Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

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T-dualità per stringhe aperte: D-brane
Loops in QFT e QST

Dalla settimana scorsa

The total number of massless scalars is 10. Leaving a singlet (the dilaton) aside, they form a (3,3) representation of SU(2)xSU(2).

The radion plays the role of a Higgs field that breaks spontaneously SU(2)xSU(2) down to U(1)xU(1) away from the special point R = R*.

4 scalari devono sparire (essere "mangiati") per fornire la terza polarizzazione di 4 bosoni con massa.

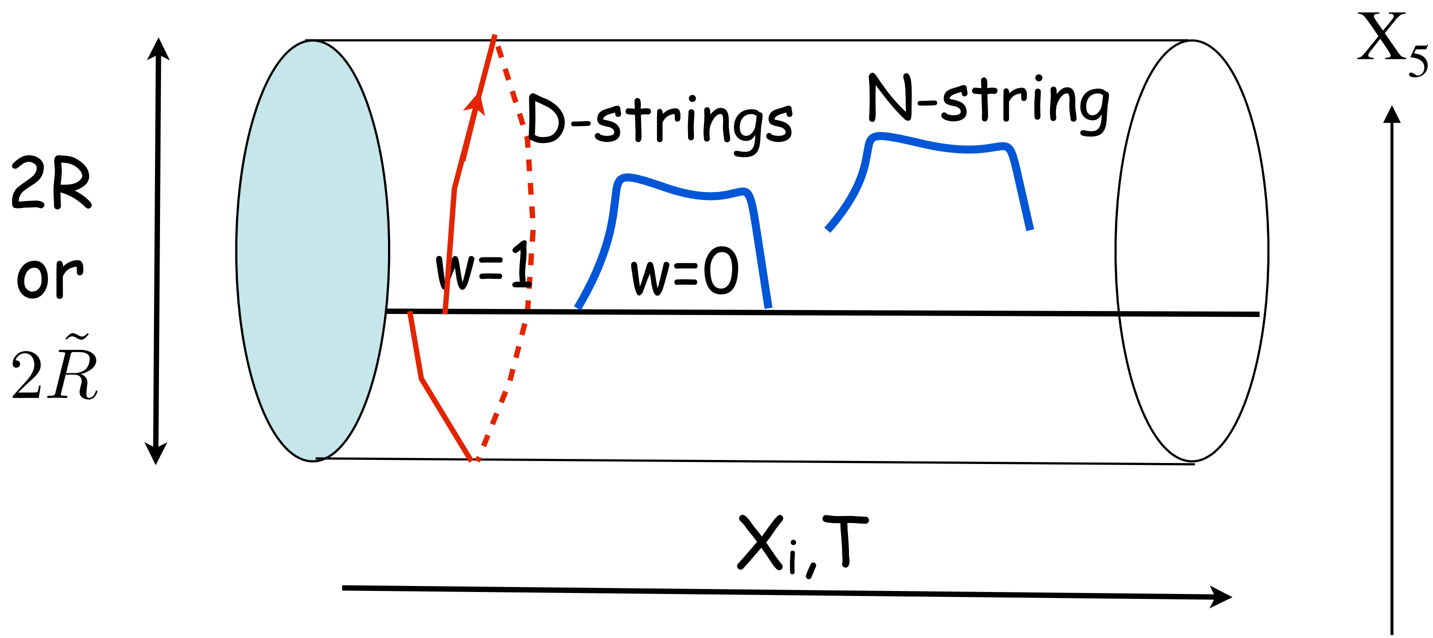
Questi si possono identificare nei due doppietti che seguono dalla decomposizione: 3 -> 0, ± 1 rispetto agli U(1) non rotti.

Complementi tecnici

1. D-stringhe: soluzioni e stati a massa nulla
2. Loops in QFT con integrale sui cammini
3. Loops in QST dall'integrale su $\gamma_{\alpha\beta}$
4. Invarianza modulare e finitezza ultravioletta

D-stringhe, D-brane

NB: T-dual N and D-strings move/wind around dual circles!

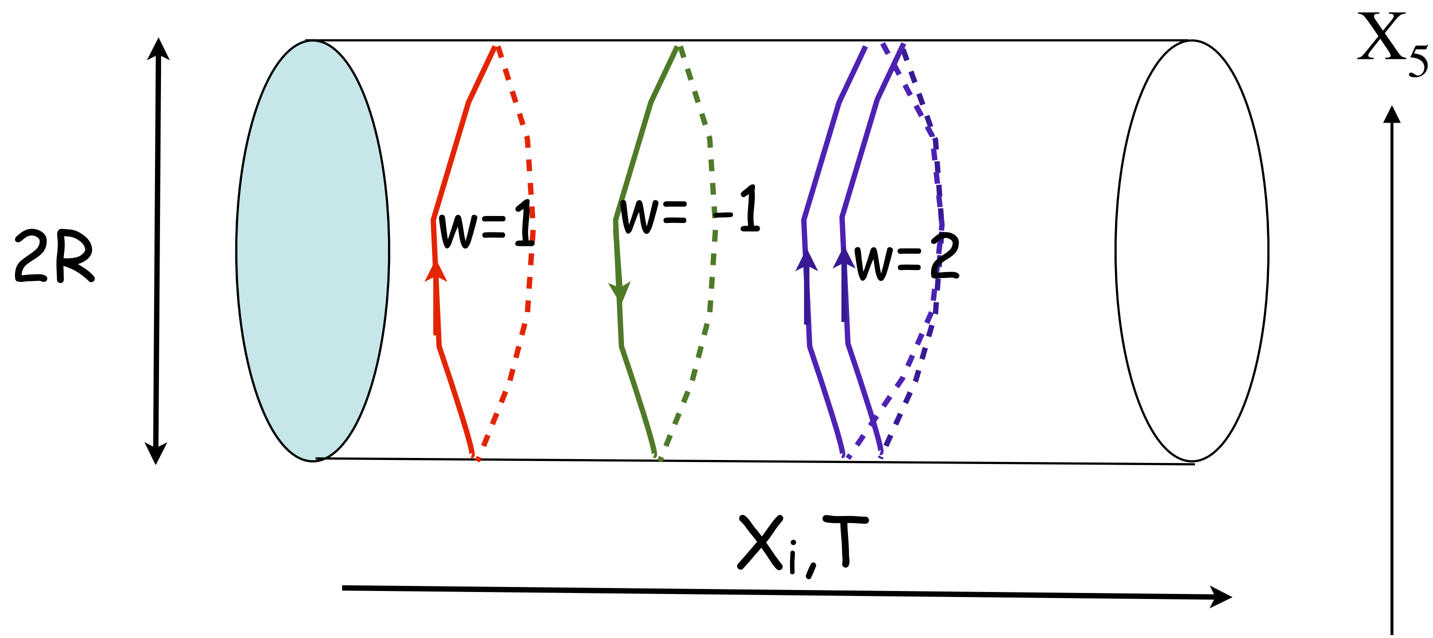


$$X_5^{(N)}(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] cos(n\sigma)$$

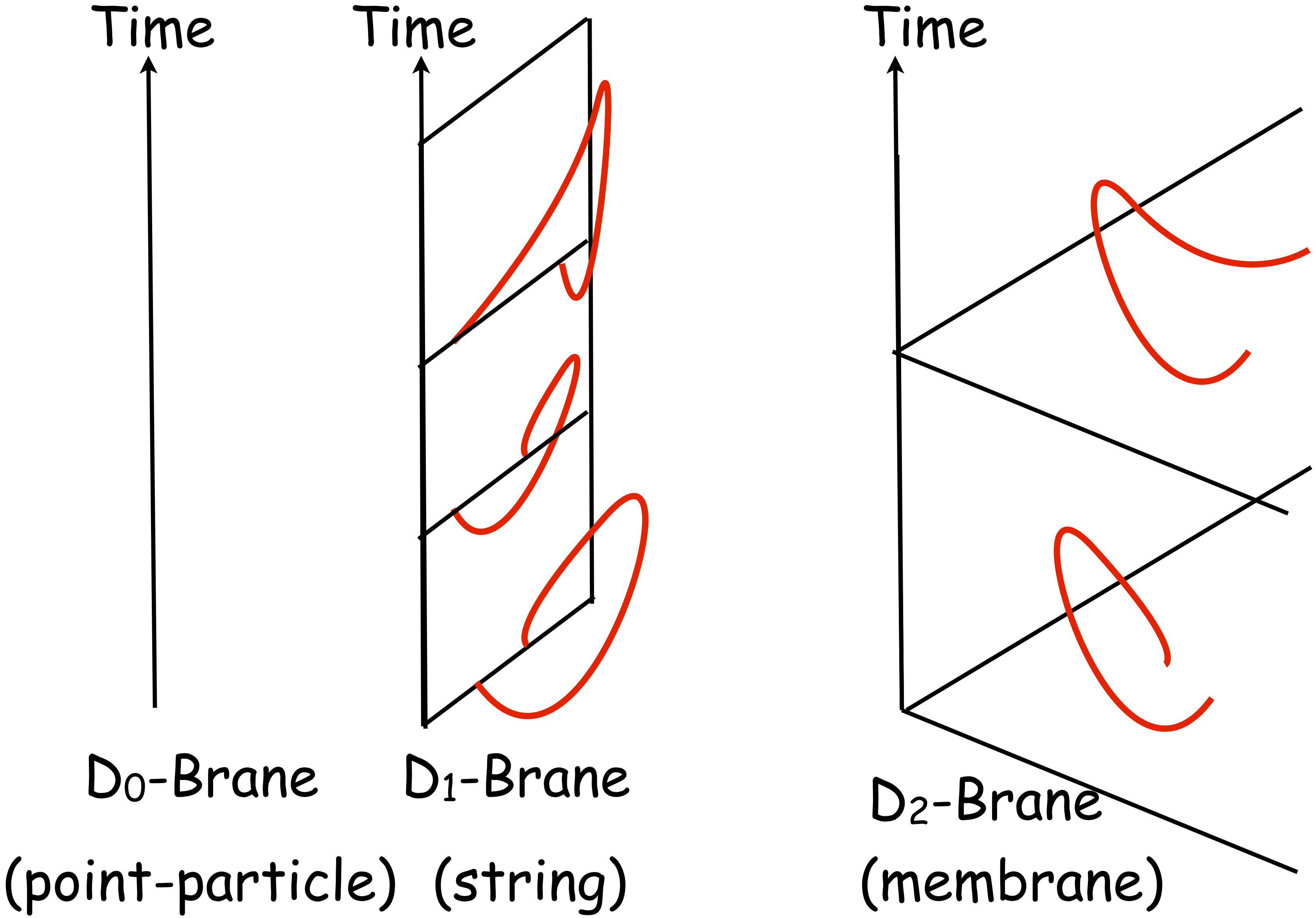
$$X_5^{(D)}(\sigma,\tau) = q_5 + 2w\tilde{R}\sigma + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] sin(n\sigma)$$

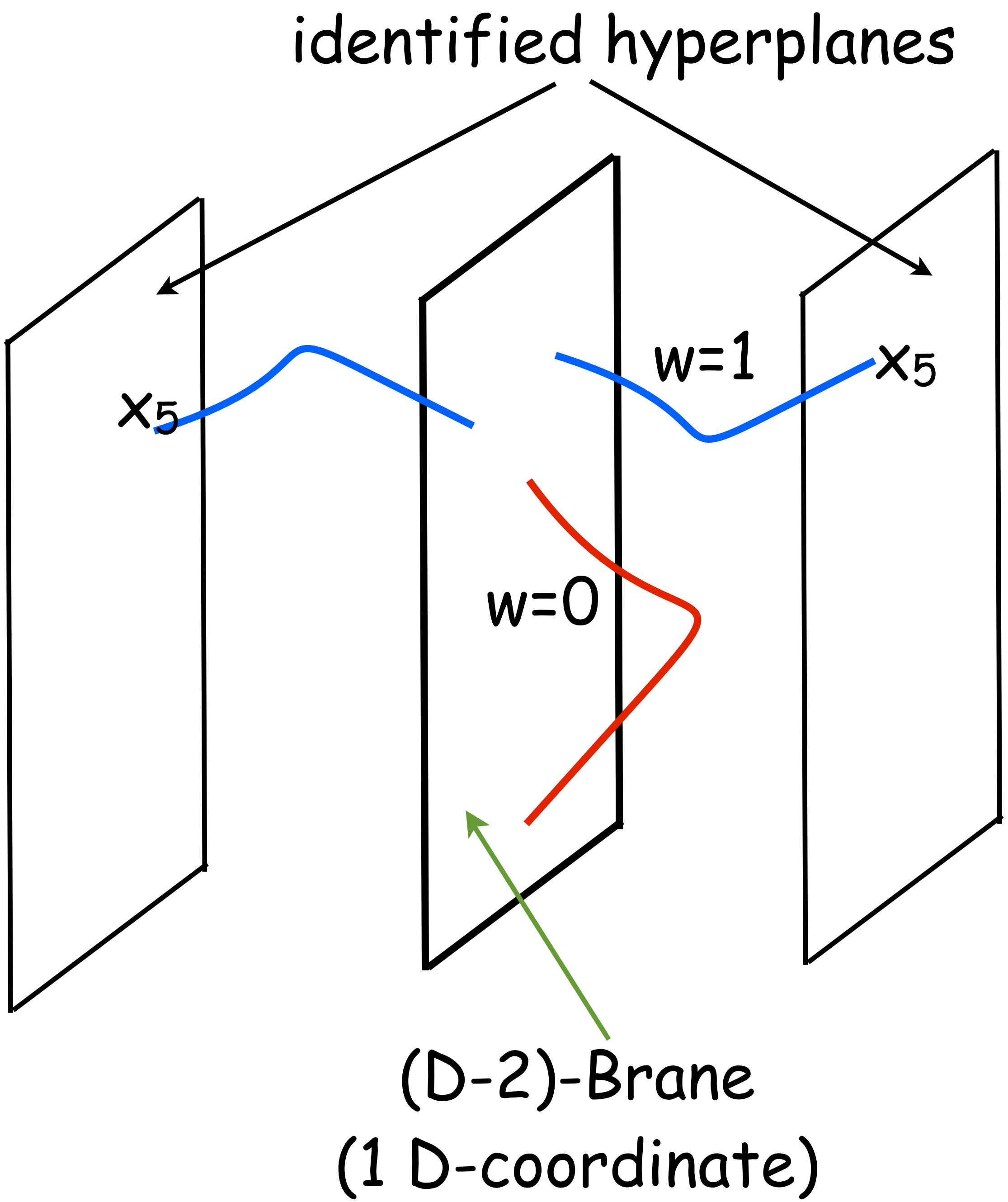
$$X_5^{(D)}(\sigma=\pi,\tau) = X_5^{(D)}(\sigma=0,\tau) + 2\pi\tilde{R}\; w$$

to be compared with the closed string case:



$$X_5(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + 2wR\sigma$$
$$+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^\dagger_{n,5}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right]$$
$$+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{\tilde{a}_{n,5}}{\sqrt{n}}e^{-2in(\tau+\sigma)} - \frac{\tilde{a}^\dagger_{n,5}}{\sqrt{n}}e^{2in(\tau+\sigma)}\right]$$


Time
Time
Time
D_0-Brane
D_1-Brane
D_2-Brane
(point-particle)
(string)
(membrane)


identified hyperplanes
x5
w=1
x5
w=0
(D-2)-Brane
(1 D-coordinate)

For the open bosonic string the mass shell condition reads:

$$L_0 = 1 \Rightarrow M^2 = \frac{\hbar^2 n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{1}{\alpha'}(N-1) \qquad N = \sum_{n,\mu} n \; a^\dagger_{n,\mu} a_{n,\mu}$$

where w=0 for the N-case and n=0 for D. For generic R the massless states are given by n=w=0, N=1, i.e. by the states $a^\dagger_{1\mu}|0\rangle$. Let us concentrate on the Dirichlet case.

If the index of the oscillator is not 5 this is a gauge boson stuck on the brane (in (D-1)-dimensions with (D-3) physical components); if the index is 5 it's a massless scalar also confined to the brane. What's the meaning of this scalar?

The answer is quite simple: the presence of the brane clearly breaks (spontaneously) translation invariance in the 5th direction. The massless scalar is the Nambu-Goldstone boson of that broken symmetry and describes the possible local deformations of the brane itself!

When there are many branes we can construct many such scalars that correspond to the inter-brane separation and play the role of Higgs bosons for the breaking of the gauge symmetry U(N) down to a subgroup:

$$U(N_1)\times U(N_2)\times U(N_3)\ldots\ U(N_n)$$

(depending on how many branes are still coincident)

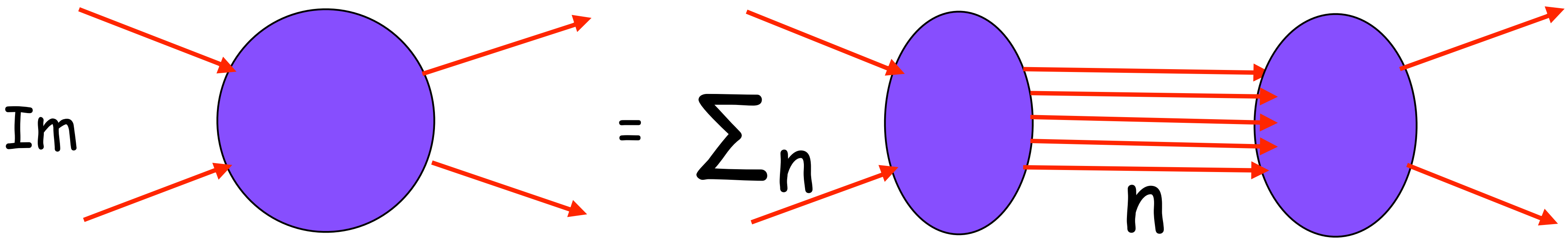
Loops in QFT e QST

Loops in QFT

In QFT loops come out naturally from its formalism
Physically, loops are needed to ensure unitarity of the S-matrix. Writing S=1 +iT unitarity gives:

$$i(T^\dagger - T) = 2ImT = T^\dagger T$$

In pictures:



Even if the blobs on the rhs are tree diagrams this equation gives loop diagrams for its lhs. Unitarity is implemented order by order in perturbation theory through Cutkowski's cutting rules for Feynman's diagrams.

Loops also follow from Feynman's path integral formalism. Schematically, if ϕ_{cl} is a classical solution of the field equations, the path (functional) integral can be expanded:

$$
\begin{aligned}
& \int d[\phi(x)] exp(-\frac{1}{\hbar} S(\phi)) \sim \\
& exp(-\frac{1}{\hbar} S(\phi_{cl})) \int d[\phi(x) - \phi_{cl}(x)] exp \left(-\frac{1}{2\hbar} S''(\phi_{cl})(\phi - \phi_{cl})^2 \right) \\
= \; & exp(-\frac{1}{\hbar} S(\phi_{cl})) \left(det S''(\phi_{cl}) \right)^{-1/2} = exp \left(-\frac{1}{\hbar} S(\phi_{cl}) - \frac{1}{2} tr[log S''(\phi_{cl})] \right)
\end{aligned}
$$

The trlog(...) is h-independent and represents a one-loop correction to the semiclassical approximation.

How do loops appear in string theory? What is the analog of Feynman's path integral in ST?

The quantum fields are NOT some fields in space-time, but the string coordinates X^μ (and ψ^μ in the superstring) and the 2D metric $\gamma_{\alpha\beta}$.

Fortunately, it turns out that in QST, at least in perturbation theory, one can introduce the equivalent of QFT's loops while staying all the time within 1st quantization.

This amounts to working with a finite number of quantum fields in D=2, an immense simplification.

But how can loops emerge for 1st quantization?

This looks impossible at first sight.

Consider a Feynman path integral approach to string quantization starting from the "Polyakov" action:

$$Z \sim \int .. \int [d\gamma_{\alpha\beta}(\xi)][dX^\mu(\xi)][d\psi^\mu(\xi)]exp(-S_P)$$

$$S_P = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\gamma^{\alpha\beta}(\xi)\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi)G_{\mu\nu}(X(\xi)) + \ldots$$

Let's concentrate again on the integral over the 2-metric $\gamma_{\alpha\beta}$.

At first sight such an integral should be trivial since 2D reparametrization plus Weyl invariance should allow to gauge-fix completely $\gamma_{\alpha\beta}$.

This statement is certainly true locally but there is a "global obstruction".

A (well-known?) theorem states that :

$$\frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma) = 2(1-g)$$

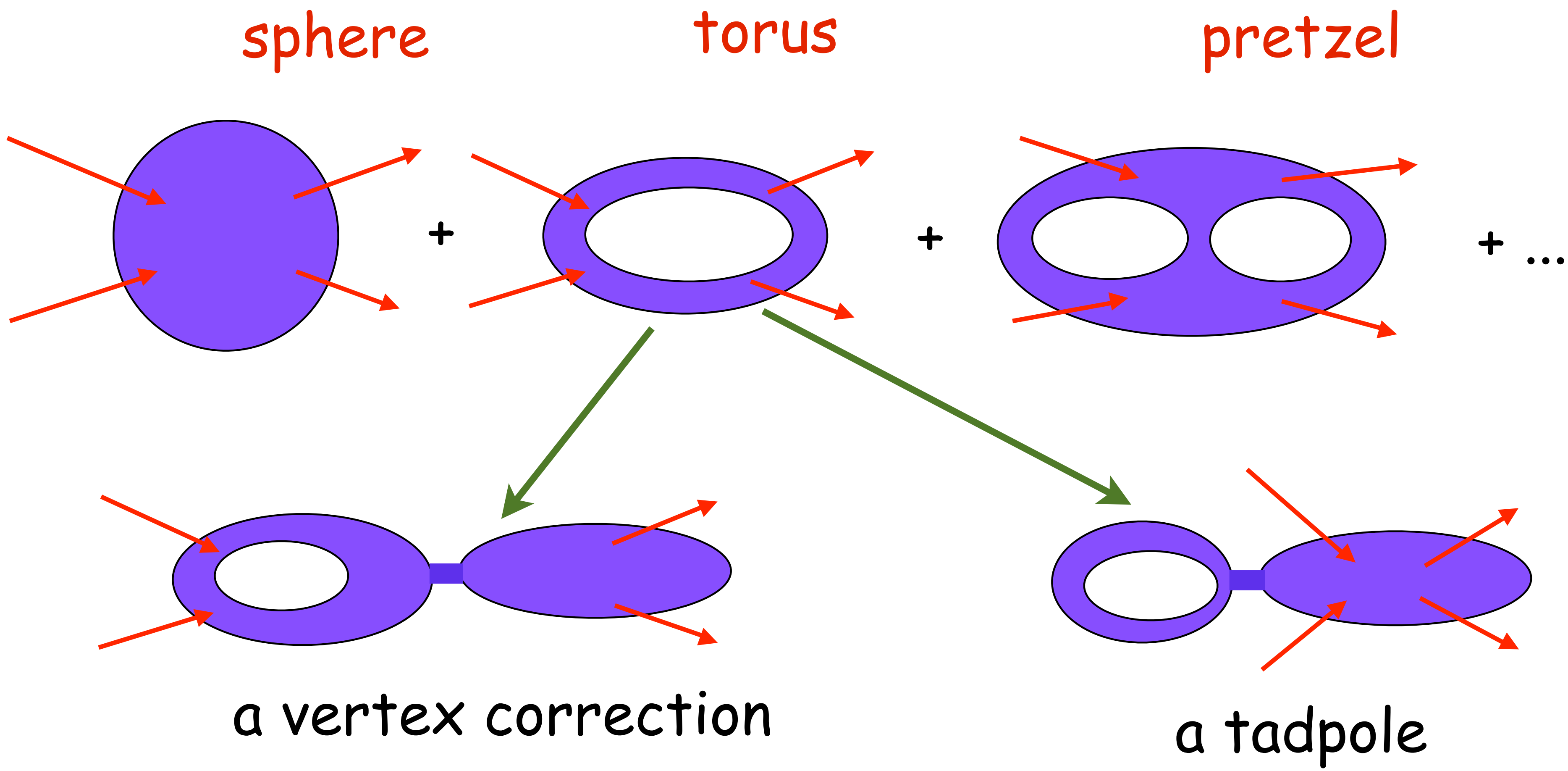
where g is the genus of the 2D Riemann surface (g=0 for the sphere, g =1 for the torus, etc.) whose geometry is given by $\gamma_{\alpha\beta}$. Fixing globally $\gamma_{\alpha\beta}$ would mean fixing g!

But why should one fix g rather than summing over it? In other words, the functional integral over the 2D metric naturally splits into a sum of functional integrals each representing Riemann surfaces of a given genus g.

Precisely this sum over g corresponds to the loop expansion in QFT! QST can introduce QFT's loops without invoking any 2nd quantization!

There is even an extra bonus: while in QFT the number of diagrams grows like a factorial of the loop order, here there is just one diagram at each order. It's DHS duality at work...

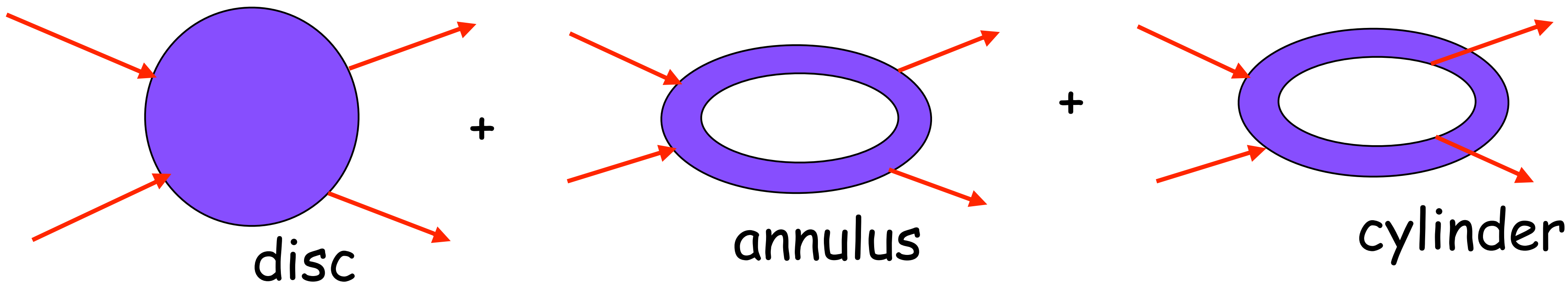
Loop expansion for closed string collisions



Closed strings attach at points on the Riemann surface. These are just our good old Koba-Nielsen variables z_i (complex numbers for closed strings) on which one has to integrate.

Open strings instead attach to boundaries of the Riemann surface, the analogue of quark loops in QCD. The sum over topologies includes a sum over different "boundaries", their total number, but also over which have strings attached to them and which do not, etc.

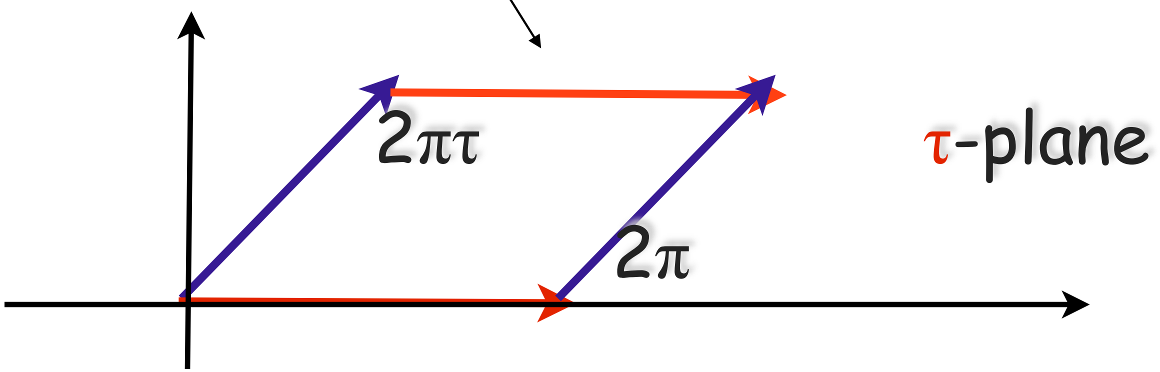
The tree level corresponds now to the disc. At one loop we find the annulus, the cylinder, the Moebius strip. One can then also add "handles" (increasing the genus) as for closed strings.



The positions at which the open strings are attached are real, ordered Koba-Nielsen variables on which one has to integrate.

For a given genus of the Riemann surface, one has now to find out what the (ordinary) integration variables are. The result:

1. For the sphere (and the disc for open strings) there is a residual invariance under projective $O(2,1)$ transformations that allows to fix 3 KN coordinates (exactly as in the DRM!).
2. For $g=1$ (torus) there is still one integration over the complex parameter τ (not to be confused with WS time) that characterizes each torus.
3. For $g>1$ there is an integration over $3(g-1)$ complex parameters that characterize the Riemann surface.



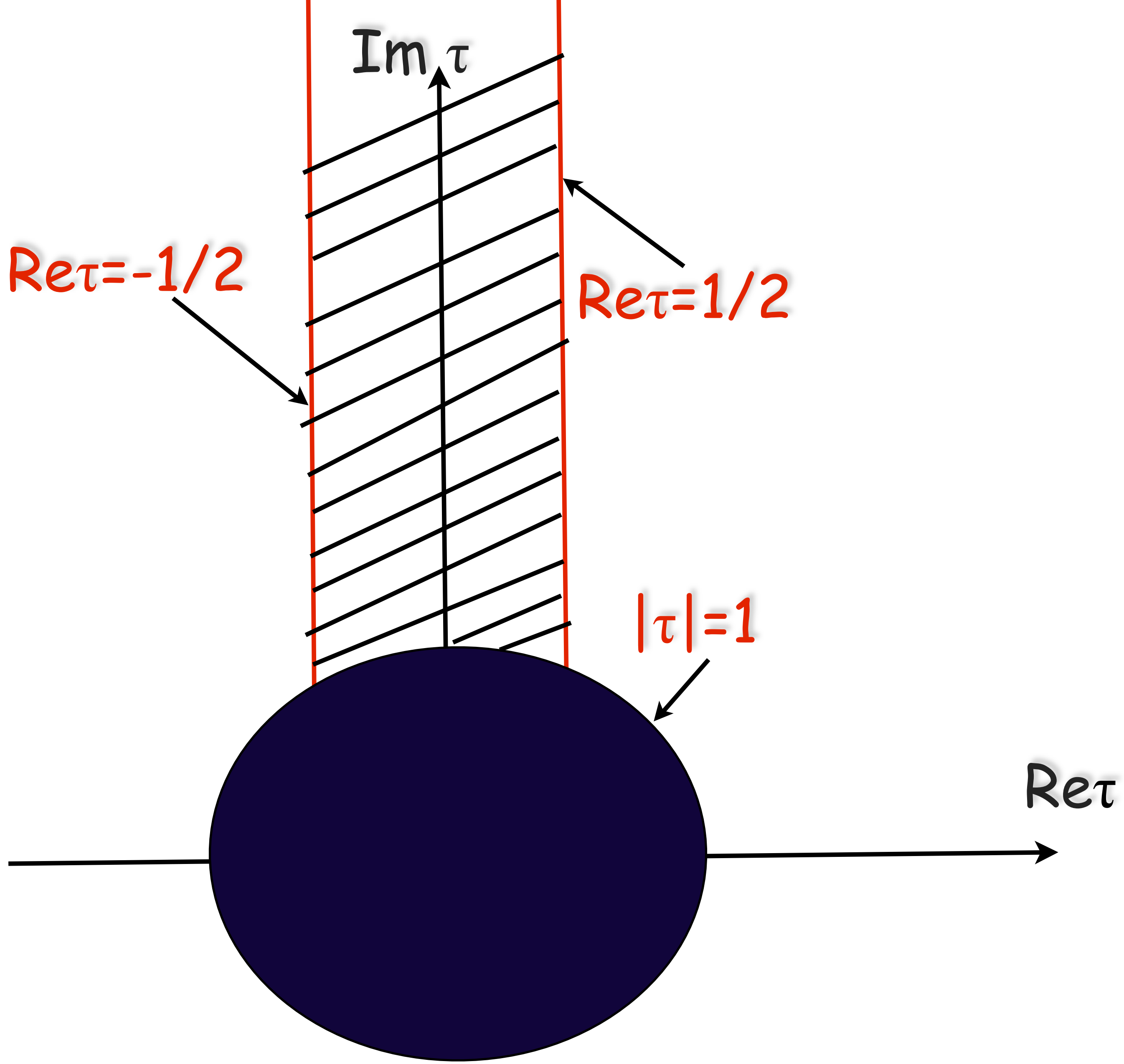
Modular Invariance

Coming back to the torus, there is still a discrete group of transformations that leaves the torus invariant. This is the group of modular transformations:

$$\tau \rightarrow \frac{p\ \tau + q}{r\ \tau + s} \quad ; \quad p, q, r, s \in Z \quad ; \quad ps - qr = 1$$

It maps the same torus in the complex-τ plane an infinite number of times leading to an infinite result if we were to integrate over the whole complex plane. We should only take one region e.g. the so-called fundamental region. This nicely avoids the point $\tau = 0$, associated with the UV region.

This is the (mathematical) way by which string theory avoids UV infinities! It amounts to a cut-off of order $M_s = l_s^{-1}$.



Fundamental region for the torus (shaded)

Perturbative unitarity can be checked when loops are added with a specific weight that depends on a single, dimensionless quantity, the so-called string coupling g_s (the analog of α in QED, to be better interpreted next week).

Modular invariance is as essential for the consistency of string theory as Weyl and reparametrization invariance.

As it turns out, imposing modular invariance at the one-loop level eliminates the gauge and gravitational anomalies (also one-loop effects!) that the GS mechanism cancels by a brute-force calculation (see below).

The search for consistent QSTs is therefore reduced to the problem of finding theories that respect modular invariance.

This is how the two consistent heterotic string theories HetO (gauge group SO(32)), HetE ($E_8 x E_8$), were found…

The Green-Schwarz 1984 breakthrough

- Remember: the SM has chiral fermions but the matter content is such that all gauge anomalies cancel.
- It looked that, instead, string theory either had anomalies (of the QFT type) or could not have chiral fermions.
- Until 1984 it "almost" looked like a no-go theorem.
- But, in the summer of 1984, Green and Schwarz found the exception to the rule: TypeI superstrings allowed for a new kind of anomaly cancellation iff the gauge group was just SO(32)!
- This "1st revolution" relaunched string theory!