Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

Gabriele Veneziano

Lezione # 13.2: 05.05.2016

Cosmologia inflazionaria di stringa: il caso omogeneo

Inflation and String Theory

There have been several attempts to incorporate standard (i.e. slow-roll) inflation in QST. It seems that slow-roll inflation is not a natural outcome of string theory.

Why not ask instead:

Which is the most natural cosmology suggested by QST?

Let us start from the field equations that follow from the effective action of string theory at tree level (small g_s) and small curvature (i.e. neglecting higher-derivative terms).

In particular let us see whether the eqns. of string cosmology show new symmetries w.r.t. those of standard cosmology.

But first a short parenthetic remark

T-duality and the dilaton

There is a subtle point about T-duality. It can be appreciated by looking at the effective action:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2}\int d^D x\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{12}H^2 + \dots\right]$$

When one dimension is compactified on a circle, the couplings in the remaining D-1 dimensions depend on a shifted dilaton:

$$g_s^{-2} = e^{-2\Phi} \to e^{-2\Phi}\int dy_5\sqrt{g_{55}} = e^{-2\Phi}2\pi R = g_{eff}^{-2}$$

As one may expect, T-duality has to be accompanied by a transformation of Φ such that the effective coupling in the non-compact dimensions remains the same. In general:

$$g_{eff}^{-2} = e^{-2\Phi}\int dy^i\sqrt{g_{ij}} = e^{-2\Phi}V_c \to g_{eff}^{-2}$$

If V_c is changed Φ has to transform to keep g_{eff} the same.

Our starting point is (hereafter $\phi = 2\Phi$)

$$\Gamma_{eff} = -\int \frac{d^D x}{l_s^{D-2}} \sqrt{-G} e^{-\phi} \left[\frac{4(D-10)}{3l_s^2} + R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

If D ≠ 10 we have no chance to get a low-curvature solution and thus we shall limit ourselves to D =10:

$$\Gamma_{eff} = -\int \frac{d^{10} x}{l_s^{8}} \sqrt{-G} e^{-\phi} \left[R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

We also limit ourselves to these massless fields representing a universal sector in all string theories.

We allow the extra dimensions to be dynamical (unfrozen).

We work in the "string frame" (fixed l_s, varying G_N, l_P) but physical consequences are frame-independent.

Homogeneous (Bianchi I) equations

It is straightforward to write down the field equations for a homogeneous (for simplicity Bianchi I) universe:

$$ds^2 = -dt^2 + \sum_i a_i^2(t)(dx^i)^2 \quad ; \quad \phi = \phi(t) \quad ; \quad B = 0$$

They take the simple form:

$$(\dot{\bar{\phi}})^2 - \sum_i H_i^2 = 0 \quad ; \quad \dot{H}_i - \dot{\bar{\phi}} H_i = 0 \quad ; \quad H_i \equiv \frac{\dot{a}_i}{a_i} \quad ; \quad \dot{\bar{\phi}} \equiv \dot{\phi} - \sum_i H_i$$

where the so-called shifted dilaton

$$\bar{\phi} = \phi - \frac{1}{2}\log(\det G_{ij})$$

satisfies, as a consequence,

$$\ddot{\bar{\phi}} - (\dot{\bar{\phi}})^2 = 0 \Rightarrow \frac{d}{dt} e^{-\bar{\phi}} = \mathrm{constant}$$

Scale-factor duality: a cosmological variant of T-duality?

There is an interesting symmetry of the string-cosmology equations under inversion of any individual scale factor $a_i(t)$, provided we keep the shifted dilaton invariant.

Indeed, under $a_i(t) \to 1/a_i(t)$, $H_i(t) \to - H_i(t)$, but our two independent equations go into themselves under this change.

This symmetry, mapping solutions into new (and generically inequivalent) ones has been called scale-factor duality (SFD) and is closely connected to T-duality (although only the latter is a true symmetry of the theory). It also holds if we add string-matter sources since those also transform nicely under SFD.

(If the $B_{\mu\nu}$ field is turned on, the discrete (Z_2^9) SFD symmetry becomes a continuous O(9,9;R) symmetry).

Generalized Kasner solutions

In the absence of other sources these equations can be easily solved. One finds:

$$a_i(t) = (\pm t)^{p_i} \quad ; \quad \phi(t) = -(1 - \sum_i p_i) \log(\pm t) + \text{const.} \quad ; \quad \sum_i p_i^2 = 1$$

These reduce to ta standard (Kasner) cosmology if we impose a constant dilaton. Note, however, that, unlike for Kasner, one can have a perfectly isotropic cosmology for a non-trivial dilaton:

$$a_i(t) = t^{\pm \frac{1}{\sqrt{d}}} \quad ; \quad \phi(t) = -(1 \mp \sqrt{d}) \log t \quad ; \quad t > 0 \quad ; \quad d \equiv D - 1 = 9$$

and similarly for $t < 0$.

Note the possibility of flipping arbitrarily the signs of each Kasner exponent if we adjust the dilaton.

This is a consequence of SFD

The pre-big bang scenario

The so-called pre big bang scenario is deeply rooted on SFD combined with the (more standard) invariance of the cosmological equations under T, the time reversal operation $t \to -t$ (NB: Also T connects inequivalent solutions!).

SFDxT acts on an individual scale factor as follows:

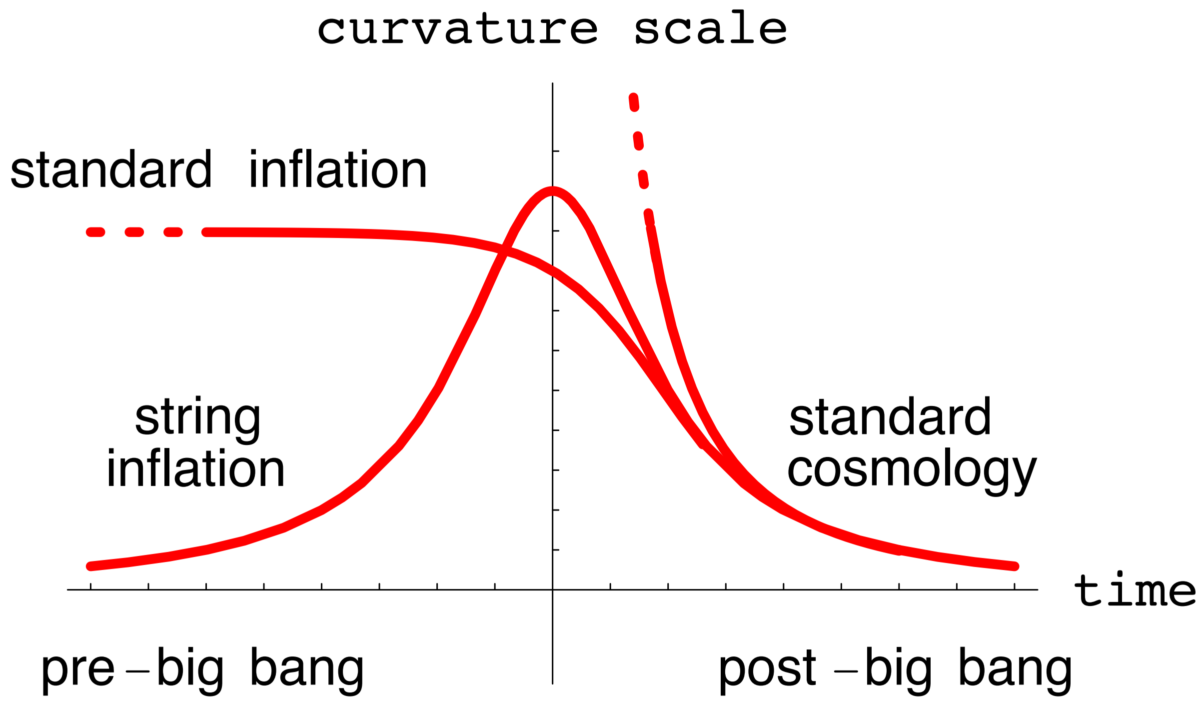
$$a_i(t) \to \tilde{a}_i(t) \equiv a_i^{-1}(-t) \Rightarrow \tilde{H}_i(-t) = H_i(t) \; ; \; \dot{\tilde{H}}_i(-t) = -\dot{H}_i(t)$$

Therefore, given a standard FLRW cosmology (an expanding & decelerating Universe at $t > 0$), SFDxT associates to it another expanding, but now accelerating, cosmology at $t < 0$. Can we put together these two SFDxT-related cosmologies?

If the answer is yes we may have a new scenario in which a long "dual" phase at $t < 0$ preceded the standard FLRW phase possibly solving the shortcomings of the latter.

Diagrams illustrating PBB idea

(GV '91, Gasperini & GV '93)



The pre-big bang accelerator

It looks as if we have obtained inflation for free in string theory! How is that possible with just a scalar field with vanishing potential?

The answer to this question lies in the peculiar way the dilaton appears in string theory. Recall that the exponential of the dilaton controls g_s and the ratio l_P/l_s.

Consider a post-big bang solution describing a decelerating expansion with a constant dilaton. Under SFDxT this solution goes into one describing a pre-big bang accelerated expamsion with a growing dilaton, hence a growing g_s and l_P/l_s.

Roughly, in the Friedman eqns. the acceleration is driven by a growing dilaton/G (DDI).

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho \; ; \; \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

The accelerated expansion is present in the string frame, i.e. if we measure distances in l_s-units. But the growth of l_P/l_s is so fast that the universe contracts if, instead, we measure distances in l_P-units.

Thus the answer to the question:

Is PBB a bouncing cosmology?

depends on the frame, i.e. on the meter we use to measure distances. The scale-factor may or may not bounce.

However, independently of the frame, PBB cosmology corresponds to a "curvature bounce" in that has a phase of growing curvature turning into one of decreasing curvature through an intermediate "string phase" during which the curvature is of order l_s^{-2}.

In any case the physical predictions are identical in the two frames.

Initial conditions & fine-tuning

So far we have assumed a "cosmological principle" for string cosmology, like it's done for the HBB cosmology.
We would like instead PBB cosmology to emerge from generic (i.e. non fine tuned) initial conditions.
This is possible if we make an assumption of "Asymptotic Past Triviality".
This is just the opposite of what is assumed in HBB cosmology (where everything started at a singularity and it's very difficult to define initial conditions).

Asymptotic Past Triviality (APT)

APT: As we go towards $t = -\infty$ the Universe gets closer and closer to the trivial vacuum of superstring theory (nearly flat D=10 spacetime and nearly vanishing string coupling, $e^{\phi} \ll 1$) but is otherwise generic (in a technical sense).

Thanks to APT we can thus use the effective action of QST at lowest order both in the genus and in the derivative expansion:

$$\Gamma_{eff} = -\int \frac{d^{10}x}{l_s^8} \sqrt{-G} e^{-\phi} \left[R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

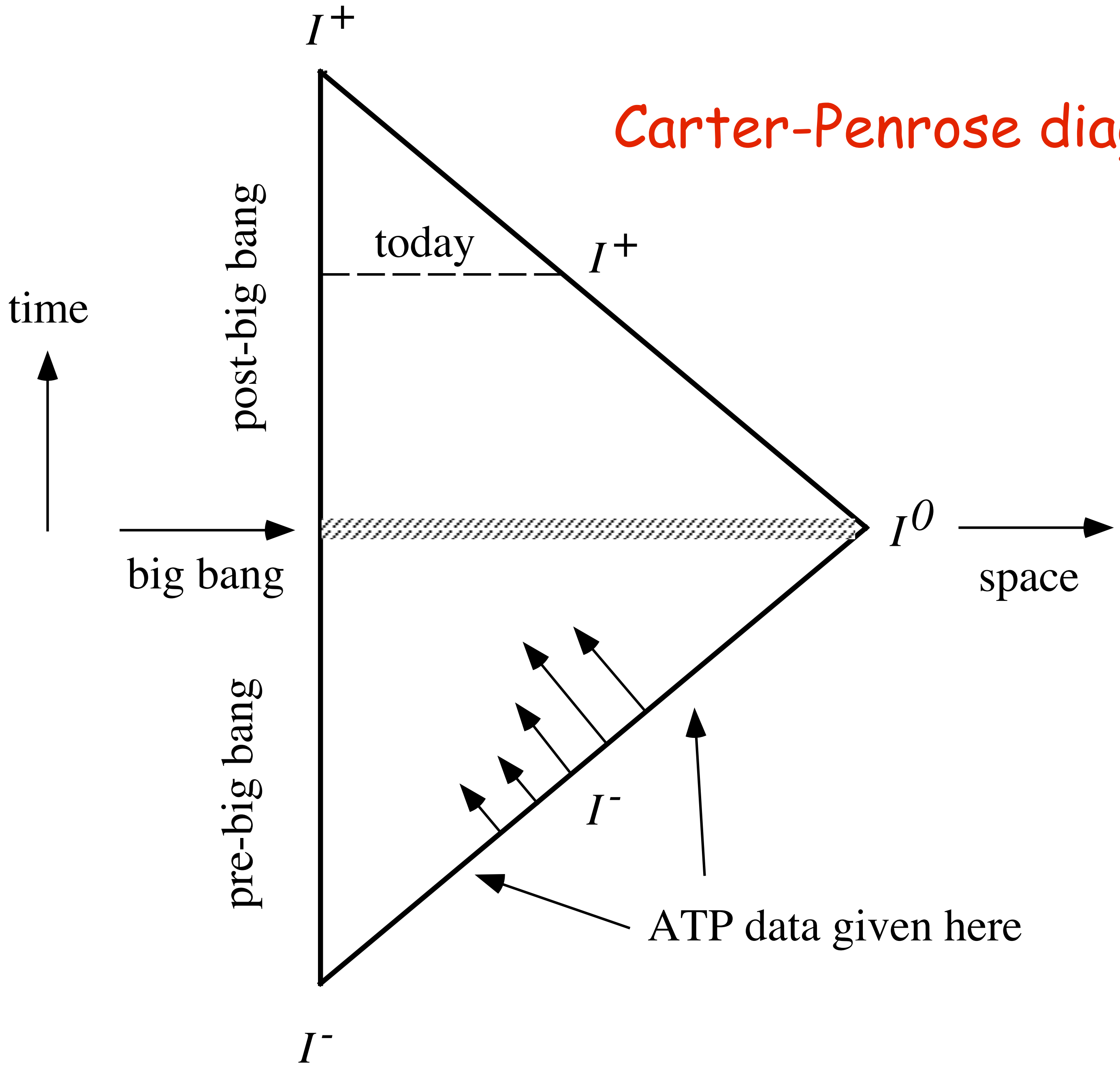
$$\Gamma_{eff} = -\int \frac{d^{10}x}{l_s^8} \sqrt{-G} e^{-\phi} \left[R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

We can write down a generic solution in the far past and check that it contains the appropriate number of arbitrary functions to be called generic. It describes, physically, a chaotic superposition of gravitational and dilatonic waves.

In the APT regime the field equations are invariant under a constant shift of ϕ and under a global rescaling of x. As a result, the generic initial data include, as free parameters, the initial value of the dilaton ϕ_{in} and the initial curvature scale.

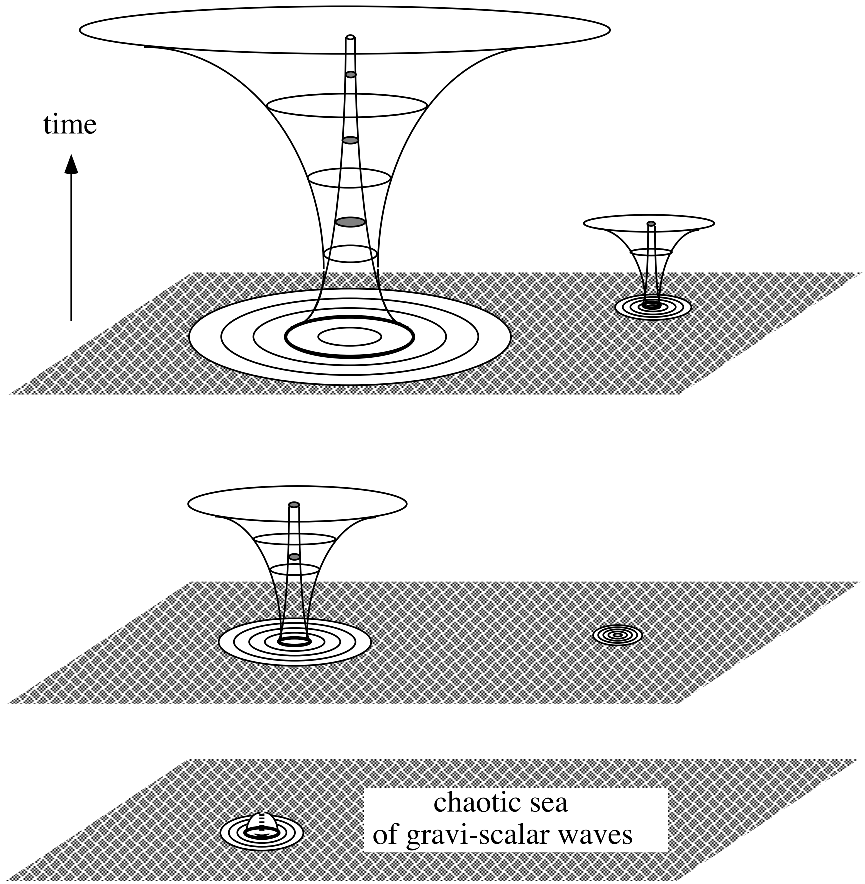
Solutions that go to the trivial vacuum in the infinite past, become increasingly complicated, curved and coupled as one moves forward in time.

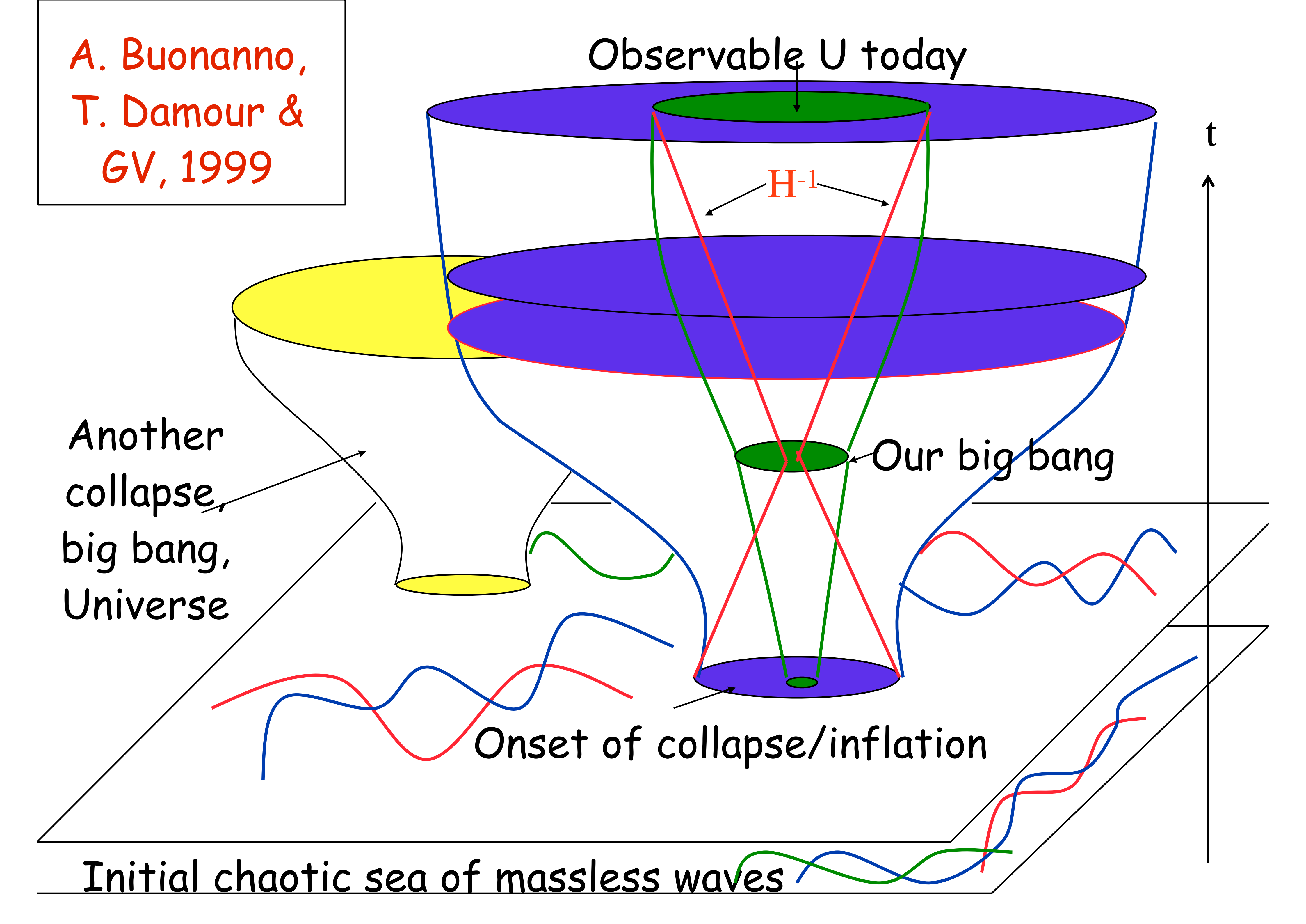
Carter-Penrose diagram



As a consequence of singularity theorems (Hawking, Penrose), the evolution generically brings about the formation of black holes in different spacetime locations with arbitrary (and randomly distributed?) values for ϕ_h and for the horizon radius R_h.

A PBB cosmology/collapse then takes place inside the horizon.





A. Buonanno,
T. Damour &
GV, 1999
Observable U today
t
H-1
Another
collapse,
big bang,
Universe
Our big bang
Onset of collapse/inflation
Initial chaotic sea of massless waves

Implementing the curvature bounce

The existence of inflationary solutions at $t < 0$ is not of much use unless we can connect this phase to a standard FLRW phase at $t > 0$.

This is the most difficult theoretical issue facing the PBB scenario.

As one approaches the singularity both the curvature and the string coupling diverge. Hence describing the bounce amounts to solving string theory when the curvature approaches the string scale and/or the string coupling becomes $O(1)$.

Strong coupling and strong curvature regimes are sometimes tractable provided the background preserves (at least part of the) supersymmetry.
Unfortunately, a time-dependent background is not supersymmetric and this makes the task very hard.
One of the very few known results concern the search for late-time de-Sitter-like (constant curvature) attractors accompanied by linear (in t) dilaton in the presence of higher derivative corrections to the tree-level effective action.
The existence of such attractors depends on the existence of real solutions to an algebraic system of n-equations in n-unknowns: generically there is a finite number of solutions.
Examples involving up to 4-derivative terms have been given, but this does not prove much because even higher order corrections cannot be neglected.

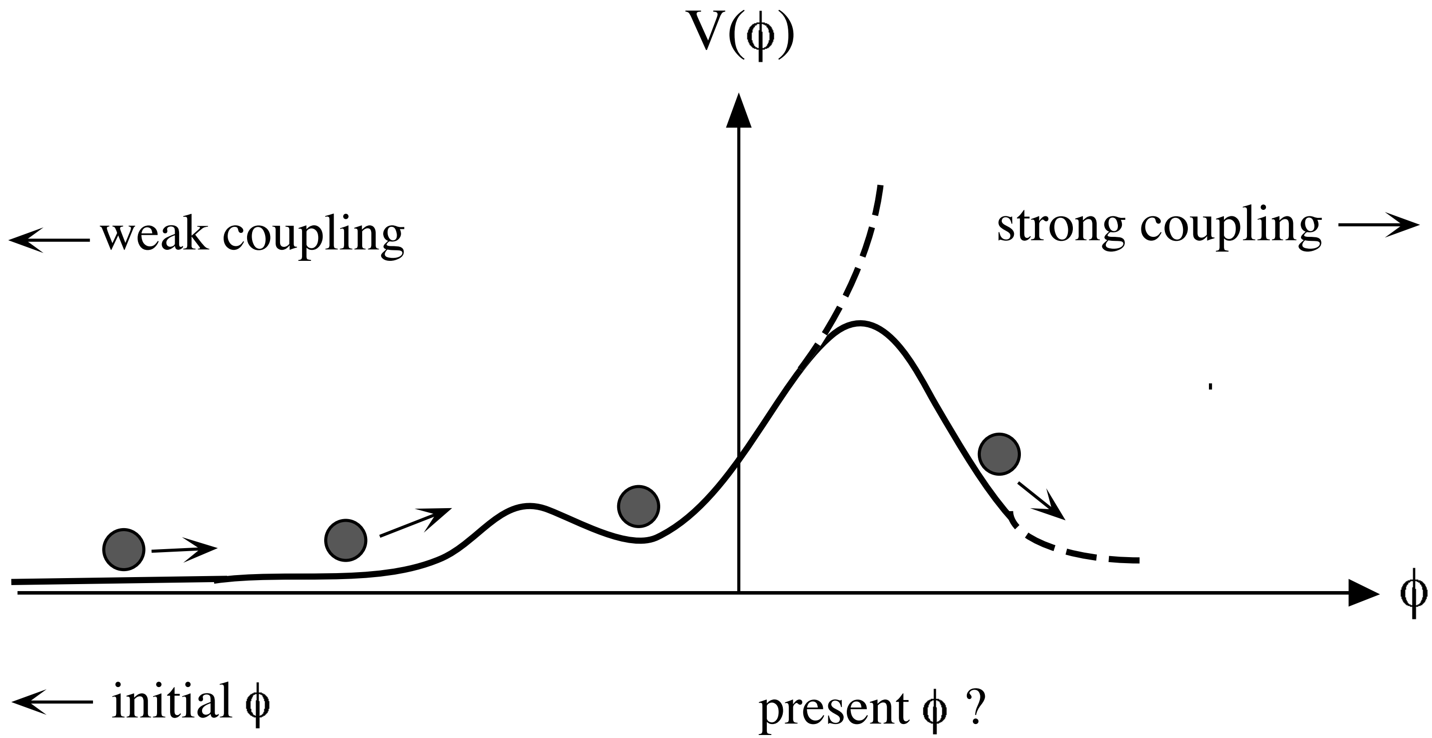
If this is what really happens:

a "string phase" would follow during which the curvature is constant (and of order l_s^{-2}) while the coupling keeps growing until higher-genus corrections become important.

It is conceivable (but not yet proven) that these loop corrections complete the transition to a FLRW phase.

Loops are related to particle production. This can warm up the Universe and account for its "initial" entropy.

Entering the strong coupling region, the dilaton can develop a non-perturbative potential (as a result of SUSY breaking) and eventually get stuck in its minimum. Thereafter the dilaton would have been constant and massive thus avoiding contradiction with precision tests of GR and with the observed time independence of various constants of Nature.


V(ϕ)
← weak coupling
strong coupling →
ϕ
← initial ϕ
present ϕ ?

Actually, in order to avoid these phenomenological problems, one has to "stabilize" also the shapes and sizes of the 6 extra dimensions of space (moduli stabilization problem) since otherwise they induce other long range forces and possible variations in the constants of Nature.

We shall see that, in order to generate an interesting spectrum of cosmological perturbations it is important to let the 6 extra dimensions contract while the other 3 expand.

It is then conceivable that, at the bounce or soon after, the extra dimensions stabilize at the self-dual size (leading incidentally to the emergence of large gauge symmetries).

Only in this case the post bang (bounce) phase will be of the conventional FLRW type.

Assuming the whole idea makes sense physically and mathematically, can we put this new cosmology to an experimental/observational test?

This is what we will discuss next week making use of how conventional and string cosmology generate the large scale structure of the Universe from Quantum Mechanics.