Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Cosmologia inflazionaria convenzianale:
il caso omogeneo

Piano delle ultime due Lezioni

5 Maggio

Cosmologia inflazionaria convenzianale e di stringa: il caso omogeneo

12 Maggio

Strutture a grandi scale in cosmologia inflazionaria convenzianale e di stringa

Cosmologia pre-inflazionaria

(senza dimostrazioni!)

Hot Big Bang cosmology

Einstein's equations, together with the cosmological principle (assumption of a homogeneous, isotropic Universe at large scales) and present observations (e.g. the redshift), lead to a very simple model known as Hot Big Bang (HBB) cosmology.

It is also named after Friedmann-Lemaître- Robertson-Walker (FLRW) who looked at solutions in the form:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right] \\ d\Omega^2 &= d\theta^2 + sin^2\theta d\phi^2 \quad ; \quad K = 0, \pm 1 \end{aligned}$$

It contains a scale-factor a(t), telling us how physical distances depend on (cosmic-proper) time, and a discrete parameter (K = 0, 1, -1) giving at any given time the spatial geometry (flat, closed, open) with curvature ${}^{(3)}R \sim K/a^2(t)$.

a(t) is related to the redshift by $(1+z) = \lambda_0/\lambda_s = a(t_0)/a(t_s)$.

Its evolution is determined by the (average) energy ρ & pressure p of matter via the two Friedman equations ($G = G_N$):

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho \; ; \; \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad H(t) \equiv \frac{\dot{a}}{a}$$

implying: $$\dot{\rho} = -3H(\rho + p) = -3H\rho(1+w) \;\; ; \;\; w \equiv \frac{p}{\rho}$$

For standard matter $\rho + 3p > 0$ (w=0 for non-relativistic matter, w = 1/3 for radiation) and this leads to a decelerating expansion and to an a(t) that goes to zero at a finite time in our past, conventionally called t=0.

At t=0, temperature, curvature and energy density become infinite, forcing the physical interpretation of t=0 as the beginning of time, the (infinitely) hot Big Bang.

Critical density and fractions

Introducing $$\rho^{(cr)} \equiv \frac{3H^2}{8\pi G} = \sum_i \rho_i + \rho_K \;;\; \rho_K = -\frac{3K}{8\pi G a^2}$$

$$\Omega_i \equiv \frac{\rho_i}{\rho^{(cr)}} \qquad\qquad \dot{\rho}_i = -3H\rho_i(1+w_i)$$

The 1st Friedman equation:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\sum_i \rho_i$$

can be rewritten in the simple form:

$$\Omega \equiv \sum_{i \neq K} \Omega_i = 1 - \Omega_K$$

NB: A spatially flat Universe is equivalent to $\Omega = 1$

Successes of HBB cosmology

1. The cosmic microwave background

(Penzias and Wilson 1965)

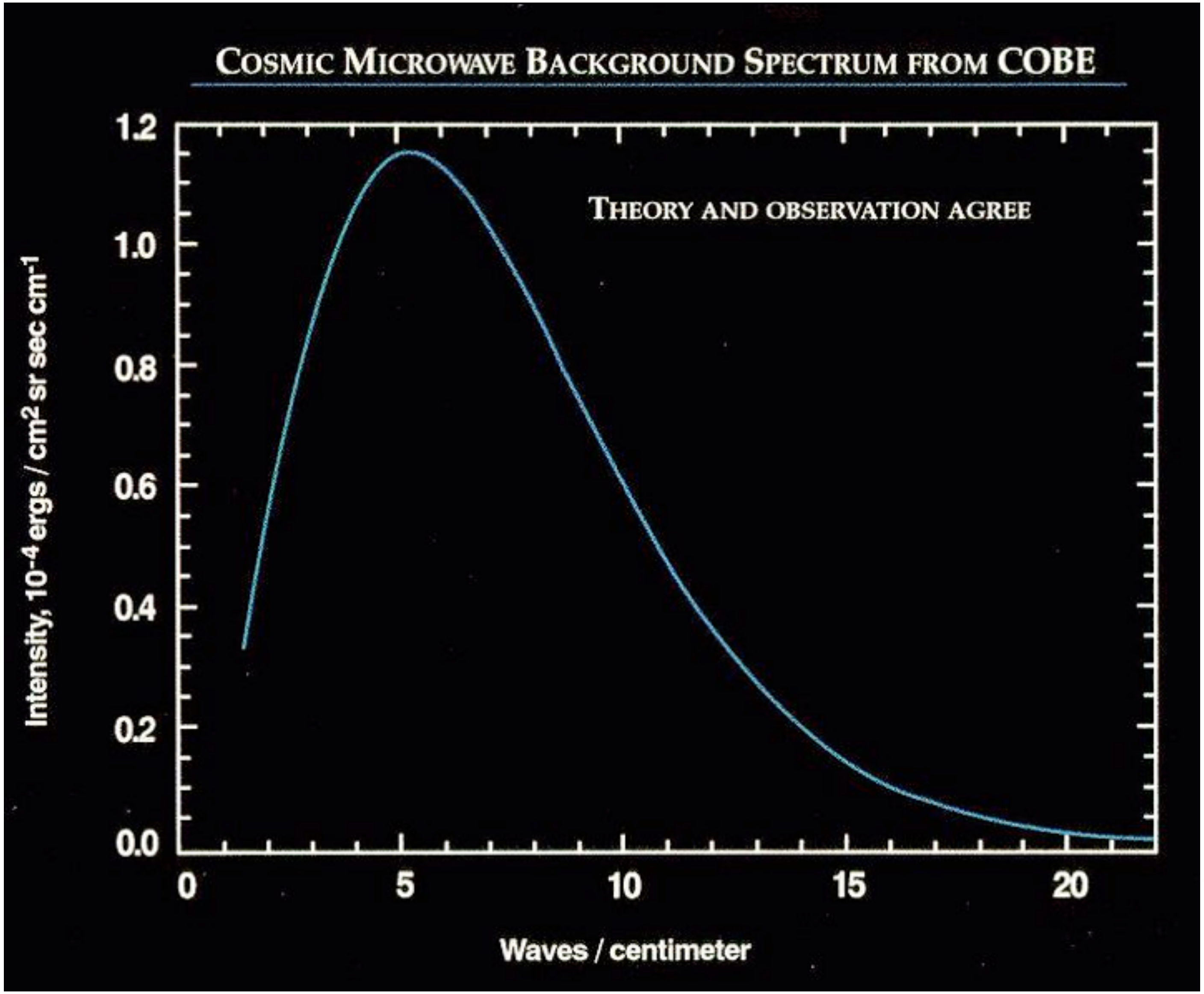
Since the 1940s, Gamow and coll. had realized that the Universe should now be filled with a black-body spectrum of electromagnetic radiation coming from the hot big bang.

The first theoretical estimate (~1950) for the present temperature was 5K in quite good agreement with the first determination of 3.5±1.0 K.

Today, the CMB spectrum is the best Planck spectrum known in Nature. Its average temperature is 2.725±0.002K.

Predicting the existence of the CMBR and its temperature was the first clear success of HBB cosmology!

$$dn(\nu) = \frac{8\pi\nu^2 d\nu}{\exp(h\nu/k_B T) - 1}$$

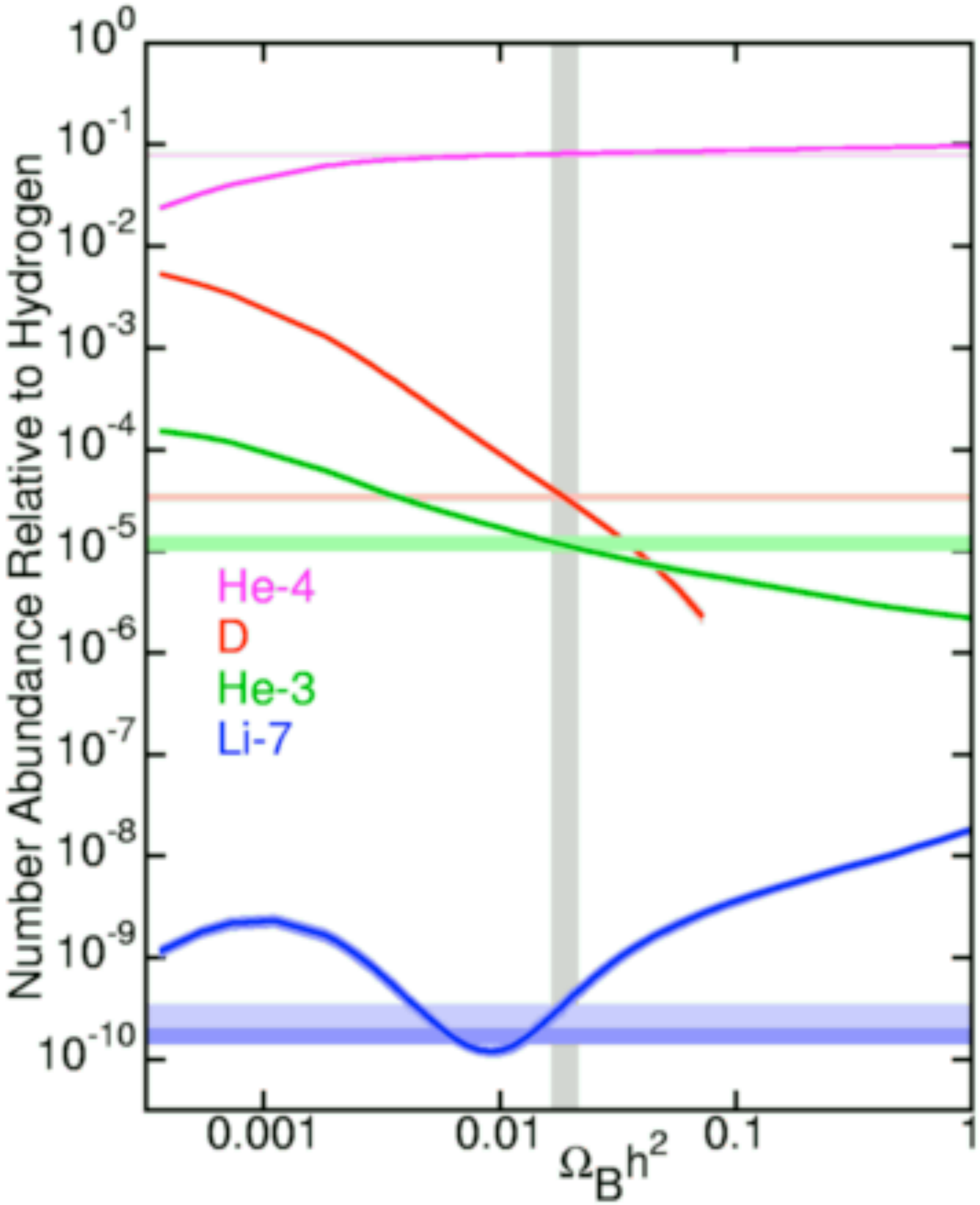


2. Primordial (BB) nucleosynthesis

A second big success of HBB cosmology is that it provides a mechanism (BBN) for producing light nuclei*) (d, He, Li, ..) out of protons and neutrons.

Temperatures of order 10^{10}K ~ MeV are needed for this to happen. The success of BBN is not just qualitative: we know the physics of the underlying processes, hence we can calculate the relative abundances of those light elements and compare them with the data.

*) Heavier elements are believed to be produced much later in very hot and dense stars, like supernovae.



Comparison with data

Horizontal bands correspond to experimental bounds;

Vertical band to allowed range for

$\Omega_B \sim 0.021\, h^{-2} \sim 0.04$

$$H(t_0) \equiv H_0 = 100\ h\ \mathrm{km}\ s^{-1}\mathrm{Mpc}^{-1} \quad ; \quad h \sim 0.72 \pm 0.05$$

Tre problemi della cosmologia pre-inflazionaria

Three shortcomings of HBB cosmology

1. Flatness problem

We know that, today, $|\Omega_K|$ cannot exceed 0.1. On the other hand Ω_K evolves in time according to:

$$\Omega_K(t) = \Omega_{K,0}\ \frac{a_0^2}{a^2}\ \frac{H_0^2}{H^2} = \Omega_{K,0}\ \left(\frac{\dot{a}_0}{\dot{a}(t)}\right)^2 \sim \Omega_{K,0}\ \left(\frac{t}{t_0}\right)^{\frac{2(1+3w)}{3(1+w)}}$$

and increases with t for a decelerated expansion ($w > -1/3$).

=> $|\Omega_K| < 10^{-32}$ at BBN & < 10^{-60} at $t = t_P \sim 10^{-43}$ sec.

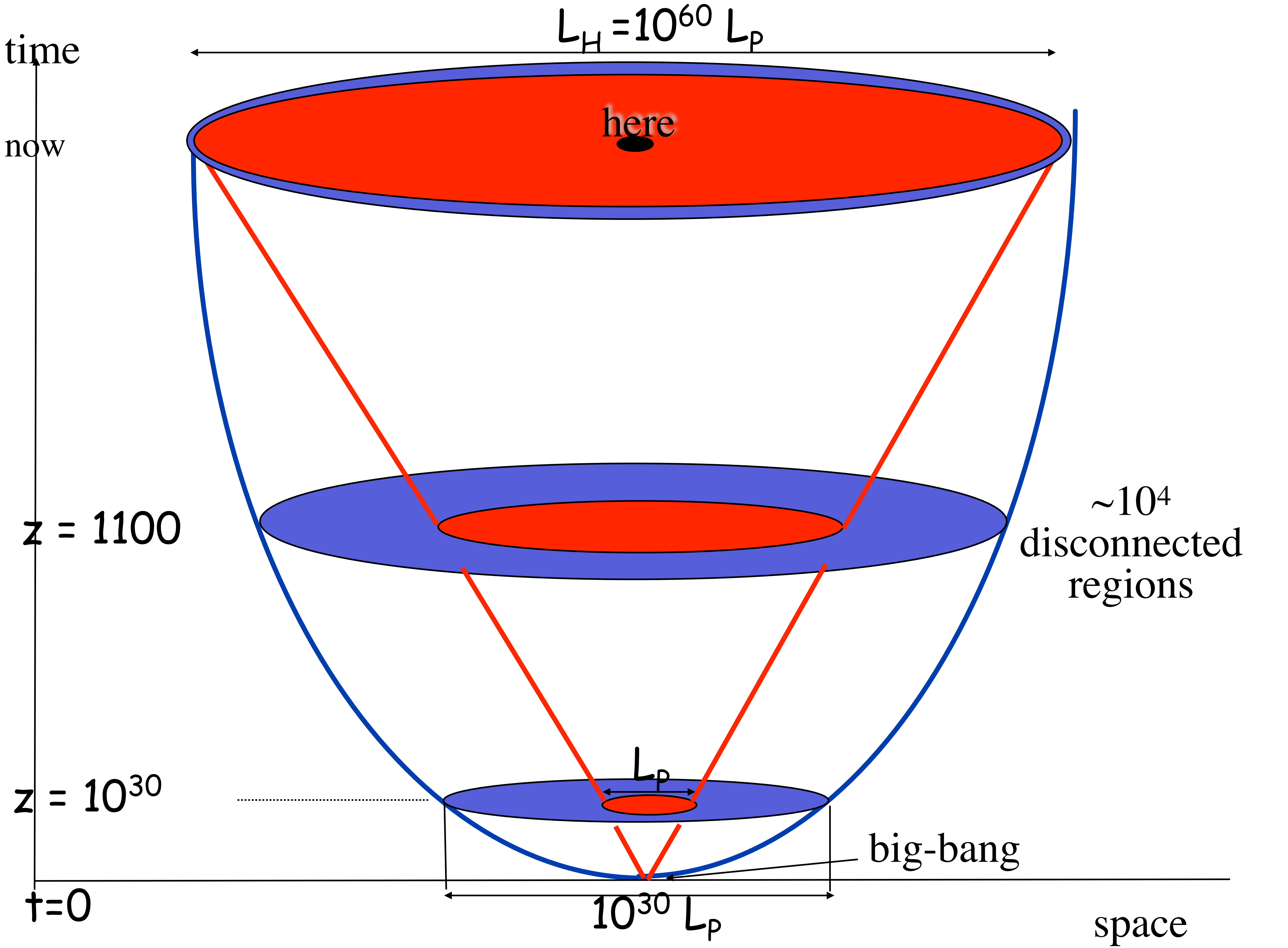
Q: Why should the Universe start with such a small spatial curvature w.r.t. the total space-time curvature?

2. Homogeneity problem

The CMB comes to us today, basically undisturbed (just redshifted) from the time of recombination (or last scattering, when atoms were formed and the Universe became transparent to photons). This happened at $z = z_{rec} \sim 1100$ i.e. when the Universe we can observe today was 1100 times smaller.

This size should be compared with another scale, the horizon, which is the distance traveled by light from $t=0$ till t_{rec}.

For standard HBB cosmology this second length scale is much smaller than the size of the Universe. The ratio is about 30 at recombination and can be as large as 10^{30} if we go back to $t = t_P \sim 10^{-43}$ sec (see picture).


$L_H = 10^{60} L_P$
time
now
here
$z = 1100$
$\sim 10^4$ disconnected regions
$z = 10^{30}$
L_P
big-bang
$t=0$
$10^{30} L_P$
space

By causality (finite c), primordial inhomogeneities can only be washed out over distances of the order of the horizon, while at recombination our Universe consisted of about 10^4-10^5 causally disconnected regions.

The puzzle is that the CMB temperature was(is) the same in each one of those causally disconnected region (directions).

Clearly, the reason why in the past the Universe was larger than the horizon is, again, that $w > -1/3$:

$$\frac{(a/a_0)}{(t/t_0)} = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}-1} = \left(\frac{t}{t_0}\right)^{-\frac{1+3w}{3(1+w)}}$$

3. Origin of large-scale structure (LSS)

The Universe, even if homogeneous on very large scales, has large- (and to an even larger extent small-) scale structures: clusters of galaxies, galaxies, stars, ...

In HBB cosmology there is no explanation for LSS. In order to explain today's structures one has to start with some tiny inhomogeneities to be put by hand on top of the FLRW Universe.

The HBB model tends to give either too much or too little LSS. Another huge fine-tuning problem.

Thus, if we accept that the Universe had a beginning, there appears to be only one way out (unless one accepts an incredible fine-tuning of the initial conditions):

Make the primordial Universe much smaller!

There is an alternative : make the Universe much older. But then we have to assume that t=0 was not the beginning of time, i.e. that the BB singularity is a classical artifact!

Cosmologia inflazionaria

The obvious solution: acceleration!

It is clear that an obvious solution to our puzzles is to insert a sufficiently long period of accelerated expansion. This is what is meant by "inflation". One demands:

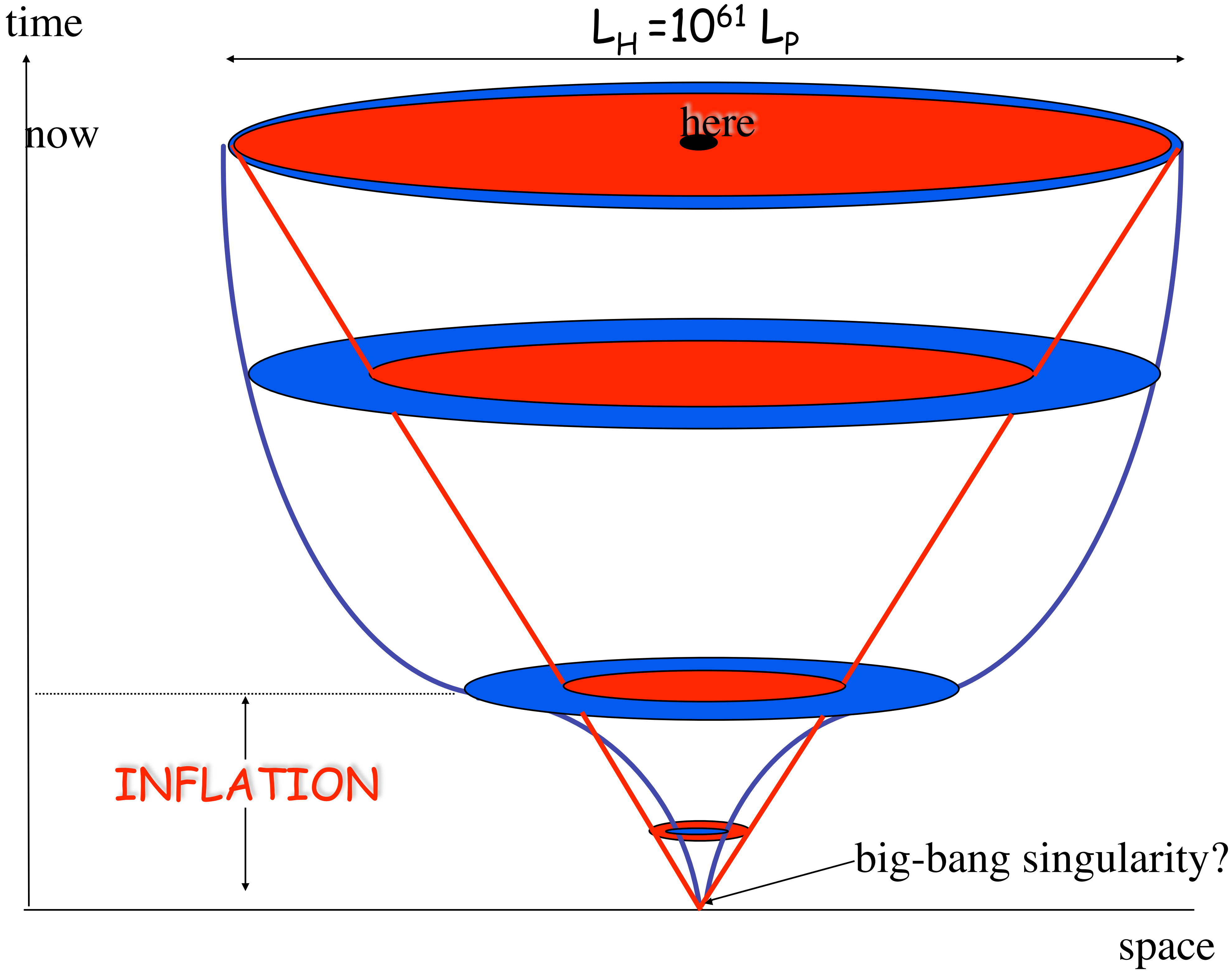
$$\frac{(a_f H_f)}{(a_i H_i)} = \frac{\dot{a}_f}{\dot{a}_i} \geq e^{N_{\min}}$$

If $N > N_{min} \sim 60$ inflation turns a generic initial Universe into a very (spatially) flat one since a^{-2} goes down faster than H^2.

Thus, $\Omega = 1$ is a generic prediction of inflation.

Also, initial inhomogeneities are stretched to scales larger than our present Horizon.

The homogeneity problem is also solved since, in the far past, our visible Universe was inside a single Hubble patch (picture).


time
$L_H = 10^{61}\ L_P$
now
here
INFLATION
big-bang singularity?
space

Inflationary models are actually agnostic about the Big Bang itself. They assume that whatever preceded inflation was kind enough to generate, at least in our patch, suitable initial conditions for the onset of inflation.

This is good news, in the sense that the predictions of inflation are insensitive to what physics preceded it.

It's also bad news since it prevents us from accessing that (probably very short-distance) physics.

Who provides the acceleration?

Ordinary matter, thanks to gravitational attraction, resists the expansion, decelerates it. In order to accelerate the expansion we need a "fluid" with $\rho + 3p < 0$ (negative enough pressure).

Quite amazingly it is relatively easy to "invent" such fluids. A positive cosmological constant is the simplest example (in fact was invented by Einstein for a similar purpose) but it's hard to get rid of. A more interesting choice is the potential energy of a nearly homogeneous and constant scalar field, called the inflaton. It has almost the same equation of state as a cosmological constant: $w \sim -1$ ($p \sim -\rho$).

At some point the inflaton starts changing rapidly in time and inflation stops. The inflaton's potential energy is dissipated, heating up the Universe (otherwise no CMB, no BBN!).

This is the actual (i.e. observable) BB according to inflation!

An analogy can be useful

Think of a waterfall. There is an enormous potential energy stocked upstream of the fall. There, the flow is slow and kinetic energy is small compared to potential energy

Eventually, however, the water gets to the edge of the fall. Suddenly, the potential energy is transformed in kinetic energy is dissipated into heat (or can be used to produce electricity).

Inflation's bonus: a quantum origin of LSS

One of the great bonuses of inflation is that, besides providing a mechanism for erasing initial inhomogeneities and spatial curvature, it can also generate a calculable (within a given inflationary model) amount of primordial perturbations.

As we shall discuss the reason for this "miracle" is quantum mechanics. This will be discussed next week.

In the next hour we shall instead address the question:

Does string theory support inflationary cosmology?