BKT transition in thin superconducting films and artificial nanostructures

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Introduction

Formulated within the class of the two-dimensional XY spin model, the Berezinskii-Kosterlitz-Thouless (BKT) transition ([1]-[3]) has been immediately object of great interest for its topological nature and its criticality. According with Mermin-Wagner theorem (1966) [4], the transition doesn't show any continuos symmetry breaking, but below a critical temperature T_{BKT} the correlation length diverges. The transition is driven by vortex excitations: at low temperatures the system shows a quasi-long-range order due to the pairing of vortices with opposite vorticity, while at high temperatures the increase of free vortices excitations destroys the large-scale coherence leading to a disordered phase.
The BKT transition has been in later years the key tool for the understanding of a wide class of phenomena, especially in condensed matter: superfluid and superconducting (SC) transitions in two-dimensional systems belong indeed to the same class of universality of the BKT one.
In superfluid films, the main hallmark of this transition is the universal and discontinuous jump of the superfluid density[5], successfully observed for the first time in superfluid He films[6].
However, in SC films such signature is much more elusive, essentially for two main reasons[7]:

- the presence of quasiparticle excitations that limits the observation of various specific features of the BKT transition to a small temperature range between T_{BKT} and BCS critical temperature T_c,
- the screening effects due to charged supercurrents which modify the logarithmic interaction between vortices needed to observe BKT physics.

The screening effects can be avoided reducing the thickness d of the SC films, in such a way that the Pearl length $\Lambda = \frac{2\lambda^2}{d}$ exceeded the sample

size. Nevertheless, the reduction of the thickness in conventional superconductors (like InO_x and NbN) implies the increase of disorder and, as consequence, the observation of BKT physics is usually restricted to samples close to the superconductor-to-insulator transition (SIT). As shown by detailed tunneling spectroscopy measurements ([13]-[15]) in the last years, these systems show a considerable increase of the intrinsic inhomogeneity of the SC properties, which leads to a drastic smearing of the superfluid-density jump predicted by the conventional BKT theory. These systems also display remarkable thermodynamic features signaling the failure of the BCS paradigm of superconductivity. In particular Cooper pairing seems to survives also above T_c and the low value of the superfluid stiffness D_s enhances phase fluctuations of the (complex) order parameter, suggesting their contribution to be dominant for the behavior of these materials.
In a recent work[16], it has been shown that the occurrence of this inhomogeneity has interesting consequences in the optical excitations: disorder renders collective modes, which are optically inactive in a clean superconductor, visible.
This phenomenum occurs essentially because of the formation, at high disorder, of percolating paths with a high value of local stiffness. In this configuration percolating superconducting paths leave aside several isolated SC islands, which acting as nano antennas contribute to the finite-frequency optical absorption.
It's needed to highlight that, in the case studied, the inhomogeneity of the local stiffness observed at high disorder shows a structure in the real space: it is characterized by the emergence of regions with similar SC properties, a deeply different scenario from the one in which the value of the stiffness has a gaussian distribution in real space.
The article of T.Cea et al. (2014)[16] presents results performed at zero temperature $T = 0$, it would be then really interesting to extend the study to the finite temperatures regime, studying the effects of this *structured* inhomogeneity on the BKT transition.
Together with these "*spontaneously*" disordered systems, there are others in which the inhomogeneity is artificially built. In the last years, there have been some experimental works ([17] [18] [19]) investigating the transport properties of artificial structures of superconducting islands placed on a metallic substrate. The results of these works are of great interest especially because a clear theoretical explanations of their main results is still missing.

My PhD Thesis will be focused on these two different disordered two-dimensional SC systems that I have just introduced:

- "Spontaneously" disordered SC films, for which I will implement Monte Carlo simulations of the disordered XY model.

- Artificial nanostructures of superconducting islands, that I will investigate also considering the effect of the interface between the normal metal and the SC islands.

Monte Carlo Simulations of "spontaneously" disordered SC films

The natural classical model to describe a two-dimensional array of superconducting grains is the two-dimensional XY model. Indeed, the energy governing the thermodynamics fluctuation is well described by a phase only hamiltonian in which the real part of the superconducting order parameter $\Psi_i = |\Delta|e^{i\theta_i}$, Δ, is considered fixed. To see even better this analogy, let's write the XY model hamiltonian:

$$H = -J \sum_{<i,j>} \mathbf{S_i S_j} \tag{1}$$

$\mathbf{S_i}$, the spin of the site i, is a classical planar vector.
If we look at the low temperature regime, we can rewrite (1) in this gaussian form:

$$H \simeq \frac{J}{2} \int d^2r(\nabla\theta(r))^2 \tag{2}$$

On the other hand, the *phase-only* hamiltonian of a superconductor, without external field, will be essentially the kinetic term of the energy:

$$H_S = \frac{1}{2}mn_s \int d^2r v_s(r) \tag{3}$$

where n_s is the number of superfluid carriers (cooper pairs) and $v_s(r)$ the superfluid velocity, which depends only on the gradient of the phase: $v_s = \frac{\hbar}{2m}\nabla\theta$. With this substitution, we can rewrite the hamiltonian in this way:

$$H_S = \frac{\hbar^2 n_s}{8m} \int d^2r(\nabla\theta(r))^2 \tag{4}$$

the analogy with the hamiltonian of the XY model is now evident: the superfluid stiffness of H_S and the renormalized coupling variable $J(T)$ are related by the relation:

$$J_s(T_{BKT}) = \frac{\hbar^2 n_s d}{4m} = \frac{2T_{BKT}}{\pi} \tag{5}$$

from the low temperature regime, when $T = T_{BKT}$, the so called *universal jump* occurs.
Performing Monte Carlo simulations on the XY spin model, the equivalent observable of the superfluid density is the helicity modulus Υ.

In Fig. 1, it is shown what I have obtained performing Monte Carlo simulations of the clean XY spin model. It is plotted the helicity modulus Υ as function of the temperature for different lattice sizes. We can see that, as expected, the universal jump occurs as soon as $T = T_{BKT}$: at this point the helicity assumes the universal value:

$$J(T_{BKT}) = \frac{2T_{BKT}}{\pi}$$

and immediately after, it drops to zero.

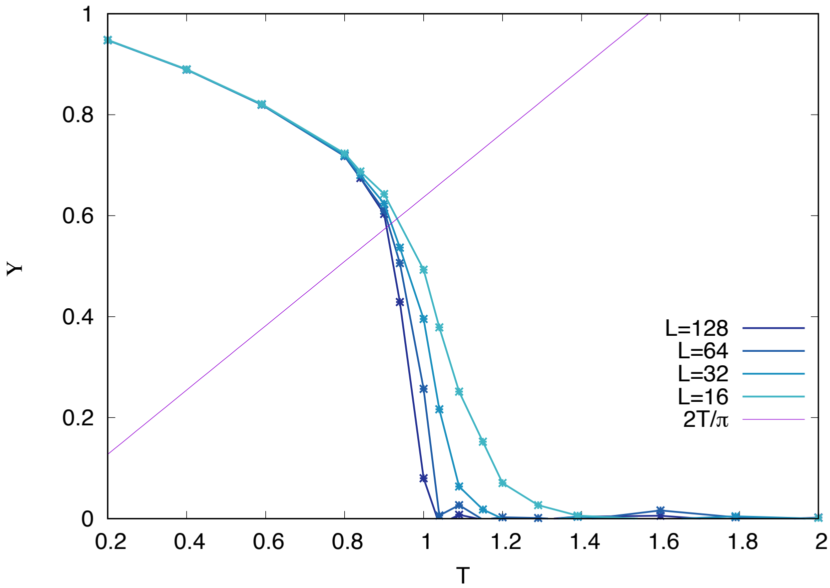

Figure 1: Helicty modulus as function of the temperature for different sizes. Result from the preliminary Monte Carlo simulations.

We are now interested to add some kind of disorder in equation (1), in order to reproduce better the properties of a superconducting two-dimensional film. In letterature the disorder is usually included in the two-dimensional XY model introducing a random magnetic field [8] [9] or through the dilution of the sites [10]. The inclusion of random coupling constant $J = J_{ij}$ with some distribution probability, without any structural distribution in real space, indeed doesn't change the thermodynamics properties of the system [11].

One of the main goals of my Phd thesis will be to perform Monte Carlo simulations of disordered SC films, exploring a different disordering method, based on the results of the work [16]. They start from an effective boson model for the disordered superconductor: the XY spin-1/2 in a transverse random field:

$$H = -2\sum_i \xi_i S_i^z - 2J \sum_{<i,j>} (S_i^+ S_j^- + H.c.)$$

From this model, through mean field calculations at $T = 0$, the Hamiltonian has been rewritten in such a way that the disorder is included in the coupling constant:

$$J_{ij} = J \sin\theta_i \sin\theta_j$$

where θ_i is the angle between the spin $\mathbf{S_i}$ and the $\hat{z}$ axis.
The idea is to use these coupling constants J_{ij} to perform Monte Carlo simulations of the system: the disorder would be then *structured* in real space.
The interesting perspectives of this numerical study are:

- the observation of the effects of this *structured disorder* on the superfluid density jump, and eventually on its smearing;
- the study of the fluctuations above the critical temperature.

Artificial Nanostructures

The second problem I will investigate concerns the understanding and the theoretical explanation of the main results obtained in two interesting experiments realized using artificial nanostructures ([17] [18] [19]). To study and try to solve this issue I will use two different approaches:

- Monte Carlo simulations of disordered aggregate systems.
- Study of the proximity effect between the metal substrate and the superconducting islands. [12].

The first approach has as starting point a previous work[20] in which the results of numerical simulations of an array of clean superconductive islands are presented. The study of this statistical model have been motivated in particular by the work of Eley at al. (2011)[17], which shows an unexpected dependence of the transition temperature of the single superconducting island (T_c^1) on the distance between them.
However the results obtained in [20] don't reproduce the experimental results of [17]. We can then ask ourselves if there could be some effect due to "spontaneous" disorder inside each island. We plan to perform numerical simulations in the case of disordered interacting spin islands.

Another relevant result of [17] is the dependence of T_c^1 on the thickness of the single island: the critical temperature T_c^1 is lower for thinner islands.

A possible approach to better understand the T_c^1 dependency both on the islands spacing and on the island thickness could be to consider the effect of the contact between the superconducting islands and the normal metal on the superconducting order parameter ψ . Indeed, the presence of this SN interface changes significantly the boundary conditions of ψ, that becomes space dependent $\psi(x)$, leading in some cases to a reduction of the critical temperature T_c.
Preliminary results show indeed that the metallic substrate induces an *antiproximity effect* on each superconducting island. This can be seen within a simplified one-dimensional model, where the variable "x" will represents or the x direction in the plane, were the islands are arranged, exploring the dependence on the island spacing, or the "z" direction taking into account the thickness of the single islands and studying the dependence of the critical temperature on it.

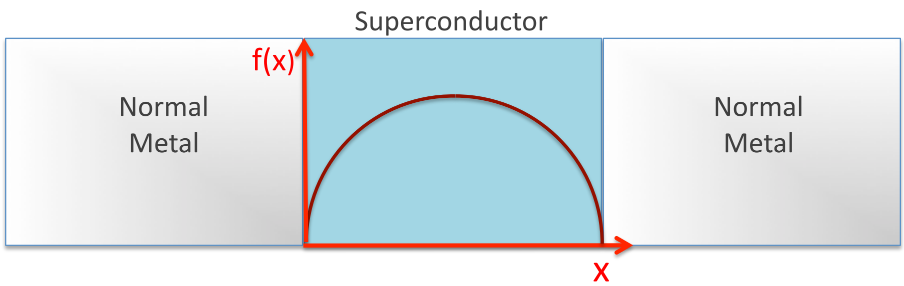

Figure 2: NSN junction in the specific case in which $f(0) = f(L) = 0$.

From the one-dimensional Ginzburg-Landau equilibrium equation, we can write the differential equation:

$$-\xi^2(T)\frac{d^2 f}{dx^2} - f + f^3 = 0 \tag{6}$$

where $f(x)$ contains the spatial variation of the order parameter: $\psi(x) = \psi_0 f(x)$. If we consider the spatial direction "x" inside the plane, we will look at the case in which the superconductor is placed between two normal metals. In this preliminary study, we have considered that the contact between them is such that the SC wave function is zero at the boundaries (Fig. 2). By symmetry, if the superconductor has linear size L, $f(x)$ will reach its maximum value for $x = \frac{L}{2}$. With these boundary conditions, we can derive a self consistent equation for f_M, defined as the maximum value taken by the function $f(x)$ ($f_M = f(\frac{L}{2})$):

$$\int_0^{f_M} \frac{df}{[(1-f^2)^2 - (1-f_M^2)^2]^{\frac{1}{2}}} = \frac{L}{2\sqrt{2}\xi(T)} \tag{7}$$

Hence, the critical temperature will be the temperature at which the value of f_M becomes zero.
The numerical solution of this equation for different values of T and L ([1]), gives the dependency of f_M on T, for different island size, and the dependency of the critical temperature T_c on the island linear dimension, as shown in Fig 3. From (7), we can also derive an analytical expression for T_c:

$$T_c = 1 - 0.74(\frac{\pi}{L})^2 \tag{8}$$

that perfectly fit the values obtained form numerical simulation, as shown in Fig. 3 (continuos line).

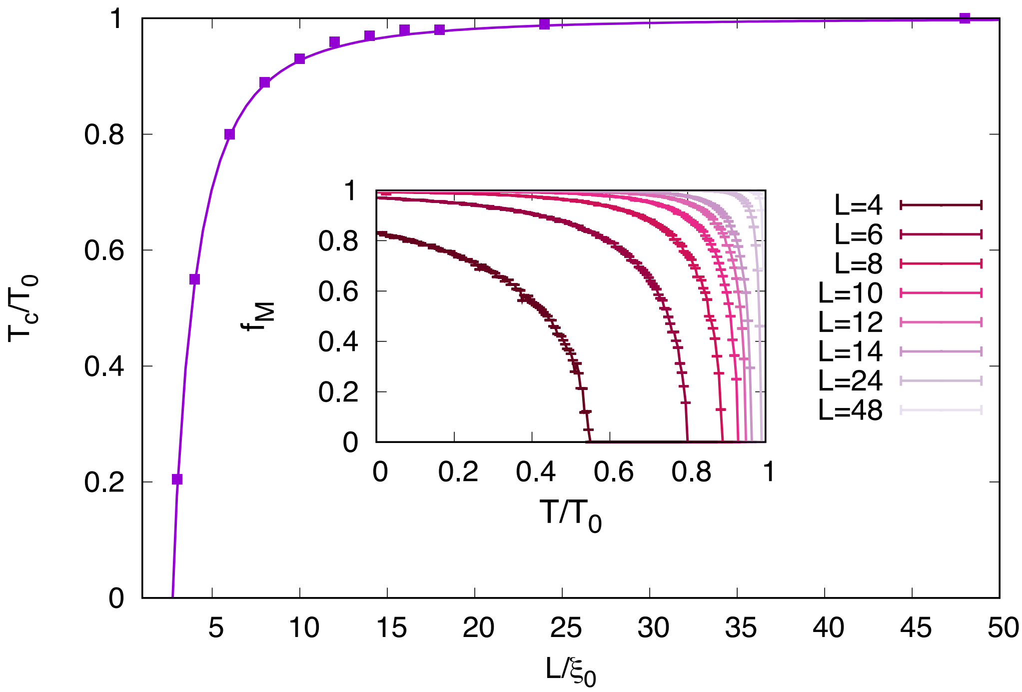

Figure 3: Critical temperature dependency on the linear size L. The points have been obtained from numerical simulation, while the continuos line from the analytical expression of T_c. In the inset the value of f_M is plotted as function of the temperature, for different linear sizes.

From this picture we can see that for $L \gtrsim 15$ the antiproximity effect almost disappears and the critical temperature becomes equal to the mean field critical one.
Since the linear size of the islands in the experiment[17] is $L \sim 10\xi_0$, the

[1]Since $\xi^2(T) = \xi_0^2 \cdot 0.74\frac{T_0}{T_0-T}$ the unit of length used in the simulation is ξ_0, while the unit of temperature is T_0.

antiproximity effect is not so relevant in this case. On the contrary, it is relevant considering instead of the linear dimension x, inside the plane, the dimension z perpendicular to it. Indeed, in [17] the two different island thickness considered are $h \sim 3\xi_0$ and $h \sim 5\xi_0$.
In this case, the junction to consider will be between a normal metal and the vacuum. However, since the condition at the interface of the vacuum will be $\frac{df}{dx}|_{x=h} = 0$, the results obtained describe also this other case, where the island thickness h will be: $h = \frac{L}{2}$.
These preliminary results suggest that the antiproximity effect, due to the contact between the normal metal and the superconducting islands, plays a significant role. The next step will be to include the effect of the islands spacing in the boundary condition of the eq. (6) , in order to look if the dependence of the critical temperature T_c^1 on the distance between the islands can be explained in these terms.

References

[1] J.M. Kosterlitz and D.J. Thouless 1973, *J. Phys. C.: Solid State Phys. 6* 1181.

[2] J.M. Kosterlitz 1973, *J. Phys. C.: Solid State Phys. 7* 1046 .

[3] V.L. Berezinskii 1970, *Sov. Phys.- JETP 32* 493.

[4] N.D. Mermin and H. Wagner 1966, *Phys. Rev. Lett. 17* 1133 .

[5] D. R. Nelson and J. M. Kosterlitz 1977, *Phys. Rev. Lett. 39* 1201

[6] D. McQueeney, G. Agnolet, and J. D. Reppy 1984, *Phys. Rev. Lett. 52* 1325

[7] P. Minnhagen 1987, *Rev. Mod. Phys. 59* 1001

[8] E. Granato and J.M. Kosterlitz 1989, *Phys Rev. Lett. 62* 7

[9] V. Alba, A. Pelissetto and E. Vicari 2009, *arXiv:0901.4682*

[10] I. S. Popov, P. V. Prudnikov and V. V. Prudinikov 2016, *Journal of Physics: Conference Series 681* 012015

[11] M. Rubinstein, B. Shraiman and D. R. Nelson 1983, *Phys. Rev. B 27* 3

[12] P.G. De Gennes 1966, *Superconductivity of Metals and Alloys*

[13] B. Sacepe, C. Chapelier, T. I. Baturina, V. M. Vinokur, M. R. Baklanov,M. Sanquer 2011, *Nat. Phys. 7* 239

[14] M. Mondal, A. Kamlapure, M. Chand, G. Saraswat, S. Kumar, J. Jesudasan, L. Benfatto, V. Tripathi, and P. Raychaudhuri 2011, *Phys. Rev. Lett. 106* 047001

[15] Y. Noat, V. Cherkez, C. Brun, T. Cren, C. Carbillet, F. Debontridder, K. Ilin, M. Siegel, A. Semenov, H.-W. Hubers, and D. Roditchev 2013, *Phys. Rev. B 88* 014503

[16] T. Cea, D. Bucheli, G. Seibold, L. Benfatto, J. Lorenzana, C. Castellani 2014, *Phys. Rev. B 89* 174506

[17] S. Eley, S. Gopalakrishnan, P. M. Goldbart and N. Mason 2011, *Nature 8* 59

[18] A. Allain, Z. Han and V. Bouchiat 2012, *Nature Materials 11* 590

[19] Z. Han, A. Allain, H. Arjmandi-Tash, K. Tikhonov, M. Feigel'man, B. Sacépé and Vincent Bouchiat 2014, *Nature Physics 10* 380

[20] I. Maccari, A. Maiorano, E. Marinari, J. J. Ruiz-Lorenzo 2015, *arXiv:1509.04593*