On non-geometric string vacua

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The ongoing thesis is devoted to the study of non-geometric structures in string theory and their phenomenological role in constructing new semi-realistic viable vacua.

1 Getting to the heart of the problem: string theory and the real world

The long-standing conundrum of string theory, as a consistent theory of quantum gravity and other fields, is to mimic 'microphysically' our world deriving all its particle and cosmological expected (and theoretically demanded) physical properties (e.g. a positive tiny cosmological costant, the standard model gauge group and fields, a low-energy supersymmetry scale breaking, an inflationary vacuum).
Fundamental (perturbative) string theories are formulated consistently in ten dimensions and in Minkowski space-time, with a large number of supersymmetry (SUSY) charges. These models are five in all, including type IIA, IIB theories (with 32 supercharges or $\mathcal{N} = 2$ SUSY) and type I, Heterotic $SO(32), E_8 \times E_8$ theories (with 16 supercharges or $\mathcal{N} = 1$ SUSY).
In particular, the massless bosonic spectra of type II theories are divided into (Neveu-Schwarz-Neveu-Schwarz) NS-NS and (Ramond-Ramond) RR sectors, respectively:

$$\text{IIA} \rightarrow \text{NS-NS} : g_{\mu\nu}, B_{\mu\nu}, \phi, \quad \text{RR} : C_\mu, C_{\mu\nu\rho},$$

$$\text{IIB} \rightarrow \text{NS-NS} : g_{\mu\nu}, B_{\mu\nu}, \phi, \quad \text{RR} : C, C_{\mu\nu}, C_{\mu\nu\rho\sigma}.$$

The RR sectors contain only form fields. The NS-NS sectors include the graviton $g_{\mu\nu}$, a 2-form $B_{\mu\nu}$ (so-called Kalb-Ramond field) and a scalar ϕ (dilaton). The forms couple electrically (and magnetically) to higher-dimensional objects known as branes. In particular, the RR fields couple to so-called *D-branes* (D is after Dirichlet) which are microscopically introduced as the endpoints of the open strings and, among other things, have proved to be good candidates to trap the Standard Model fields in four dimensions [**1**].
In order to make contact with four-dimensional physics one needs to take into account for compactifications on (topologically non-trivial) internal six-dimensional tiny-size manifolds breaking part of the supersymmetries (i.e. complex *Calabi-Yau* 3-folds or singular limits of them, known as *orbifolds*).
In this sense, many efforts during the last decade have been aiming to address the problem of *moduli stabilisation*, i.e. to furnish a dynamical physical explanation to the many massless fields (moduli) propagating in the four-dimensional space-time and parameterising, among other things, the geometry (rough, shape and size) of the internal compact spaces. The opportun solution to the underlying problem string theorists invented is known as *flux compactification* and consists in turning on 3-form field strengths of the gauge fields of the massless spectrum of the theory on the compact space in order to generate a four-dimensional potential giving mass to the moduli fields [**2**].
The scalar potential of the $\mathcal{N}=1$, $D=4$ low-energy effective theory is specified by two fundamental ingredients, namely the Kähler potential K and the superpotential W. The first one depends only on the moduli of the compactification, the latter one is holomorphic in the fields and is given by the celebrated Gukov-Vafa-Witten (GVW) formula:

$$W = \int_{CY} G_3 \wedge \Omega. \tag{1}$$

Here the IIB 3-forms $H_3 = dB_2$ and $F_3 = dC_2$ have been put together to define the complex 3-form $G_3 = F_3 - \tau H_3$, with $\tau = C + ie^{-\phi}$.[1] Also, the 3-form fluxes threading non-trivial γ_3 CY 3-cycles have to satisfy the following Dirac quantisation conditions:

$$\frac{1}{(2\pi)^2\alpha'}\int_{\gamma_3} F_3 = m_{\gamma_3} \in \mathbb{Z}, \qquad \frac{1}{(2\pi)^2\alpha'}\int_{\gamma_3} H_3 = n_{\gamma_3} \in \mathbb{Z}. \tag{2}$$

Of course, we don't forget, as integrability conditions, the *Bianchi identities* (BI) $dH_3 = dF_3 = 0$.
The $\mathcal{N}=1$, $D=4$ scalar potential V is found to be

$$V = e^K (G^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2), \tag{3}$$

with $G_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$, $D_a \equiv \partial_a + \partial_a K$ and a, b denoting the (complex) moduli fields (the bar means complex conjugation).

[1] Ω is the holomorphic 3-form always allowed on the CY.

It turns out that not all the moduli fields can be stabilised at tree level. And indeed, it's possible to show that the so-called Kähler moduli ρ decouple from (3) and the potential becomes:

$$V = e^K \sum_{a,b\neq\rho} (G^{a\bar{b}} D_a W D_{\bar{b}} \bar{W}). \tag{4}$$

In order to stabilise all the moduli fields perturbative and non-perturbative corrections to the scalar potential have been considered in recent now famous scenarios, i.e. KKLT, LVS [**3**].
At the minimum of the scalar potential one has to impose the following conditions on the (tree-level stabilising) complex moduli fields:

$$D_a W = 0. \tag{5}$$

Actually, the above procedure of moduli stabilisation completely changes if so-called *non-geometric fluxes* are included. These new fluxes can be defined as sourced by non-Riemannian (also named non-geometric or/and exotic) branes and generally modify in a phenomenologically interesting way the superpotential W. In this context, the next section is preparatory to write about our current research interests at best. There, we introduce the notion of non-geometry in string theory, in particular how non-Riemannian geometric structures can be built up consistently making optimum use of T-duality (which is a symmetry relating two different compactified theories). Above all, we introduce in some detail the concept of non-geometric fluxes. In the final section, we write about our current research: this concerns the generalisation of the supergravity four-dimensional (super)potentials to embedding a particular class of non-geometric fluxes, known as P-fluxes (more technically, these fluxes form the spinor-vector/"gravitino" representation **352** of the orthogonal group $O(6,6)$), beyond what has been done until now turning on mostly so-called non-geometric NS-NS fluxes (embedded in the **220** representation of $O(6,6)$). Our challenge will be to find explicit four-dimensional vacua which use these fluxes in a smart way to address the problem of moduli stabilisation (and viable inflation).

2 The uncomfortable/intriguing presence of a non-geometry

It's actually well-known that string theory can be formulated in consistent ways in non-geometric backgrounds [**4**].
Here non-geometry refers to the fact that some consistent deformations of four-dimensional (effective low-energy) supergravity theories introducing massive terms (gaugings) in the action, cannot be uplifted geometrically to a fundamental ten dimensional theory. On its own, this provides an interesting window on frameworks beyond the scope of supergravity.
From the point of view of string theory, one shoud stress that the very notion of classical geometry breaks down at the string scale $l_s \sim \sqrt{\alpha'}$ (the unique full-dimensional parameter

of the theory) that, in fact, is a critical length for compactifications. The last claim is supported by the *modus operandi* of a fundamental perturbative symmetry of string theory known as T-duality.
In its most simple formulation, T-duality establishes the equivalence between a compactification on a circle of radius R and a circle of radius α'/R (with the trick to exchange simultaneously the momentum of the string along the circle with a new - purely stringy - momentum related to the strings' windings around the circle). An important example of this duality is that relating type IIA and IIB theories in $D = 9$ dimensions.
The circle compactification introduces a modulus field in the lower dimensional theory , e^{ψ}, which parameterizes $\mathbb{R}^{+}$. One can easily show by T-duality that the true moduli space of the theory, i.e. the independent values taken by the modulus field, is given by $\mathrm{R} \geq \sqrt{\alpha'}$, where the string length scale is a fixed point of the symmetry. This explains the critical behaviour of the theory at the string scale l_s.
T-duality provides also the most simple way to construct non-geometric string backgrounds. A typical example is the chain of five-dimensional branes which one can construct starting from a particular solution of the supergravity equations of motion, known as NS5-brane, and performing three T-dualities along the spatial directions orthogonal to it [**5**]. Schematically, one finds:

$$\text{NS5-brane} \rightarrow \text{KK-monopole} \rightarrow Q\text{-brane} \rightarrow R\text{-brane}. \tag{6}$$

Apart from its name (and its specific physical properties), the KK (Kaluza-Klein) -monopole is a solution of the equations of motion of general relativity and then it should not worry the reader. In order to explain the reason for the names of the last two branes, let us reveal that one could explain the same chain of dualities like a chain of fluxes. Indeed, the NS5-brane has the major characteristic to be magnetically charged under the 2-form gauge field B_2, that is to say that there is a non-zero 3-form flux threading the spatial directions orthogonal to the NS5-brane, i.e. $H_3 = dB_2$. The KK-monopole, on the other hand, has a twisted-torus structure that in generic backgrounds consists of a one-form basis η_M satisfying

$$d\eta^M = \omega^M_{NP}\eta^N \wedge \eta^P. \tag{7}$$

Actually, there is only one non-vanishing value of ω for the KK-monopole and this can be thought of as the (pure-gravitational) flux related to the brane. In (generic) components, the complete chain of fluxes is the following (Q, R are nothing but the most common choices of the fluxes' names in literature):

$$H_{\bar{M}\bar{N}\bar{P}} \rightarrow \omega^{\bar{M}}_{\bar{N}\bar{P}} \rightarrow Q^{\bar{M}\bar{N}}_{\bar{P}} \rightarrow R^{\bar{M}\bar{N}\bar{P}}. \tag{8}$$

These fluxes are actually embedded in the **220** of $O(6,6)$. In a recent paper we have classified independently all these fluxes via the existence of dualities with respect to special representations in supergravity known as mixed-symmetry potentials [**6**]. Also, we have shed light on P-fluxes which form a different T-dual chain of fluxes S-dual[2] to the first one and are embedded in the **352** of $O(6,6)$: this is the vector-spinor or "gravitino" representation of $O(6,6)$ and in type IIB theory corresponds to $\Theta_{M\dot{\alpha}}$. In $SL(6,\mathbb{R})$ components, the IIB P-fluxes are found to be:

$$P_a, P_a^{b_1b_2}, P_a^{b_1b_2b_3b_4}, P_a^{b_1b_2b_3b_4b_5b_6}, P^{ab_1b_2}, P^{ab_1b_2b_3b_4}, P^{ab_1b_2b_3b_4b_5b_6}. \tag{9}$$

3 The detailed project plan

The first four-dimensional string models taking into account for non-geometric fluxes have been developed in [**7**].

Here the non-geometric fluxes Q, its S-dual P and R have been introduced (together with the geometric ones ω) as gaugings of the four-dimensional low-energy theory in order to be consistent with the various perturbative and non-perturbative dualities related to type II theories.

Let us be a bit more specific on the structure of these models in order to clarify the status of our current research work.

In [7] type IIA/IIB models compactified on the toroidal orbifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ have been considered. The two (modding out) $\mathbb{Z}_2$ act on the internal coordinates restricting the cohomological basis (i.e. non-trivial closed forms) of the torus T^6 and reducing the number of supersymmetries. For the sake of completeness, we should also add that these type II models are considered to be orientifolded, in the sense that another $\mathbb{Z}_2$ projection is taken into account in order to reduce the supersymmetry to $\mathcal{N} = 1$ in four dimensions. Physically, this orientifold projection is equivalent to introduce *O-planes* as (non-dynamical) topological defects localised on the internal space. Since O-planes carry a negative RR charge (and tension), in order to build up a consistent neutral vacuum one needs also to add a stack of D-branes to the background (and/or also turning on 3-form fluxes, contributing to the charge like D-branes). The allowed number of D-branes and the (quantised) values of the fluxes are regulated by so-called *RR-tadpole cancellation conditions* and are actually encoded in the supergravity action.

By means of a consistent choice of the orientifold projection, type IIB theory with O3-planes and type IIA theory with O6-planes (only) can be considered. It turns out that these two theories have to match each other after performing three T-dualities on the internal space:

$$\text{IIB with O3} \leftrightarrow \text{IIA with O6}. \tag{10}$$

Moreover, it's possible to show that type IIB/O3 theory allows only for Q-flux components and, after considering its S-dual completion (P-flux), the GVW superpotential generalises to:

$$W_{O3} = \int [G_3 + (Q - \tau P)\mathcal{J}_c] \wedge \Omega. \tag{11}$$

[2]S-duality is a distinctive non-perturbative symmetry of Type IIB string theory.

Here

$$\mathcal{J}_c = C_4 + \frac{i}{2}e^{-\phi} J \wedge J, \tag{12}$$

J being the so-called Kahler 2-form determined by the CY metric.
The O3/IIB superpotential (with $P = 0$) matches after T-dualities with that of type IIA/O6:

$$W_{O6} = \int [e^{J_c} \wedge F_{RR} + \Omega_c \wedge (H_3 + \omega J_c + Q{J_c}^2 + RJ_c^3)]. \tag{13}$$

Here F_{RR} is the sum of all the RR field strengths, i.e. $F_{RR} = F_0 + F_2 + F_4 + F_6$, and

$$J_c = B + iJ, \tag{14}$$

$$\Omega_c = C_3 + iRe(C\Omega). \tag{15}$$

The results (11) and (13) are very remarkable in that they generalise the (super)potentials induced by the standard NS-RR fluxes, enabling to look for new generalized vacua. Also, adding non-geometric fluxes in principle allows to stabilise all the moduli fields at tree level. Specific applications of (11) to moduli stabilisation, and to cosmological inflation, appeared in [**7**] and [**8**]. In [**9**] an analogous work has been carried out for type IIA/O6 on the toroidal orbifold $T^6/\mathbb{Z}_4$. In each of these models, authors have to take into account several consistency conditions between fluxes, D-branes and O-planes, namely: tadpole cancellation conditions and BI [**10**]. Of course, the structure of these vacua deserve to be studied in more detail and would be interesting to analyse it in other specific models. Interestingly, models using exotic branes (as far as we know) have still not been considered and it might be interesting to generalise the tadpole cancellation conditions, in that these branes are directly coupled to non-geometric fluxes.
With that in mind, we can go a step further and ask ourselves if there is any way to generalise the superpotentials (11) and (13) including other kinds of fluxes (namely, P-fluxes). It's well-known that the allowed gaugings of the $D = 4$, $\mathcal{N} = 8$ supergravity theory (i.e. the one with the maximum number of supersymmetries) are embedded in the **912** representation (embedding tensor) of $E_{7(7)}$ (which is the global symmetry of the theory)[**11**]. Performing an orientifold projection $\mathbb{Z}_2$, $D = 4$, $\mathcal{N} = 4$ supergravity is automatically obtained, with global symmetry $SO(6,6) \times SL(2,\mathbb{R}) \subset E_{7(7)}$, the embedding tensor reduced to $\mathbf{912} \to (\mathbf{220}, \mathbf{2}) + (\mathbf{12}, \mathbf{2})$. In [**12**] this kind of generalised analysis for the superpotentials has been carried out in the type IIB/O3 case (and with the $\mathbb{Z}_2$ of the orbifolding further restricting the number of available supersymmetries and fluxes). It would be very interesting to understand what does that mean for implementing new moduli stabilisation scenarios. Furthermore, an analogous analysis for the IIA case has not yet been done and this would be an important generalisation of the results found in [**13**].
Also, the generalized compactifications, in the vacuum expectation value of the moduli fields, can be described in terms of so-called free-acting asymmetric (non-geometric) orbifolds and can be studied from the point of view of conformal field theories (CFTs) [**14**].

We would like to understand in more detail the link between non-geometric orbifolds and compactifications using non-geometric fluxes.

References

[1] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.
E. Kiritsis, *"D-branes in standard model building, gravity and cosmology,"* Fortsch. Phys. 52 (2004) 200 [arXiv:hep-th/0310001].

[2] J. Polchinski and A. Strominger, *"New Vacua for Type II String Theory,"* Phys. Lett. B388, 736 (1996) [arXiv:hep-th/9510227].
S. B. Giddings, S. Kachru and J. Polchinski, *"Hierarchies from fluxes in string compactifications"* [arXiv:hep-th/0105097].
S. Gukov, C. Vafa and E. Witten, *"CFT's from Calabi-Yau four-folds,"* Nucl. Phys. B 584, 69 (2000) [Erratum-ibid. B 608, 477 (2001)] [arXiv:hep-th/9906070].
A. Giryavets, S. Kachru, P. K. Tripathy, and S. P. Trivedi, *"Flux compactifications on Calabi-Yau threefolds,"* JHEP 04 (2004) 003, hep-th/0312104.
M. Grana, *"Flux compactifications in string theory: A comprehensive review,"* Phys. Rept. 423 (2006) 91?158, hep-th/0509003.
M. R. Douglas and S. Kachru, *"Flux compactification,"* Rev. Mod. Phys. 79, 733 (2007) [arXiv:hep-th/0610102].

[3] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *"De Sitter vacua in string theory,"* Phys. Rev.D 68, 046005 (2003) [arXiv:hep-th/0301240].
J. P. Conlon, F. Quevedo and K. Suruliz, *"Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,"* JHEP 0508, 007 (2005) [arXiv:hep-th/0505076].
J. P. Conlon, *"Moduli stabilisation and applications in IIB string theory,"* Fortsch. Phys. 55, 287 (2007) [arXiv:hep-th/0611039].

[4] C. M. Hull, *"A geometry for non-geometric string backgrounds,"* JHEP 0510 (2005) 065, arXiv:hep-th/0406102.
A. Dabholkar and C. Hull, *"Duality twists, Orbifolds, Fluxes,"* JHEP 0309, 054 (2003) [arXiv:hep-th/0210209].
A. Dabholkar and C. Hull, *"Generalised T-duality and non-geometric backgrounds,"* (2006) arXiv:hep-th/0512005.

[5] J. de Boer and M. Shigemori, *"Exotic branes and non-geometric backgrounds,"* Phys. Rev. Lett. 104, 251603 (2010) [arXiv:1004.2521 [hep-th]].

J. de Boer and M. Shigemori, *"Exotic Branes in String Theory,"* Phys. Rept. 532, 65 (2013) [arXiv:1209.6056 [hep-th]].

[6] E.A. Bergshoeff, V.Penas, F. Riccioni and S. Risoli, *"Non-geometric fluxes and mixed-symmetry potentials,"* 10.1007/JHEP11(2015)020, arXiv:1508.00780.

[7] G. Aldazabal, P. G. Camara, A. Font, and L. E. Ibanez, *"More dual fluxes and moduli fixing,"* Journal of High Energy Physics 2006 (2006), no. 05 070.
J. Shelton, W. Taylor and B. Wecht, *"Non-geometric flux compactifications,"* JHEP 0702, 095 (2007) [hep-th/0607015].
J. Shelton, W. Taylor, and B. Wecht, *"Generalized Flux Vacua,"* JHEP 0702 (2007) 095, hep-th/0607015.

[8] C. Damian, L. R. Diaz-Barron, O. Loaiza-Brito, and M. Sabido, *"Slow-Roll Inflation in Non-geometric Flux Compactification,"* JHEP 06 (2013) 109, 1302.0529.
F. Hassler, D. Lst and S. Massai, *"On Inflation and de Sitter in Non-Geometric String Backgrounds,"* arXiv:1405.2325 [hep-th].
R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, E. Plauschinn, Y. Sekiguchi, and F. Wolf, *"A Flux-Scaling Scenario for High-Scale Moduli Stabilization in String Theory,"* Nucl. Phys. B897 (2015) 500?554, 1503.07634.

[9] M. Ihl, D. Robbins, and T. Wrase, *"Toroidal orientifolds in IIA with general NS-NS fluxes,"* JHEP 0708 (2007) 043, [arXiv:0705.3410].

[10] P. Shukla, *"Revisiting the two formulations of Bianchi identities and their implications on moduli stabilization,"* (2016) [arXiv:1603.08545v1].

[11] B. de Wit, H. Samtleben and M. Trigiante, *"On Lagrangians and gaugings of maximal supergravities,"* Nucl. Phys. B 655 (2003) 93 [arXiv:hep-th/0212239].
B. de Wit, H. Samtleben and M. Trigiante, *"Gauging maximal supergravities,"* Fortsch. Phys. 52 (2004) 489 [arXiv:hep-th/0311225].
F. Riccioni and P. West, *"The E11 origin of all maximal supergravities,"* [arXiv:hep-th/0705.0752].

[12] G. Aldazabal, E. Andrés, P.G. Cámara, M. Graña, *"U-dual fluxes and Generalized Geometry,"* (2010) [arXiv:1007.5509v1].

[13] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, *"Superpotentials in IIA compactifications with general fluxes,"* Nucl. Phys. B 715 (2005) 211 [arXiv:hep-th/0411276].
G. Villadoro, F. Zwirner, *"$\mathcal{N} = 1$ effective potential from dual type-IIA D6/06 orientifolds with general fluxes,"* (2005) [arXiv:hep-th/0503169v3].

[14] C. Condeescu, I. Florakis and D. Lust, *"Asymmetric Orbifolds, Non-Geometric Fluxes and Non-Commutativity in Closed String Theory,"* JHEP 1204 (2012) 121 [arXiv:1202.6366 [hep-th]].
C. Condeescu, I. Florakis and D. Lust, *"Gauged supergravities and non-geometric Q/R fluxes from asymmetric orbifold CFT's,"* JHEP 1204 (2012) 121 [arXiv:1202.6366 [hep-th]].