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La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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Stringhe in un fondo non banale
Azione efficace nello spazio-tempo

Strings in non-trivial backgrounds

We have already seen how to generalize the Polyakov action to a non-trivial space-time background metric $G_{\mu\nu}(x)$.

$$S_G = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\gamma^{\alpha\beta}(\xi)\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi)G_{\mu\nu}(X(\xi))$$

and also to a 2-form $B_{\mu\nu}=-B_{\nu\mu}$

$$S_B = -\frac{T}{2}\int d^2\xi\epsilon^{\alpha\beta}\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi)B_{\mu\nu}(X(\xi))$$

with $\varepsilon^{\alpha\beta}$ the Levi-Civita symbol in D=2.

Classically, the local D=2 symmetries are present for any $G_{\mu\nu}$ and $B_{\mu\nu}$.

Can we write anything else that satisfies, classically, the 2D local symmetries, and in particular Weyl invariance?

The only possibility appears to be:

$$S_\Phi = \frac{1}{4\pi}\int d^2\xi \sqrt{-\gamma} R(\gamma) \Phi(X(\xi))$$

but only if the field $\Phi(x)$, called the dilaton, is a constant.

In that case, as already discussed, the integral has a topological meaning:

$$\frac{1}{4\pi}\int d^2\xi \sqrt{-\gamma} R(\gamma) = 2(1-g)$$

It is related to the genus g of the Riemann surface described by the metric $\gamma_{\alpha\beta}$. Thus, if Φ is constant, $S_\Phi = 2\Phi(1-g)$; if it isn't, S_Φ is non-trivial and classically not Weyl-invariant.

We have to preserve the local WS symmetries at the quantum level. Maybe, for once, QM can help?

Let's then put all 3 terms together. The action for a string in a metric, antisymmetric and dilaton background becomes:

$$S = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\left[\partial_\alpha X^\mu \partial_\beta X^\nu \left(\gamma^{\alpha\beta}G_{\mu\nu} + \frac{\epsilon^{\alpha\beta}}{\sqrt{-\gamma}}B_{\mu\nu}\right) - \frac{1}{2\pi T}R(\gamma)\Phi\right]$$

N.B. In the quantum action S/h, only the combinations $G_{\mu\nu}/l_s^2$ and $B_{\mu\nu}/l_s^2$ (with dimensions length^{-2}) appear. This will provide useful checks below.

Now the Big Q:

Under what conditions for the background fields can we satisfy the conditions of 2D-rep. and Weyl invariance at the quantum level?

This is, in general, a highly non trivial problem. We know one solution: Minkowski spacetime, vanishing B, and constant Φ, provided D takes a critical value (D=26, 10).

This is the string we have been discussing so far with just one small additional point.

When the above action is inserted in Feynman's path integral it will weight the contribution of a Riemann surface of genus g by a factor exp(-2Φ(1-g)) hence by a factor exp(2Φ) for each extra string loop. Therefore, exp(2Φ) plays, in QST, the same role that α plays in QED (or in the SM): it is the (dimensionless) loop-counting parameter.

In order to look for more general solutions we have to resort to some kind of perturbation theory around the "trivial" backgrounds. The best (only?) known of these expansions is the one valid for slowly varying fields, the so-called α' expansion whose expansion parameter is:

$$l_s^2\ \partial^2 \sim l_s^2\ G^{\mu\nu}\partial_\mu\partial_\nu \sim \hat{G}^{\mu\nu}\partial_\mu\partial_\nu$$

A non trivial calculation (->2nd hour) leads to the following equations to leading non-trivial order in α' (l_s^2):

$$\beta^{\Phi} = \frac{D - D_c}{3} + l_s^2 \left(\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} D_\mu D^\mu \Phi - \frac{1}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) + O(l_s^4) = 0$$

$$\beta^{G}_{\mu\nu} = l_s^2 \left(R_{\mu\nu} + \frac{1}{4} H_{\mu\rho\sigma} H_{\nu}^{\ \rho\sigma} - 2 D_\mu D_\nu \Phi \right) + O(l_s^4) = 0$$

$$\beta^{B}_{\mu\nu} = l_s^2 \left(D^\rho H_{\mu\nu\rho} - 2 \partial^\rho \Phi H_{\mu\nu\rho} \right) + O(l_s^4) = 0 \quad ; \quad H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic}$$

We can now reinterpret the meaning of $D = D_c$.

If $D \neq D_c$, there is no solution to the above equations with nearly constant backgrounds. All solutions must necessarily involve fields whose space-time variations are so large to compensate for the extra factor l_s^2.

However, in that case, we cannot neglect higher-order corrections (e.g. $O(l_s^4)$) and be sure that we do have a solution. There is fortunately an exception.

The linear-dilaton solution

Take a Minkowskian background with $D \neq D_c$, $B=0$, and $\Phi = Q_\mu X^\mu$ where Q_μ is a constant (space or time like) vector.

At the order we are computing we find that all the β-functions are zero provided we take:

$$Q_\mu Q^\mu = \frac{D_c - D}{3 l_s^2}$$

This shows how a classically non-Weyl-invariant term in the action can be used to give back Weyl-invariance at the quantum level!

In fact this solution turns out to be exact (at $g=0$ level) since a linear dilaton keeps the action quadratic. Necessarily, however, the effective coupling of string theory grows large (either at space- or at time-like infinity) and one has to worry about loop corrections.

The effective action of QST

A very interesting property of our (β-function) equations is that they are (functional)derivatives of a function(al).

That means that they correspond to the e.o.m. that follow from an "effective" action Γ_{eff}

Up to the order we have considered we find Γ_{eff} to be:

$$\Gamma_{eff} = l_s^{2-D} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D_c - D)}{3 l_s^2} - R(G) + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{12} H^2 + \ldots \right]$$

1. For the dots, see below.
2. The dilaton appears with the "wrong" sign, but there is nothing wrong with this (-> 2nd hour).
3. (D_c-D) plays the role of a cosmological constant

Other interesting properties of Γ_{eff}

1. The dilaton appears through an overall factor multiplying something that can only depend on its derivatives. This is as expected since, if Φ is constant, the only dependence on Φ must be through an overall factor $\exp(-2\Phi(1-g))$.
2. Γ_{eff} contains no arbitrary dimensionless parameters and just a single dimensionful one, l_s. Actually, even l_s can be eliminated if one uses, instead of G, B, Φ, the rescaled fields $l_s^{-2} G$, $l_s^{-2} B$, Φ. Again, this is as expected.
3. Γ_{eff} is general covariant. It is also invariant under $B \to B + d\Lambda$. B only enters through its field strength $H = dB$. We'll come back next week to the stringy symmetries of Γ_{eff}.

The two meanings of Γ_{eff}

The effective action has two distinct meanings. The first is the one we have just said: it generates (as its own e.o.m.) the conditions to be satisfied by the background fields in order to preserve the 2D local symmetries of string theory at the quantum level.

The second meaning is a more familiar one for an effective action: Γ_{eff} can be used to compute classical solutions, ground states, the couplings of various massless particles and their scattering amplitudes as an expansion in powers of energy.

It is fascinating that these two (apparently unrelated) concepts get interconnected in string theory. How come?

(-> 2nd hour for a hint)

A theory of gravity but not Einstein's!

In D dimensions, the analogue of the Einstein-Hilbert action (w/ a cosmological constant) takes the form:

$$\frac{1}{\hbar}S_{EH} = \left(\frac{1}{l_P}\right)^{D-2}\int d^Dx\sqrt{-g(x)}\left(\Lambda - \frac{1}{2}R(g)\right) \quad ; \quad 8\pi G_N\hbar \equiv l_P^{D-2}$$

while in QST we found:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{12}H^2 + \dots\right]$$

Besides H, are they equivalent up to some field redefinition? The answer is obviously no: even if we set H=0, QST gives a scalar-tensor theory of a Jordan-Brans-Dicke kind!

A massless dilaton induces long-range interactions that, in general, violate the equivalence principle: the dilaton, having spin zero, couples (non universally!) to mass rather than to energy and produces violations of UFF. An $m > 10^{-4}$ eV needed to be safe.

This is a real threat to QST, making it vulnerable even to long-distance/low-energy experiments.

In fact, at tree-level, string theory is already completely ruled out by present precision tests of the EP

(cf. JBD theory with a small ω).

Hopefully cured by loop and/or non-perturbative corrections

Unification of gravity and gauge interactions

For an Einstein-Maxwell theory in 10D:

$$\frac{1}{\hbar}S_{EH} = \left(\frac{1}{l_{10}}\right)^8 \int d^{10}x\sqrt{-g_{10}(x)}\left(\Lambda - \frac{1}{2}R_{10}(g)\right) + \frac{1}{4\alpha_{10}}\int d^{10}x\sqrt{-g_{10}(x)}F^2$$

$$8\pi G_{10}\hbar \equiv l_{10}^8$$

while in QST we find:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^8 \int d^{10}x\sqrt{-G_{10}}e^{-2\Phi}\left[R_{10}(G) - 4\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{12}H^2 - \frac{1}{4}l_s^2F_{10}^2 + \ldots\right]$$

For a constant Φ we can identify:

$\Lambda = 0$, $l_{10}^8 = \exp(2\Phi)l_s^8$, $\alpha_D = \exp(2\Phi)l_s^6$ => $l_{10}^8 = \alpha_D\, l_s^2$,

The latter eq. unifies gravity and gauge interactions!

What happens if we now compactify the extra dimensions?

(-> 2nd hour)

The two expansions of Γ_{eff}

We have (roughly) seen how quantization of the string produces potential 2D anomalies that have a natural expansion in powers of l_s.

We have also seen that integrating over the 2D metric produces another expansion, in powers of exp(2Φ).

Therefore Γ_{eff} has a double perturbative expansion:

$$\begin{aligned}\Gamma_{eff} &= -\left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2}+R(G)-4\partial_\mu\Phi\partial^\mu\Phi+\frac{1}{12}H^2+O(l_s^2)\right]\\ &+ \left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}\,[\ldots]+O(e^{2\Phi})\end{aligned}$$

The expansion in powers of exp(2Φ) has a QFT analogue. The one in powers of l_s does not since the point-particle limit corresponds to l_s =0. The short-distance modifications due to $l_s \neq 0$ remove the UV infinities of the QFT-like expansion!

These latter modifications make loop corrections well defined in the UV. Indeed, one gets their correct order of magnitude, exp(2Φ), by computing loops as in a QFT but with a short distance cutoff given by the string length.

Here is a prototypical quantum-gravity loop correction:

$$\left(\frac{\text{loop}}{\text{tree}}\right) \sim G_N \Lambda_{UV}^{D-2} \rightarrow \left(\frac{l_P}{l_s}\right)^{D-2} = exp(2\Phi)$$

which is of the same order as a gauge-loop correction. Unification of gravitational and gauge interactions at $E \sim M_s$ survives loop corrections…

But, as we move towards the IR, loops behave differently, as in QFT…