Phenomenology of theories with deformed Planck-scale symmetries in the neutral kaon system

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The efforts in building a physical theory that combines the basic tenets of quantum mechanics and general relativity have been confronting for more than 30 years with a cumbersome intellectual puzzle commonly known under the name of *information paradox*.
This, as initially proposed by Hawking in 1976 [1], suggests that physical information could permanently disappear in a black hole, allowing a pure initial state to evolve into a mixed state. The mixed nature of this state is ultimately due to the existence of an unobservable region inside the event horizon of the black hole and to the entangled nature of the vacuum state of the quantum field. In other words, this would imply that unitarity of quantum mechanics is lost when gravitational effects are taken into account, which would represent a truly fundamental modification of the laws of physics as we currently know them.
However, soon after Hawking's proposal many criticisms began to arise. It was noted that, in this framework, conservation laws become decoupled from symmetry principles, so that, for instance, rotation invariance would no longer imply conservation of angular momentum. Also, a study by Banks, Peskin, and Susskind (BPS) [2] argued that conservation of energy and momentum could not be preserved without losing locality.
However, as was later noted by Srednicki [3], this kind of non-locality is actually harmless because it does not lead to macroscopic violations of causality, even if it was contextually pointed out that purity loss is incompatible with the weakest possible form of Lorentz covariance. Since then, the possibility of a nonunitary evolution of density matrices has been somewhat abandoned, even if with the latest notable exception of Weinberg [4].
Nonetheless, more recently, much interest in a large area of research has been devoted to the fate of Lorentz (and Poincaré) symmetry at the Planck scale, focusing on the idea that conjectured new effects of a quantum gravity theory might result in a deformation or breaking of this symmetry, which could become manifest at low energies in the form of UV corrections [5].
The main aim of this project is directed to reconsider the possibility of a fundamental decoherence inspired by theories with deformed relativistic symmetries, with particular reference to the model of deformed translations proposed by Arzano in [6].
Considering the fact that Lorentz symmetry violation could be linked to that of $\mathcal{CPT}$ symmetry, and that the neutral kaon system $K_0 - \bar{K}_0$ has a remarkably delicate balance of scales which provides opportunities for very sensitive tests ($\sim 10^{-20}$ GeV) of $\mathcal{CPT}$ symmetry and decoherence effects [7–9], the efforts of this project will be directed to investigate the possible phenomenological output of the model [6] in the kaon system and focus on constraints to impose to this model by the experimental results obtained by the KLOE experiment at the DAΦNE e^+e^- collider, the Frascati ϕ-factory.
It must be stressed that such kind of phenomenology is essentially new in the horizon of quantum-gravity induced phenomena. It has never been experimentally tested before and is fair to say that it must thoroughly be put under scrutiny since there could be the possibility to put new bounds on the scale at which deformation of symmetries becomes not negligible.

Purity loss as a consequence of deformed symmetries

The basic formalism for a theory in which pure states can evolve into mixed states prescribes to take a density matrix ρ as a starting point to describe the physical system, rather than the state $|\psi\rangle$, a vector in the Hilbert space. The eigenvalues of ρ represent probabilities, and so must be real, positive or zero, and sum to one. Thus ρ must be hermitian and satisfy $\mathrm{Tr}\rho = 1$. The state is said to be pure if $\rho = |\psi\rangle\langle\psi|$ for some Hilbert space vector $|\psi\rangle$; in this case $\mathrm{Tr}\rho^2 = \mathrm{Tr}\rho = 1$. If $\mathrm{Tr}\rho^2 < 1$, then ρ cannot be written as $|\psi\rangle\langle\psi|$ for any $|\psi\rangle$, and the state is said to be mixed. The usual evolution equation prescribed by ordinary quantum mechanics for ρ is the Von Neumann equation

$$\dot{\rho} = -i[H, \rho],$$

and this leaves both Tr ρ and Tr ρ^2 constant in time. To allow loss of purity, we must modify this equation. One assumes that the new evolution equation for ρ is still linear, and still first order in time derivatives:

$$\dot{\rho}_{ab} = \tilde{H}_{ab}^{cd}\rho_{dc}.$$

The generalized hamiltonian $\tilde{H}$ must be constrained to preserve hermiticity, positivity, and trace of ρ. As shown by BPS in [2], the most general equation of this kind is:

$$\dot{\rho} = -i[H, \rho] - \frac{1}{2}h_{\alpha\beta}(Q^\alpha Q^\beta \rho + \rho Q^\beta Q^\alpha - 2Q^\alpha \rho Q^\beta), \tag{1}$$

where $h_{\alpha\beta}$ is a Hermitian matrix of coupling constants and the Q^α form a basis of Hermitian matrices with $Q^0 = 1$. Eq. 1 is known as the Lindblad equation.

BPS showed that if $h_{\alpha\beta}$ has nonnegative eigenvalues and is real and symmetric, then conservation of energy requires that H commute with each of the Q^α. In a quantum field theory there are very few operators which commute with the hamiltonian, and all of them are global, i.e. they are integrals over all space of a local density. However this choice, as said above, has been shown by BPS to lead to a breakdown of locality.

It was only 10 years after that Sredincki reconsidered the problem in [3] and proved that such nonlocality is harmless and does not lead to violations of locality. He moved from this premise and went ahead arguing that, if energy is conserved, loss of purity is incompatible with the weakest possible form of Lorentz covariance. The model [6] encompasses this problem showing that, in the framework of quantum Poincaré algebras, deformed time translation generators have an adjoint action which leads to a covariant[1] Lindblad equation.

To try to better motivate this point we remind that in ordinary quantum mechanics symmetry generators act in a particularly simple way on their eigenstates. For instance the translation generators act on the basis of the Hilbert space given by their eigenstates as

$$P_\mu|k\rangle = k_\mu|k\rangle.$$

Using the notion of dual representation of a Lie algebra is possible to define also the action on the dual space spanned by bras[2]

$$P_\mu\langle k| = -k_\mu\langle k|.$$

It is also defined the action of the generator on a composite state $\mathcal{H}\otimes\mathcal{H}$, which follows the Leibniz rule

$$P_\mu(|k_1\rangle \otimes |k_2\rangle) = P_\mu|k_1\rangle \otimes |k_2\rangle + |k_1\rangle \otimes P_\mu|k_2\rangle.$$

With these one can extend to the action on other operators, e.g. the projection operator $\pi_k = |k\rangle\langle k|$

$$P_\mu(\pi_k) = P_\mu(|k\rangle\langle k|) = P_\mu|k\rangle\langle k| - |k\rangle\langle k|P_\mu = [\pi_k, P_\mu].$$

Now, symmetry deformation in the UV of the type considered in DSR theories [10] suggests that action of P^μ on $\mathcal{H}\otimes\mathcal{H}$ and $\mathcal{H}^*$ is modified, according to the theory of Hopf algebras. An interesting example to fix this idea is the case of a massive particle coupled to gravity in three space-time dimensions, where it is known that the three-momentum of the particle is an element of a non-abelian group [11].

To generalize to this case the action of the generators of translation we now have to consider a basis of kets $|k(g)\rangle$ for the Hilbert space labeled by coordinates on a non-Abelian Lie group.

We will have, similarly as before

$$P_\mu|k(g)\rangle = k_\mu(g)|k(g)\rangle.$$

[1]The term covariant here must be intended having in mind that Poincaré symmetry is deformed in such models.

[2]This action "from the left" must be distinguished from the usual action of the hermitian conjugate "from the right".

However, when acting on the dual representation, the nontrivial structure of momentum space must be taken into account:

$$P_\mu \langle k(g)| = k_\mu(g^{-1})\langle k(g)|.$$

But perhaps what is truly different to the previous case is the action on multi-particle states (coproduct)

$$P_\mu(|k_1(g)\rangle \otimes |k_2(h)\rangle) = P_\mu(g\cdot h)|k_1\rangle \otimes |k_2\rangle \equiv \Delta P_\mu |k_1\rangle \otimes |k_2\rangle \neq \Delta P_\mu |k_2\rangle \otimes |k_1\rangle,$$

since the coordinates live in a non-Abelian group and the group homomorphism properties dictate a non-Abelian composition rule for momentum eigenvalues

$$k_\mu(g) \oplus k_\mu(h) \equiv k_\mu(gh) \neq k_\mu(h) \oplus k_\mu(g) \equiv k_\mu(gh).$$

This is just a well defined realization of representation of deformed symmetries, which are studied in the context of noncommutative field theories connected to the quantization of relativistic point particle coupled to gravity. A natural consequence of these facts is that the action of the generators on operators will be deformed accordingly. It can been shown [6] that for the deformed translation generators associated to the momentum space of a massive particle in 2+1 gravity the coproduct reads, to first order in $\kappa = 1/4\pi G$

$$\Delta P_\mu = P_\mu \otimes \mathbb{1} + \mathbb{1} \otimes P_\mu + \frac{1}{\kappa}\epsilon_{\mu\nu\sigma} P^\nu \otimes P^\sigma + \mathcal{O}\left(\frac{1}{\kappa^2}\right).$$

This mathematical machinery leads to a deformed action of generators on operators, in particular on the density matrix for which it reduces to an evolution equation of a Lindblad form:

$$\dot\rho = -i[P_0,\rho] - \frac{1}{2} h_{ij}(P^i P^j \rho + \rho P^j P^i - 2P^j \rho P^i), \tag{2}$$

where the matrix h_{ij} is given by

$$h = \frac{i}{\kappa}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},$$

that is not positive definite, which means that positivity of the evolution of ρ is not guaranteed.
This picture can be generalized also to the four-dimensional case where the deformed translation generators from a quantum deformation of the Poincaré algebra known as κ-Poincaré act on the density matrix to give (here κ is a constant $\sim E_{Planck}$)

$$\dot\rho = -i[P_0,\rho] + \frac{1}{\kappa} P_m \rho P_m - \frac{i}{\kappa}\rho \vec{P}^2 \tag{3}$$

which can be shown to satisfy a deformed notion of covariance. It can be noted that such equation does not affect the evolution of pure one particle states but only that of superpositions of states, such as multi-particle and mixed states.

The neutral kaon system as a probe for quantum gravity tests

The neutral kaon system is one of the most sensible probes of a possible breakdown of conventional quantum mechanics because it contains phenomena which depend on quantum coherence over macroscopic distances [7, 8]. A neutral kaon oscillates forth and back between itself and its antiparticle, because the physical quantity, strangeness, which distinguishes antikaons from kaons, is not conserved by the weak interactions which governs the time evolution and permit $K^0 \leftrightarrow \bar{K}^0$ transitions. A neutral kaon is by itself a two-state system. This imply that an initially pure $|K^0\rangle$ or $|\bar{K}^0\rangle$ state will gradually evolve into a state of mixed strangeness:

$$|K^0(t)\rangle \to a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle,$$

and in general, we can write a generic state $|\psi(t)\rangle$ of the system as a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$ plus possible decay products $|f_j\rangle$

$$|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle + \sum_j c_j(t)|f_j\rangle.$$

This equation can be solved for the unknown functions $a(t)$, $b(t)$ by using a perturbation approximation (the Wigner-Weisskopf approximation) which yields

$$\psi(t) = e^{-iHt}\psi_0 \tag{4}$$

where $\psi(t)$ is the column vector with components $a(t)$ and $b(t)$, ψ_0 equals ψ at $t = 0$, and H is the time-independent 2×2 matrix, whose components refer to the two-dimensional basis $|K^0\rangle$, $|\bar{K}^0\rangle$. Since kaons may decay, H is not hermitian and e^{-iHt} is not unitary. This motivates the expression of this effective hamiltonian H in terms of the hermitian mass M and decay Γ matrices

$$H = M - \frac{i}{2}\Gamma, \tag{5}$$

which, according to the perturbation approximation are related to the weak interaction $\mathcal{H}_w$

$$M_{\alpha\alpha'} = m_0\delta_{\alpha\alpha'} + \langle\alpha|\mathcal{H}_w|\alpha^{'}\rangle + P\sum_f \frac{\langle\alpha|\mathcal{H}_w|\beta\rangle\langle\beta|\mathcal{H}_w|\alpha^{'}\rangle}{m_0 - E_\beta} \tag{6}$$

$$\Gamma_{\alpha\alpha'} = 2\pi\sum_\beta \langle\alpha|\mathcal{H}_w|\beta\rangle\langle\beta|\mathcal{H}_w|\alpha^{'}\rangle\delta(m_0 - E_\beta), \tag{7}$$

where P stands for the principal part prescription and m_0 is the mass of the neutral kaon. These relations are of great value since they enable to state directly the symmetry properties of $\mathcal{H}_w$ in terms of experimentally observable relations among the elements of H.

As said, the neutral kaon system is an optimal candidate for testing decoherence, i.e. the time evolution of a pure state into an incoherent mixture of states. It is therefore possible and extremely interesting to put experimental limits on decoherence effects at the level of Planck's scale region in order to test various theoretical quantum gravity models.

The density matrix formalism correctly treats pure and mixed states in a unique consistent framework. The evolution of a single kaon described by the density matrix ρ is supposed to obey [7] a modified Von Neumann equation:

$$\dot{\rho} = -iH\rho + i\rho H^\dagger + \delta\not{H}\rho, \tag{8}$$

where H is the usual neutral kaon effective hamiltonian and the extra term $\delta\not{H}$ would induce decoherence in the system. For a suitable basis choice and expanding ρ in terms of Pauli spin matrices σ^i, the extra term can be represented by a 4×4 matrix:

$$\delta\not{H} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & \beta & \gamma \end{pmatrix}, \tag{9}$$

where α, β and γ are real parameters which violate $\mathcal{CPT}$ symmetry and can be measured with a sensitivity $\mathcal{O}(m_K^2/M_{\text{Planck}}) \sim 2 \times 10^{-20}$ GeV.

It is evident that eq. (8) has the same structure of the Lindblad type of eq. (2). But, even more interestingly, the corrections which one naively expects from (3) for a non-relativistic system are exactly in the range of accessible sensitivities in neutral kaon experiments.

Goal of the project

The principal goal of the project will be then to investigate thoroughly how one can apply the model [6] to the case of the non-relativistic neutral kaon system and how it will be possible to parameterize departures from ordinary quantum mechanics evolution induced by gravity. In this way it will possible to extract constraints on the model with Plack-scale deformed symmetries from the measurements of α, β and γ at the Frascati ϕ-factory. This could lead to a phenomenological test for this kind of quantum gravity candidate theories which of course would be of great impact.

To this end,as a first step I will implement the Lindblad evolution (3) for the case of correlated kaon pairs as those produced at the Frascati $\phi-$factory to see quantitatively how momentum dependence makes it different from the ordinary evolution. This would permit a parameterization of the non-ordinary terms similar (hopefully) to that of $\delta \not{H}$, which could be directly tested at the DAΦNE collider and confronted with the α, β and γ parameterization.
Also, as a next level, much care will be devoted to the fate of $\mathcal{CPT}$ symmetry, due to the central importance of this symmetry in the building of the theoretical framework of local quantum field theory with Poincaré invariance adopted to describe phenomena in the absence of gravity. Such a symmetry, in the presence of gravity, may be itself deformed or broken and its behavior has not yet been determined [12].
Finally, there is also the opportunity to broaden the spectrum of phenomenological tests of the model considering studies of other $\mathcal{CPT}$ violations with entangled neutral kaons such as the ones that can be found in [13].

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