Cattedra Enrico Fermi 2015-20116

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Lezione # 2.1: 12.11.2015

Interazioni forti anni sessanta
Elementi della teoria della matrice S

Dalla prima lezione...

“Cercherò di tenere una via di mezzo, evitando un eccessivo formalismo matematico, ma cercando di far passare le principali idee fisiche”

“Aggiusterò il tiro, e di conseguenza anche il programma, secondo la risposta del pubblico”

Andremo più lentamente la prima ora lasciando alcuni dettagli più tecnici, per chi è interessato, alla seconda

Sempre dalla prima lezione
la tavola completa
(per completezza)

Gauge quantum numbers in theSM
(one family of left-handed fermions)

	SU(3)	SU(2)	U(1)
(u,d) = Q	3	2	1/6
(	1	2	-1/2
u	3*	1	-2/3
d	3*	1	+1/3
e	1	1	+1
(	1	2	1/2

+ the c.c. fields, including $\Phi^*= (\phi^{0*}, \phi^-)$ + two more fermion families + sterile neutrinos?

In questa prima parte del corso ci concentreremo sulle interazioni forti spengendo a mano quelle elettrodeboli. Questo ci da, apparentemente, un'enorme semplificazione

Gauge quantum numbers in QCD (one family of left-handed quarks)

	SU(3)
u	3
d	3
u	3*
d	3*

+ r.h. antiparticles + 8 gluons of SU(3).

looks deceptively simple …

La QCD, sebbene predittiva e colma di successi per una classe di osservabili, resta una teoria estremamente complessa. Vedremo come prese il sopravvento sulla stringa adronica degli anni sessanta-settanta ma anche come, in un certo limite, essa stessa debba ridursi a una teoria di stringhe, teoria ancora tutta da scoprire

VERO INIZIO DEL CORSO

Status in the mid sixties
(with Michelin-star grading)

1965	FORCE
*** (QED)	EM
*(*) (Fermi)	WEAK
* (Models)	STRONG
** (GR)	GRAVITY

STRONG INTERACTIONS in the 60s

No Theory, rather:

A handful of models capturing one or another aspect of hadronic physics e.g.

- Short range i.e. no massless particles (Yukawa 1935)
- Symmetries, conservation laws (parity, $SU(2)_I$, $SU(3)_F$) making them, in principle, simpler.
- Many metastable states (resonances) extending to large m and J: an ever increasing zoo?

Why did we take the wrong way?

A QFT approach looked hopeless:

The game at the time was to associate fields with particles (stable or not) you do observe. But then:

1. Too many d.o.f. => too many fields
2. High-J QFT's are pathological (J=2 is already bad-enough!)

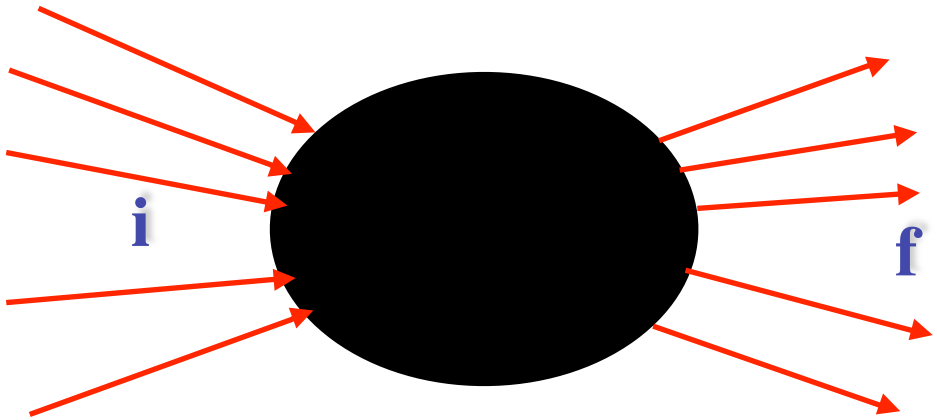
An S-matrix approach looked more promising.

In line with "stick to what you measure" philosophy

The S-matrix (Heisenberg 1943)

$$S_{fi} = \text{complex number} \; ; \; |S_{fi}|^2 = \text{probability for } i \to f$$

Equivalently: S = operator evolving |i> to S|i>



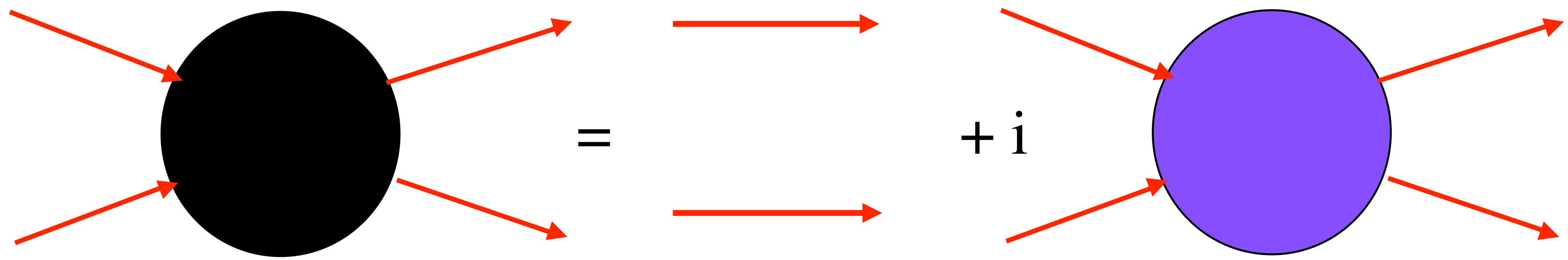
- **Symmetries:** easy to implement on S
- **Causality** => analyticity, dispersion relations
- **Crossing** : see below
- **Conservation of Probability:** $\sum_f |S_{fi}|^2 = 1$

Unitarity constraint: $SS^\dagger = 1$

Cluster decomposition (valid for short range forces) gives S as a trivial plus a non-trivial (scattering) term:

$$S_{fi} = \delta_{fi} + iT_{fi} = \delta_{fi} + i\delta^{(4)}\left(\sum p_i - \sum p_f\right) A_{fi}$$

In pictures for 2->2 scattering (denoting T by a blue blob):

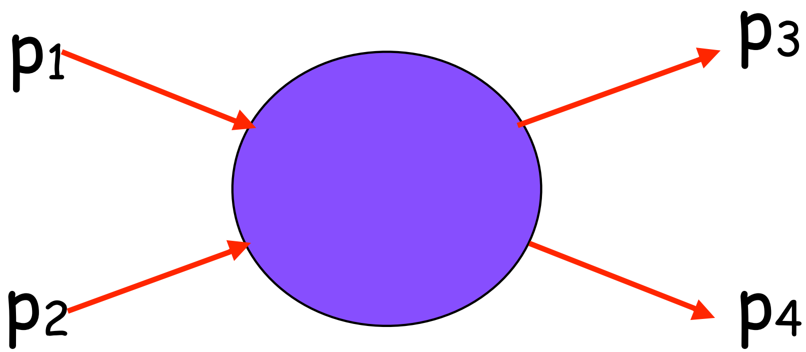


where A is an ordinary function of variables associated with the external momenta. Lorentz invariance restricts the quantities A can depend upon. Finally, impose energy-momentum conservation and the fact that the external (initial and final) particles have definite mass (on-shell cond.[s]).

Bottom line: relativistic 2-body scattering amplitude *A* depends on just 2 independent variables, s and t.

Better: 3 variables with one linear relation:

$$
\begin{aligned}
s = -(p_1 + p_2)^2 &= -(p_3 + p_4)^2 \\
t = -(p_1 - p_3)^2 &= -(p_2 - p_4)^2 \\
u = -(p_1 - p_4)^2 &= -(p_2 - p_3)^2 \\
s + t + u &= \sum m_i^2
\end{aligned}
$$



s,t,u are the so-called Mandelstam variables

Their meaning will become clearer in a moment

NB: we use the metric (-,+,+,+): $p^2 = -p_0^2 + p_1^2 + p_2^2 + p_3^2$

Analyticity

A consequence of causality is that $A(s,t)$ is an analytic function of its arguments.

An analytic function $f(z)$ satisfies many amazing properties (in particular it can be uniquely "continued").

It can have singularities in the form of poles

$f(z) \sim (z - c)^{-1}, (z - c)^{-2}, \ldots$

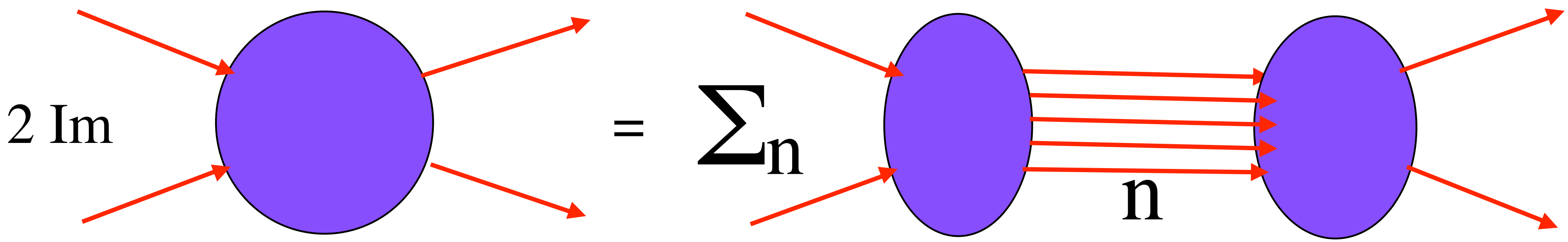
or branch points:

$f(z) \sim (z - c)^{\alpha}$ if α is not an integer.

Unitarity. Recalling:

$$S = 1 + iT = 1 + i\delta^{(4)}\left(\sum p_i - \sum p_f\right) A$$

$$S^\dagger S = 1 \Rightarrow -i(T - T^\dagger) = 2ImT = T^\dagger T$$



This (non-linear) equation basically determines the singularities of $A(s,t)$ (more generally of A_{fi} , 2nd hour)

Crossing symmetry

$$
\begin{aligned}
s = -(p_1 + p_2)^2 &= -(p_3 + p_4)^2 \\
t = -(p_1 - p_3)^2 &= -(p_2 - p_4)^2 \\
u = -(p_1 - p_4)^2 &= -(p_2 - p_3)^2 \\
s + t + u &= \sum m_i^2
\end{aligned}
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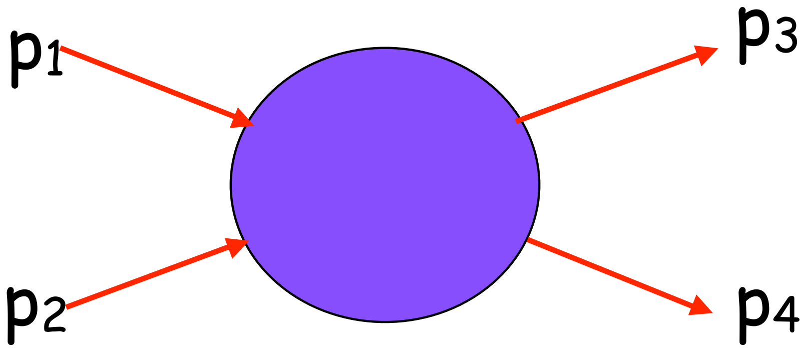


The same analytic function describes, in different regions, 3 distinct processes:

s-channel: 1+2->3+4 ; s > 0, t, u <0

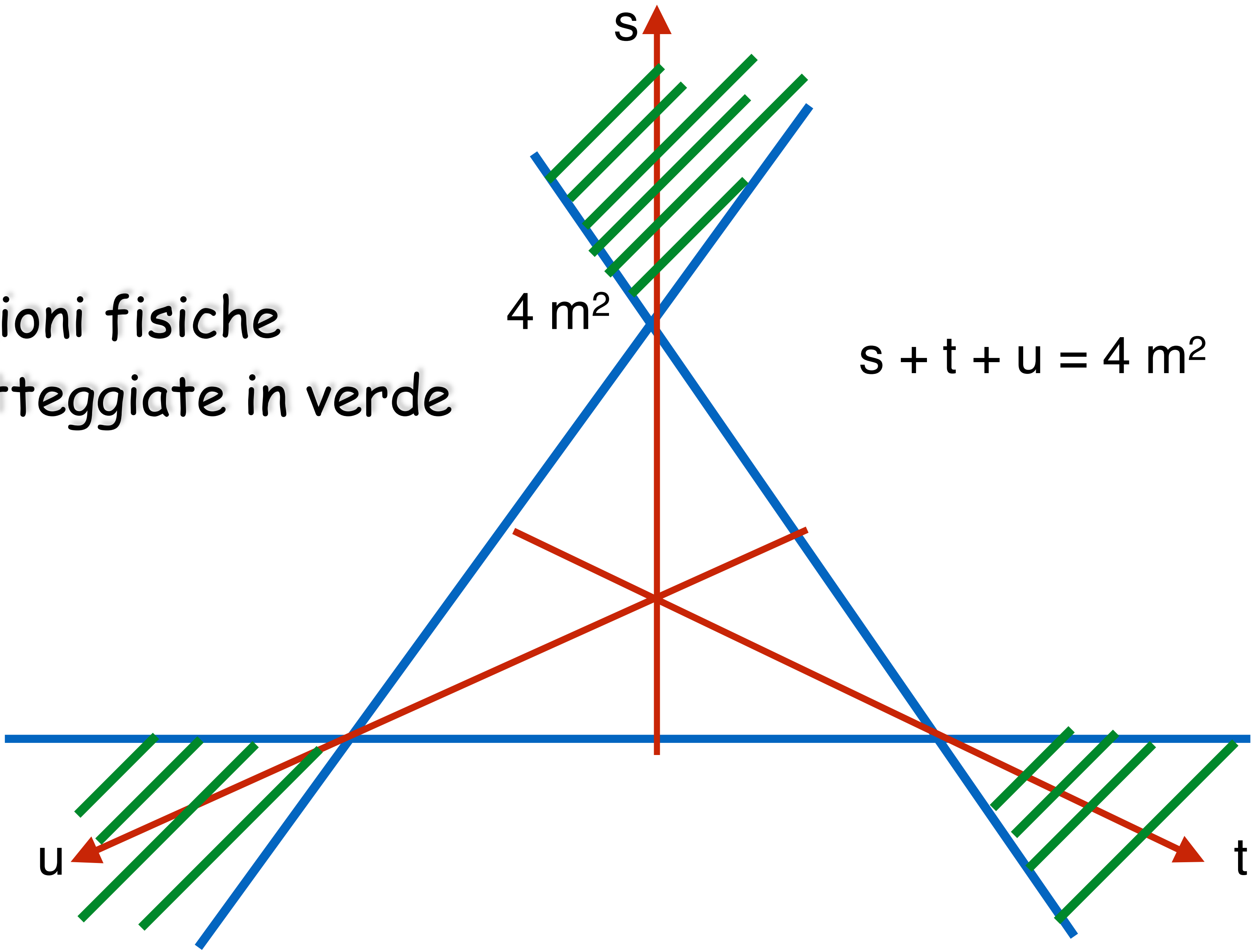
t-channel: 1+3*->2*+4 ; t > 0, s, u <0

u-channel: 1+4*->3+2* ; u > 0, s, t < 0

where a star denotes the antiparticle

Mandelstam plane (equal masses)

Regioni fisiche
tratteggiate in verde



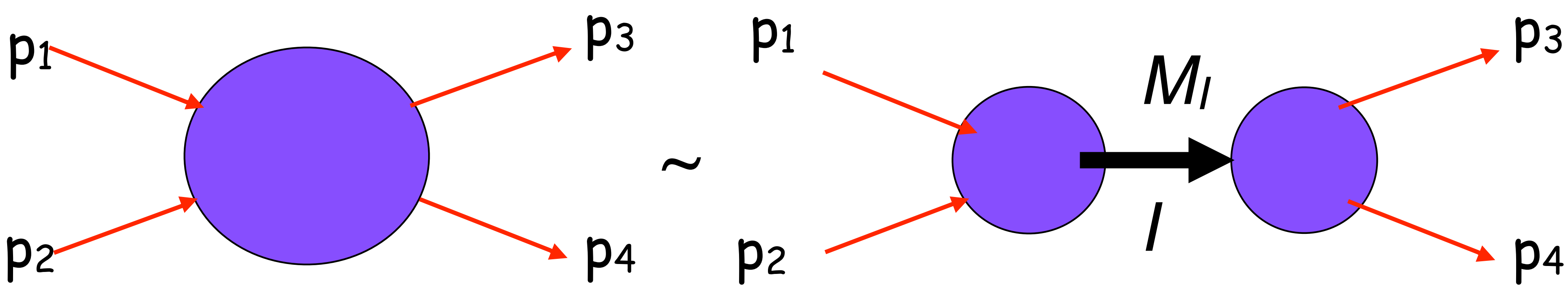
Poles in A

A single particle appearing as intermediate state in a channel produces a pole in the corresponding Mandelstam variable at the (mass)^2 of that particle.

Unitarity forces the residue of the pole to "factorize".

Example of s-channel intermediate state:

$$A(s,t) \sim -\frac{g_{12I}\; g_{I34}}{s - M_I^2 + i\epsilon}$$



What about the iε?

Stable vs. unstable particles

A particle cannot be stable if its mass is above threshold of a channel it is coupled to.

If so the pole is really off the real axis. Unitarity gives the value of ε (second hour). Breit-Wigner formula:

$$A(s,t) \sim -\frac{g_{12I}\ g_{I34}}{s - M_I^2 + iM_I\Gamma_I}$$

where Γ_I is the inverse (in natural units) of the lifetime of the unstable particle (in agreement with the UP of QM) i.e. the decay rate:

$$\Gamma_I = \sum_f \Gamma_{If}\ ;\ \Gamma_{If} \propto \int |g_{If}|^2 \delta(M_I - E_f)$$

Stable vs. unstable particles

Let us compute Im *A*:

$$\mathrm{Im} A(s,t) = g_{12I} \; \frac{M_I \Gamma_I}{(s - M_I^2)^2 + M_I^2 \Gamma_I^2} \; g_{I34}$$

N.B. For small Γ_I the integral of Im *A* (over s) is independent of Γ_I and we may take $\Gamma_I \to 0$.

This is the so-called narrow-resonance-approximation to be extensively used later. Careful use necessary.

fine prima ora?

Organizing the hadronic zoo

A) Group theory:

- $SU(2)_I$, $SU(3)_F$, same-J particles (e.g. p <-> n)

B) Regge theory of complex J

- For combining different-J particles (Regge)
- For describing high-energy scattering (Chew-Mandelstam)
- Most important for the birth of string theory

Sketch of Regge's theory of complex J

Consider non-relativistic potential scattering.
Expand the scattering amplitude in partial waves:

$$A(E,\theta) = \sum_{J=0}^{\infty} A_J(E) P_J(cos\theta)$$

In 1959 Tullio Regge had the bold idea of looking at $A_J(E)$ as an analytic function of complex J. He found that, quite generically, there were poles in J at an energy-dependent position i.e. at $J = \alpha(E)$:

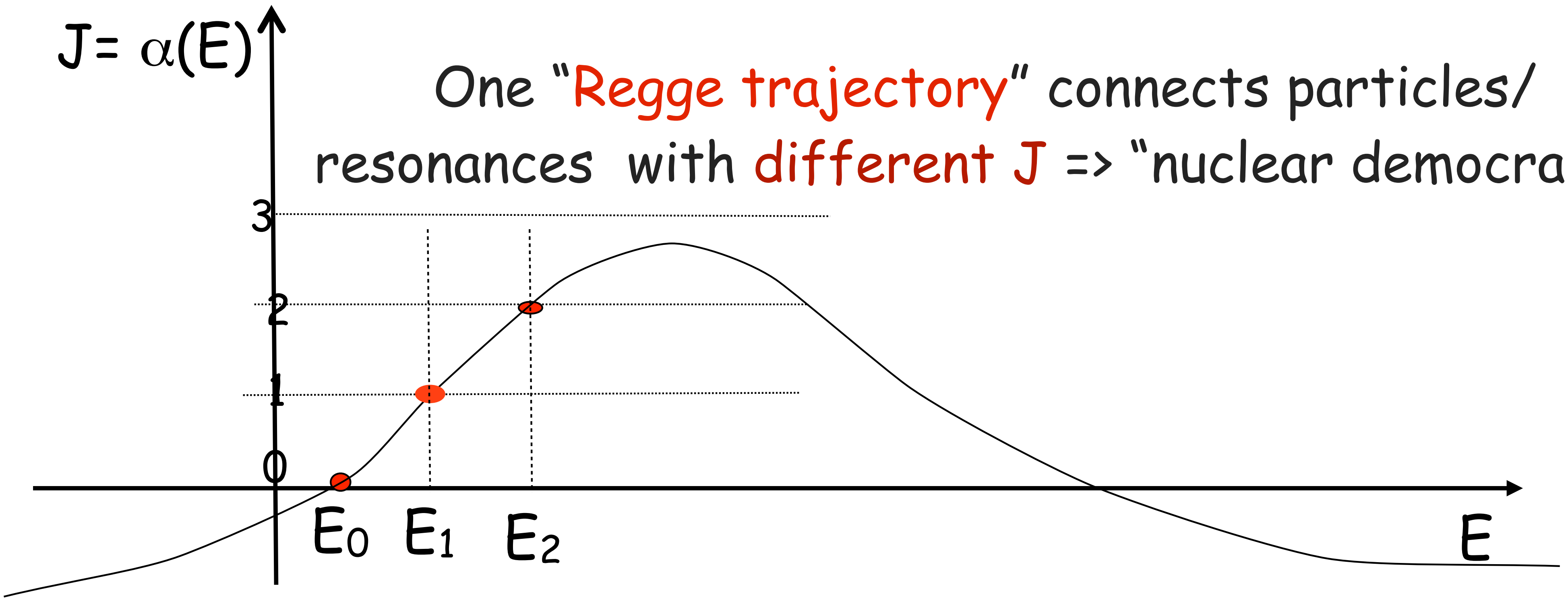
$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

Assuming $\alpha(E)$ to go to a positive integer n at $E = E_n$.

$$\alpha(E_n) = n \Rightarrow A(E,\theta) = \frac{\beta(E_n)}{n - \alpha(E)} P_n(cos\theta) \sim -\frac{\beta(E_n)}{\alpha'(E - E_n)} P_n(cos\theta)$$

i.e. just the contribution of a single resonance of energy E_n.

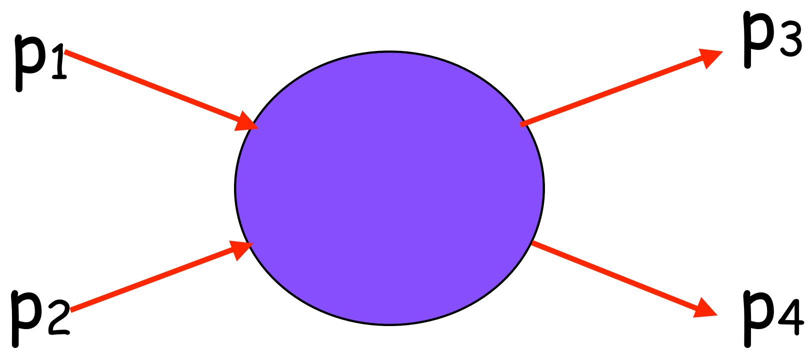


One "Regge trajectory" connects particles/ resonances with different J => "nuclear democracy".

Chew-Mandelstam's application of Regge theory in relativistic scattering

In s-channel region expand $A(s,t)$ in t-channel partial waves:

$$\begin{aligned} s = -(p_1+p_2)^2 &= -(p_3+p_4)^2 \\ t = -(p_1-p_3)^2 &= -(p_2-p_4)^2 \\ u = -(p_1-p_4)^2 &= -(p_2-p_3)^2 \\ s+t+u &= \sum m_i^2 \end{aligned}$$



$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) \quad ; \quad cos\theta_t = 1 + 2s/t$$

At large $s > 0$ & fixed $t < 0$, the contribution of the J^{th} term grows like s^J. The series is badly divergent but, under some assumptions, can be extended analytically

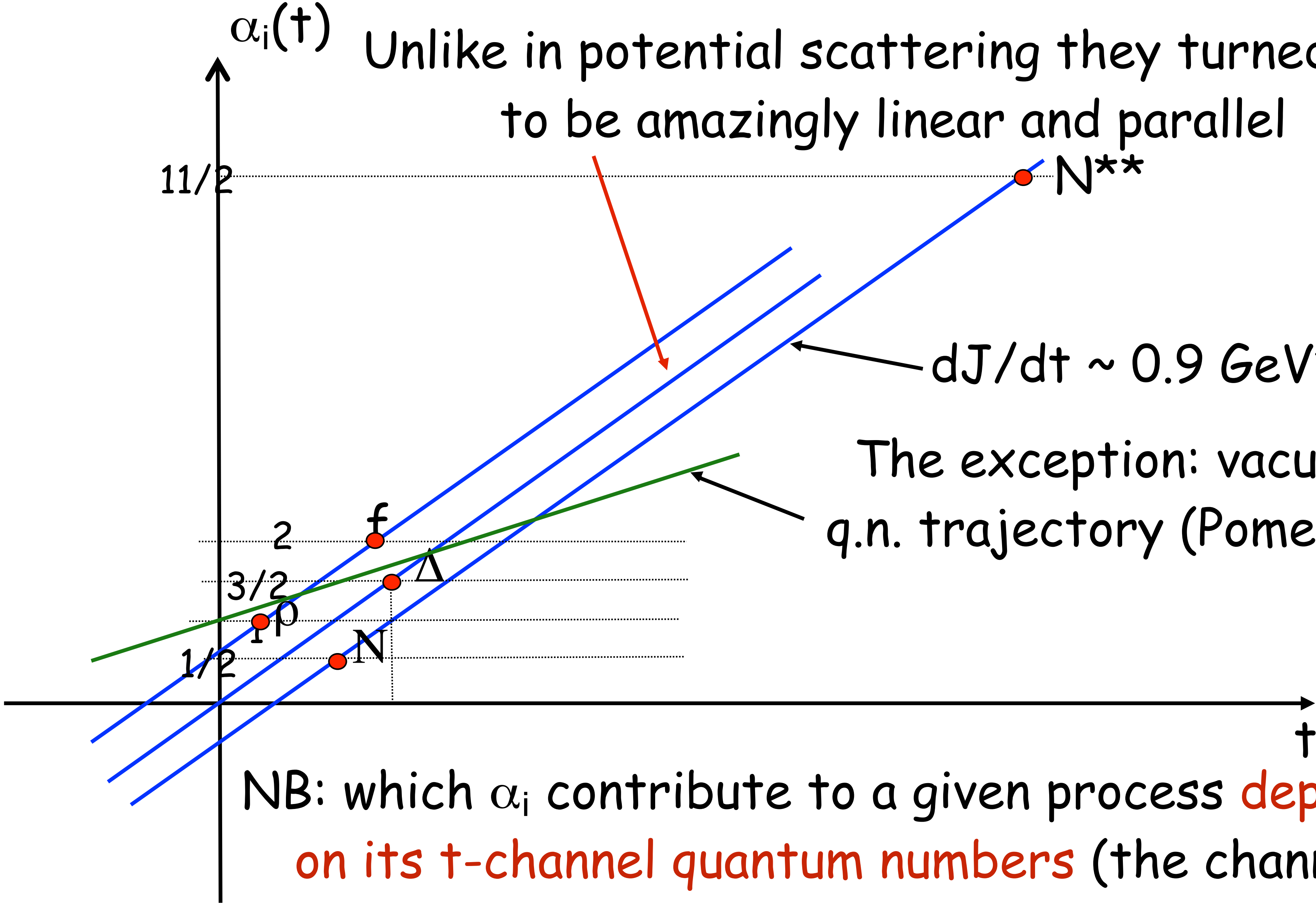
Leaving the bloody details to the next hour here is the result:

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$

Note that Regge theory gives, in general a complex scattering amplitude while usual single particle exchanges do not. Interpreting correctly this imaginary part turned out to be crucial (next week?).

A very interesting experimental discovery of the sixties was the unexpected shape of the Regge trajectories:

Unlike in potential scattering they turned out to be amazingly linear and parallel

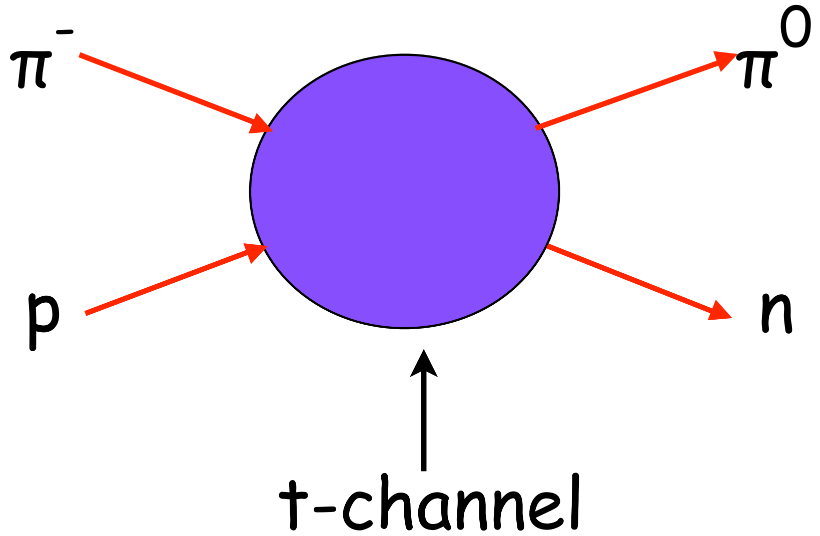


The exception: vacuum q.n. trajectory (Pomeron)

NB: which α_i contribute to a given process depends on its t-channel quantum numbers (the channel whose Mandelstam variable is kept fixed)

Examples

1. pion-nucleon charge exchange

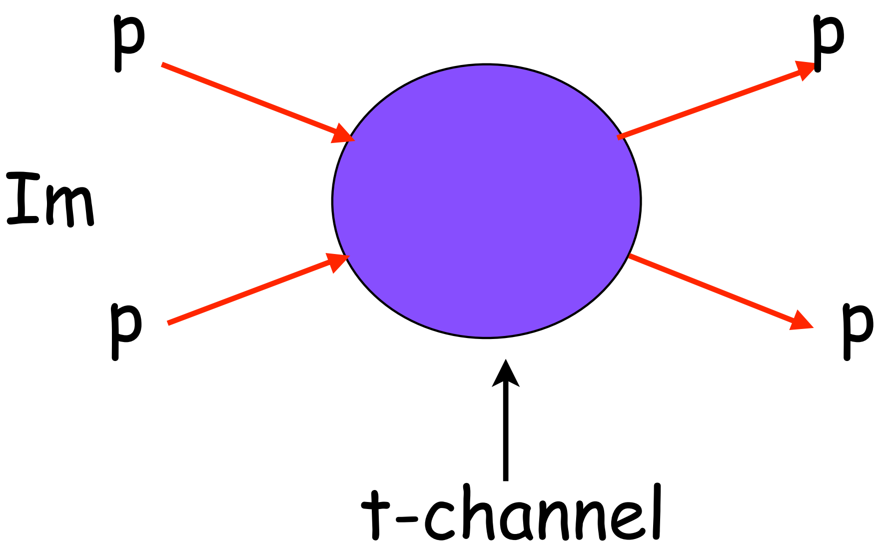


I=1 trajectories of both signatures can contribute

$$A(s,t) \sim \frac{\beta_\rho(t)[e^{i\pi\alpha_\rho} - 1]}{sin(\pi\alpha_\rho(t))} s^{\alpha_\rho(t)} + \frac{\beta_{A2}(t)[e^{i\pi\alpha_{A2}} + 1]}{sin(\pi\alpha_{A2}(t))} s^{\alpha_{A2}(t)}$$

Fitting data gives $\alpha_\rho(0) \sim \alpha_{A2}(0) \sim 0.57$ explaining quite well the scattering data above a few GeV. Distinctive prediction: shrinkage of forward peak

2. proton-proton total cross section (LHC)



I=0, 1 trajectories of both signatures can contribute

Highest one has vacuum quantum numbers

$$\sigma_T = \frac{1}{s} Im A(s,0) \sim \frac{1}{s} Im \frac{\beta_{\mathcal{P}}(0)[e^{i\pi\alpha_{\mathcal{P}}} + 1]}{sin(\pi\alpha_{\mathcal{P}}(0))} s^{\alpha_{\mathcal{P}}(0)} + \dots = \beta_{\mathcal{P}(0)} s^{\alpha_{\mathcal{P}}(0)-1} + \dots$$

Fitting data gives $\alpha_P(0) \sim 1.07$ violating a famous (Froissart) bound ($\log^2 s$): the story must be more complicated!

Argomenti seconda ora

1. Check of unitarity of BW formula.
2. Experiments measure production cross sections (production rates) times branching ratios and neglect the weak decay of hadrons when computing such rates. Why?
3. Pole in BW formula appears to be below the real axis. In reality it is below a cut in the complex s-plane (associated with the multi particle states into which the resonance decays) i.e. the pole lies on the 2nd Riemann sheet (there is also a sheet in which the pole is above the real axis, but the physical scattering amplitude is defined as $A(s,t)$ evaluated on the real axis + i ε and that pole is far away and ineffective).

4. Real analyticity, branch points, cuts: a multi-particle state appearing as intermediate state in a channel produces a branch point in the corresponding Mandelstam variable at the minimal invariant mass of that system.

5. Unitarity connects the discontinuity across the corresponding cut to a product of an amplitude A_1 and a complex-conjugate amplitude A_2^* both involving the intermediate state and the particles defining the channel.

6. Derivation of Regge behaviour.