Phenomenology of theories with deformed Planck-scale symmetries in the neutral kaon system

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The efforts in building a physical theory that combines the basic tenets of quantum mechanics and general relativity have been confronting for more than 30 years with a cumbersome intellectual puzzle known under the name of *information paradox*.
This, as initially proposed by Hawking in 1976 [1], suggests that physical information could permanently disappear in a black hole, allowing a pure initial state to evolve into a mixed state. The mixed nature of this state is ultimately due to the existence of an unobservable region inside the event horizon of the black hole and to the entangled nature of the vacuum state of the quantum field. In other words, this would imply that unitarity of quantum mechanics is lost when gravitational effects are taken into account, which would represent a truly fundamental modification of the laws of physics as we currently know them.
However, soon after Hawking's proposal many criticisms began to arise. It was noted that conservation laws become decoupled from symmetry principles, so that, for instance, rotation invariance would no longer imply conservation of angular momentum. Also, a study by Banks, Peskin, and Susskind (BPS) [2] argued that conservation of energy and momentum could not be preserved without losing locality. Since then, the possibility of a nonunitary evolution of density matrices has been somewhat abandoned.
In this project I will reconsider this possibility in the context of theories with deformed relativistic symmetries, based on the model proposed by Arzano in [3]. Moreover, since it has been shown [4, 5] that the neutral kaon system $K_0 - \bar{K}_0$ is suitable for testing experimentally the basic principles of quantum mechanics such as the possible breaking of CPT symmetry, which could manifest as a result of Lorentz symmetry violation, I will investigate the possible phenomenological output of the model and focus on constraints to impose to this model by the experimental results obtained by the KLOE experiment at the DAΦNE e^+e^- collider, the Frascati ϕ-factory.

Purity loss as a consequence of deformed symmetries

The basic formalism for a theory in which pure states can evolve into mixed states prescribes to take a density matrix ρ as a starting point to describe the physical system, rather than the state $|\psi\rangle$, a vector in the Hilbert space. The eigenvalues of ρ represent probabilities, and so must be real, positive or zero, and sum to one. Thus ρ must be hermitian and satisfy $\text{Tr}\rho = 1$. The state is said to be pure if $\rho = |\psi\rangle\langle\psi|$ for some Hilbert space vector $|\psi\rangle$; in this case $\text{Tr}\rho^2 = \text{Tr}\rho = 1$. If $\text{Tr}\rho^2 < 1$, then ρ cannot be written as $|\psi\rangle\langle\psi|$ for any $|\psi\rangle$, and the state is said to be mixed. The usual evolution equation for ρ is the Von Neumann equation

$$\dot{\rho} = -i[H, \rho],$$

and this leaves both Tr ρ and Tr ρ^2 constant in time. To allow loss of purity, we must modify this equation. One assumes that the new evolution equation for ρ is still linear, and still first order in time derivatives:

$$\dot{\rho}_{ab} = \tilde{H}_{ab}^{cd}\rho_{dc}.$$

The generalized hamiltonian $\tilde{H}$ must be constrained to preserve hermiticity, positivity, and trace of ρ. As shown by BPS in [2], the most general equation of this kind is:

$$\dot{\rho} = -i[H, \rho] - \frac{1}{2}h_{\alpha\beta}(Q^\alpha Q^\beta \rho + \rho Q^\beta Q^\alpha - 2Q^\alpha \rho Q^\beta), \tag{1}$$

where $h_{\alpha\beta}$ is a Hermitian matrix of coupling constants and the Q^α form a basis of Hermitian matrices with $Q^0 = 1$. Eq. 1 is known as the Lindblad equation.
BPS showed that if $h_{\alpha\beta}$ has nonnegative eigenvalues and is real and symmetric, then conservation of energy requires that H commute with each of the Q^α. In a quantum field theory there are very few operators which commute with the hamiltonian, and all of them are global, i.e. they are integrals over all space of a local density. However this choice, as said above, has been shown by BPS to lead to a breakdown of locality.
It was only 10 years after that Sredincki reconsidered the problem in [6] and proved that such nonlocality is quite harmless and does not lead to observably macroscopic violations of locality. He moved from this premise and went ahead arguing that, if energy is conserved, loss of purity is incompatible with the weakest possible form of Lorentz covariance. The model by Arzano encompasses this problem showing that, in the framework of quantum Poincaré algebras, deformed time translation generators have an adjoint action which leads to a covariant Lindblad equation.
To try to motivate this point we remind that in ordinary quantum mechanics symmetry generators act in a particularly simple way on their eigenstates. For instance the translation generators act on the basis of the Hilbert space given by their eigenstates as

$$P_\mu|k\rangle = k_\mu|k\rangle$$

and on the dual space spanned by bras

$$P_\mu\langle k| = -k_\mu\langle k|.$$

If we now consider a basis of kets $|k(g)\rangle$ for the Hilbert space labeled by coordinates on a non-Abelian Lie group we will have again

$$P_\mu|k(g)\rangle = k_\mu(g)|k(g)\rangle.$$

However, when acting on the dual representation, the nontrivial structure of momentum space must be taken into account:

$$P_\mu\langle k(g)| = k_\mu(g^{-1})\langle k(g)|.$$

Besides, since the coordinates live in a non-Abelian group, the group homomorphism properties dictate a non-Abelian composition rule for momentum eigenvalues

$$k_\mu(g) \oplus k_\mu(h) \equiv k_\mu(gh) \neq k_\mu(h) \oplus k_\mu(g) \equiv k_\mu(gh).$$

This is just a well defined realization of representation of deformed symmetries according to the theory of Hopf algebras, which are studied in the context of noncommutative field theories connected to the quantization of relativistic point particle coupled to gravity. In three space-time dimensions it has been shown [3] that this machinery leads to a deformed evolution equation for the density matrix of a Lindblad form:

$$\dot{\rho} = -i[P_0, \rho] - \frac{1}{2}h_{ij}(P^iP^j\rho + \rho P^jP^i - 2P^j\rho P^i), \tag{2}$$

where the matrix h_{ij} is given by

$$h = \frac{i}{\kappa}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

and is not positive definite (here $\kappa = 1/4\pi G$). Therefore positivity of the evolution of ρ is not guaranteed.

Kaon interferometry and phenomenological outcome

The neutral kaon system is one of the most sensible probes of a possible breakdown of conventional quantum mechanics because it contains phenomena which depend on quantum coherence over macroscopic distances [4]. Decoherence, i.e. the time evolution of a pure state into an incoherent mixture of states, necessarily implies by means of a theorem CPT violation. It is therefore possible and extremely interesting to put experimental limits on decoherence effects at the level of Plack's scale region in order to test various theoretical quantum gravity models.
The evolution of a single kaon described by the density matrix ρ obeys a modified Liouville equation:

$$\dot{\rho} = -i[H, \rho] + i\delta \not{H}\rho, \tag{3}$$

where H is the usual neutral kaon effective hamiltonian and the extra term $\delta \not{H}$ would induce decoherence in the system. For a suitable basis choice and expanding ρ in terms of Pauli spin matrices σ^i, the extra term can be represented by a 4×4 matrix:

$$\delta \not{H} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & \beta & \gamma \end{pmatrix} \tag{4}$$

where α, β and γ are real parameters which violate CPT symmetry and can be measured. It is evident that eq. 3 has the same structure of the Lindblad type of eq. 2.

The principal aim of the project will be then to investigate thoroughly how these two pictures are quantitatively related and how it will be possible to extract constraints on the model with Plack-scale deformed symmetries from the measurements of α, β and γ at the Frascati ϕ-factory. This could lead to a phenomenological test for this kind of quantum gravity candidate theories which of course would be of great interest for the scientific research in this branch of physics. To do this I plan to study how the theoretical model with deformed symmetries can be rewritten in the case of the kaon system and in particular to determine the form of the effective hamiltonian in order to permit a direct confrontation involving the α, β and γ parameters.

References

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