Cattedra Enrico Fermi 2015-2016

La teoria delle stringhe:
l'ultima rivoluzione in fisica?

Gabriele Veneziano

Lezione # 10.2: 14.04.2016

Simmetrie nello spazio-tempo:
T-dualità per stringhe chiuse

Complementi tecnici

1. Transformazioni stringhesce di coordinate
2. KK per l'azione di EH
3. KK per l'azione della particella puntiforme.
4. KK per l'azione della stringa
5. Spettro a massa nulla a R = R*

Ci si può domandare, a questo punto, se le trasformazioni di coordinate della RG esauriscano o meno le simmetrie spazio-temporali della teoria.

L'enorme degenerazione degli stati della stringa suggerirebbe un gruppo di simmetria altrettanto enorme che va ben oltre quello della RG.

Questo problema è ancora irrisolto.

Consideriamo, ad esempio, la TGC "stringhesca":

$$X^\mu \to X^\mu + \xi^\mu_{\rho\sigma}(X)\gamma^{\alpha\beta}\partial_\alpha X^\rho \partial_\beta X^\sigma$$

una trasformazione in uno spazio di stringhe e non di punti…

È questa una simmetria della stringa?

Purtroppo non è possibile verificarlo con calcoli di Matrice-S.

KK in QFT

(da Einstein-Hilbert a Einstein-Maxwell)

In the original KK theory the extra dimension of space is a circle of radius R. The e.m. potential A_μ becomes, essentially, the $g_{\mu 5}$ component of the 5-d metric, while g_{55} plays the role of a scalar field associated with the proper size of the circle (the "radion"). More precisely, by writing the 5d-metric as:

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\sigma} (dx^5 + A_\mu dx^\mu)^2$$

we find

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5}\ R_5 \Rightarrow \frac{\pi R}{\kappa_5^2} \int d^4x \sqrt{-g_4}\ e^\sigma \left[R_4 - \frac{e^{2\sigma}}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

giving ($F = R\, F_{can}$)

$$l_P^2 \equiv 8\pi G = \frac{l_5^3}{2\pi\rho}\ ;\ \alpha_4 = \frac{l_P^2}{\rho^2}\ ;\ \rho \equiv e^\sigma R\ ;\ \kappa_5^2 = l_5^3\ ;\ (\hbar = 1)$$

KK for point-particle lagrangian/ hamiltonian

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\sigma}(dx^5 + A_\mu dx^\mu)^2$$

(A = R A_{can})

$$S_5 = -\frac{1}{2}\int d\tau \left(e^{-1}\dot{x}^a \dot{x}^b g_{ab}(x) + e\; m_5^2\right)$$

$$S_4 = -\frac{1}{2}\int d\tau \left(e^{-1}\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x) + e\; m_4^2\right) - q\int d\tau\; \dot{x}^\mu A_\mu(x)$$

KK for string lagrangian/hamiltonian

$$ds_5^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dx^5 + A_\mu dx^\mu)^2$$

$$B_{\mu 5} = -B_{5\mu} = \tilde{A}_\mu \; ; \; B_{55} = 0$$

$$S_5 = -\frac{1}{2}\int d\sigma d\tau (\dot{X}^a \dot{X}^b - X'^a X'^b) G_{ab}(x) + (\dot{X}^a X'^b - \dot{X}^b X'^a) B_{ab}$$

So far R was generic. Something quite remarkable happens, however, if we choose a particular value for R:

$$R = R^* \equiv \sqrt{\hbar\alpha'} = \frac{l_s}{\sqrt{2}}$$

In this case there are ways of getting massless strings on top of those we had for generic R:

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2)\ ;\ N - \tilde{N} + nw = 0$$

$$n = w = \pm 1\ ;\ N = 0, \tilde{N} = 1 \quad \text{or}$$

$$n = -w = \pm 1\ ;\ N = 1, \tilde{N} = 0$$

These are 4 massless vectors, two left and two right-moving. Together with the 2 previous ones they form the 6 gauge bosons of an SU(2)xSU(2) gauge group w/ the two factors corresponding to left and right-moving states. Note that the 4 new gauge bosons carry themselves momentum and winding and are therefore themselves charged, a characteristic of non-abelian gauge theories.

The above solutions also provide 4 massless scalars (when we take the oscillator in the 5th direction).

Actually there are 4 more massless scalars corresponding to taking the oscillator vacuum and

(n = ± 2, w=0) or (n =0, w= ± 2).

The total number of massless scalars is thus 10. Leaving a singlet (the dilaton) aside, they form a (3,3) representation of SU(2)xSU(2). The radion corresponds to a particular direction in both SU(2)'s and plays the role of a Higgs field that breaks spontaneously SU(2)xSU(2) down to U(1)xU(1) away from the special point R = R*.

The mass of the gauge bosons corresponding to the broken generators is linear in (R-R*).