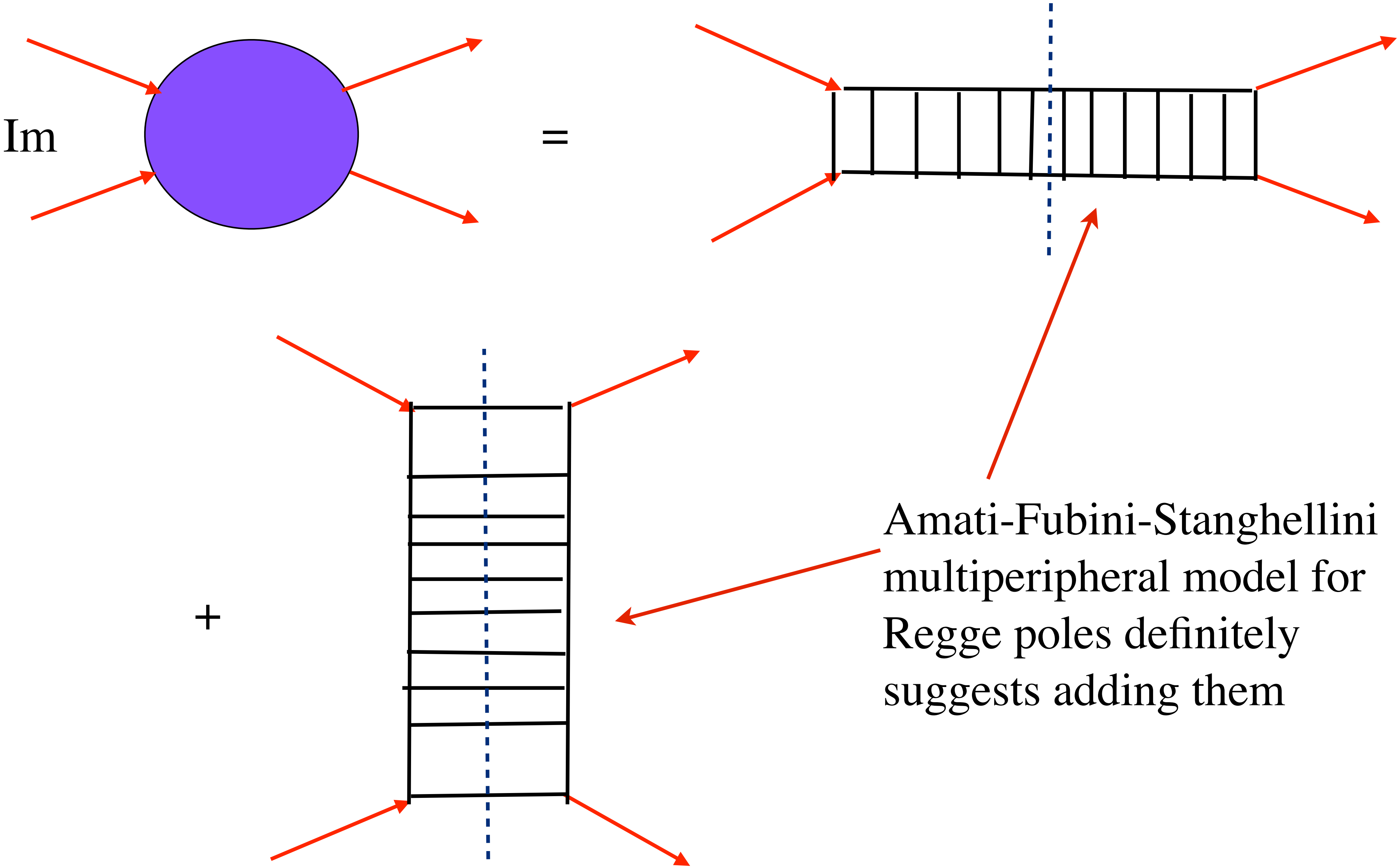




Likewise…





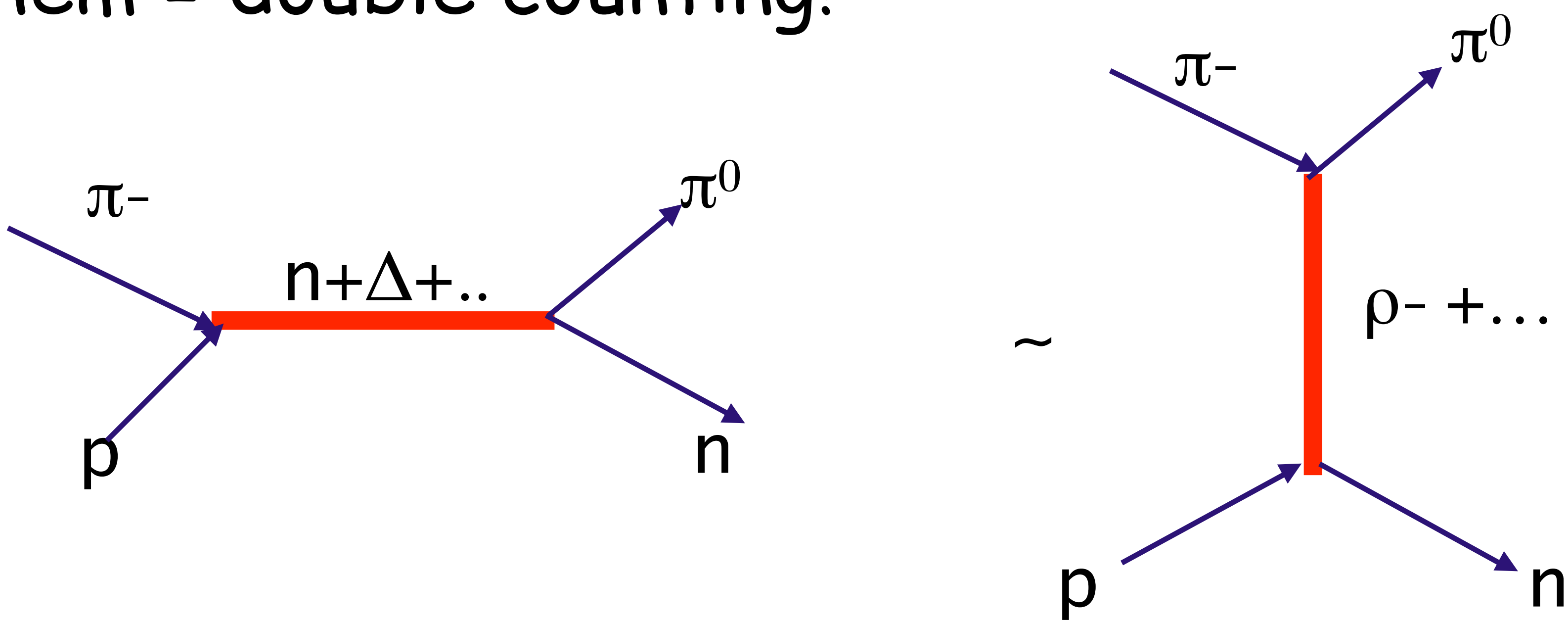

Im
=
+
Amati-Fubini-Stanghellini multiperipheral model for Regge poles definitely suggests adding them

DHS duality

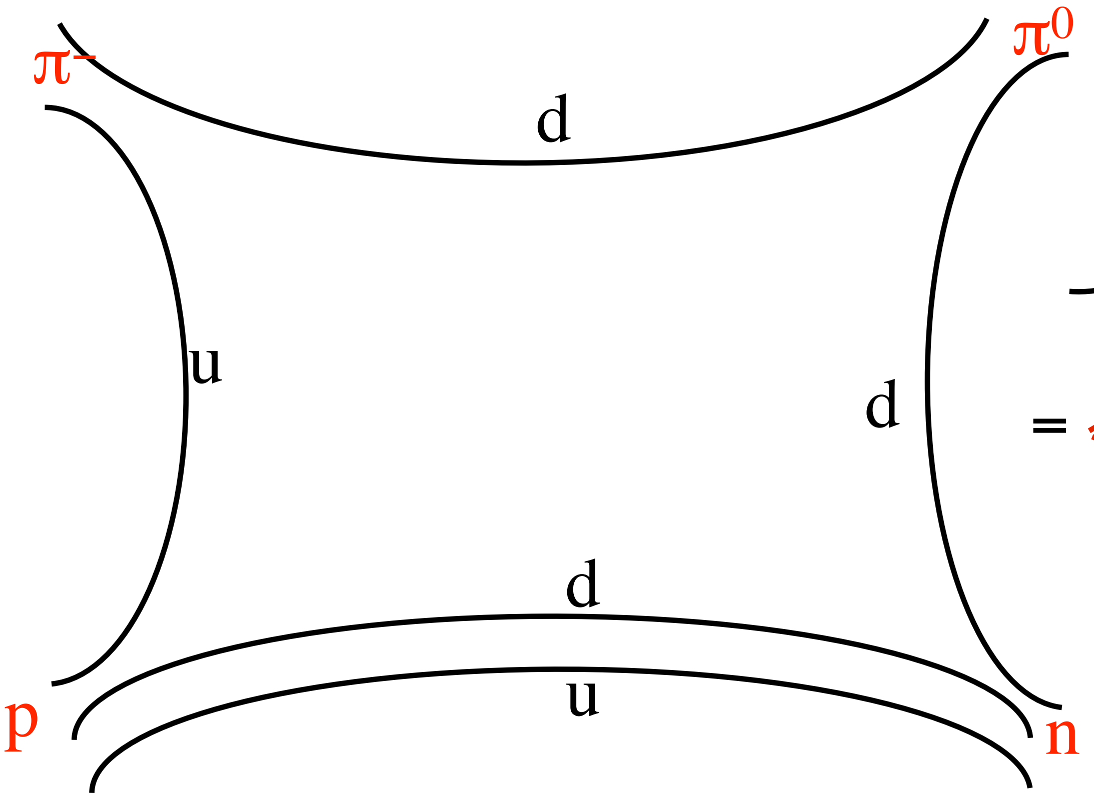
Erice, 1967: Gell Mann bringing news from Caltech:

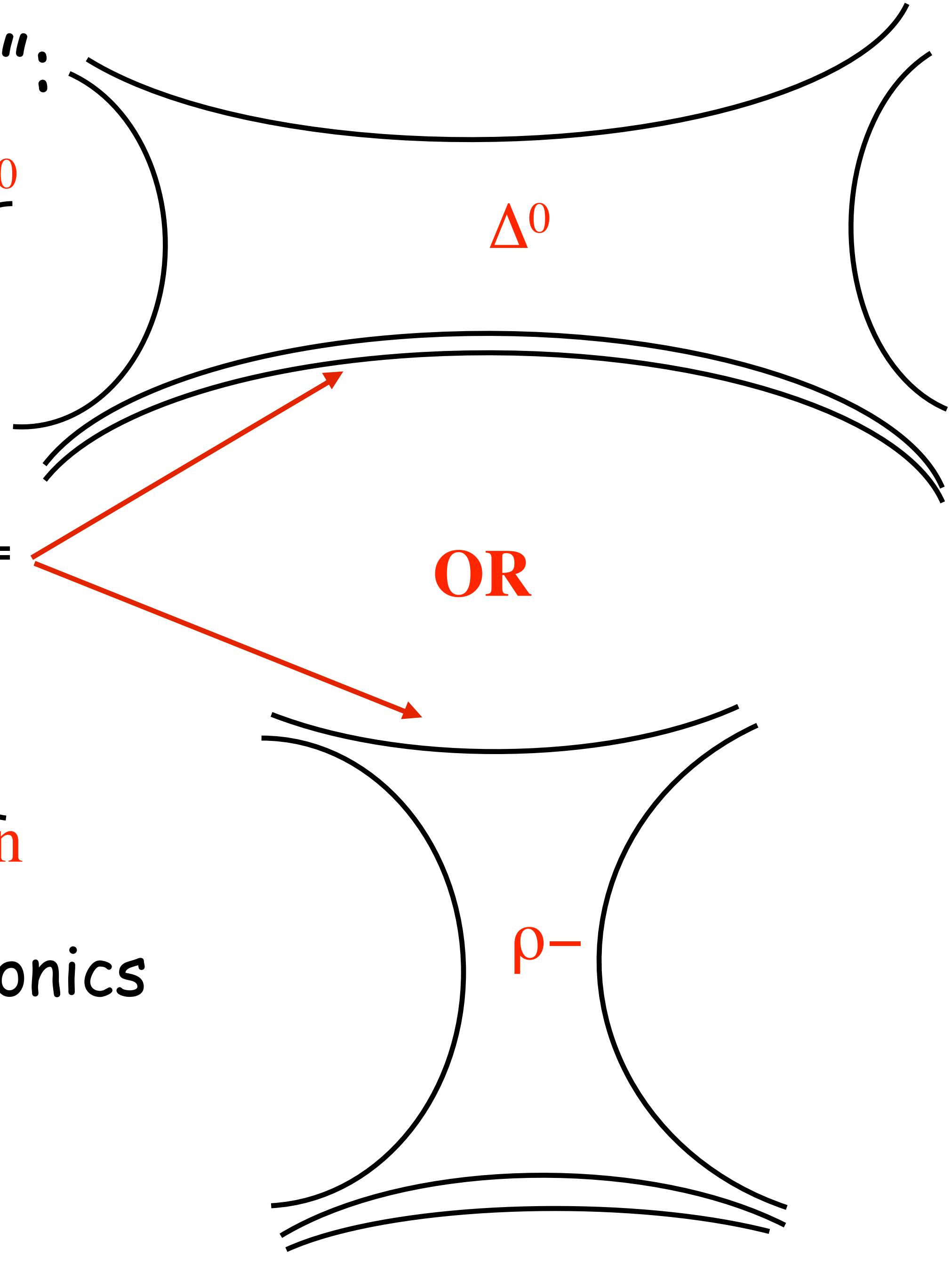
Dolen-Horn-Schmit (DHS) duality: s-and t-channel descriptions are roughly equivalent, complementary, DUAL (Cf. QM's particle/wave duality)

Adding them = double counting!



DHS duality prompted Harari and Rosner to invent "duality diagrams":





NB: Quarks were just a mnemonics for QN's in those days

N.B. The Pomeron is dual to multi-particle states

π N scattering looked too complicated
We* decided to consider a simpler case:

$\pi\,\pi \to \pi\,\omega$ (Very symmetric & very selective in QN's)
(ρ, ρ^*..)

Between the fall of 1967 and the summer of 1968 we made much progress in finding accurate solutions to this "Cheap Bootstrap".

*) Ademollo, Rubinstein, Virasoro, GV (+Bishari & Schwimmer)
with much advice and encouragement by Sergio Fubini

Weizmann Institute, 1967