"Enrico Fermi" Chair
2021/2022

Lectures at Sapienza Università di Roma, January-May 2022

A History of the Science of Light
From Galileo's telescope to the laser and the quantum information technologies

Serge Haroche,
Ecole Normale Supérieure and
Collège de France, Paris

Lecture 14. May 5th 2022

Lecture 14:
Juggling with atoms and photons in a box

Outline of Lecture 14

Cavity QED as laboratory realization of quantum physics thought experiments

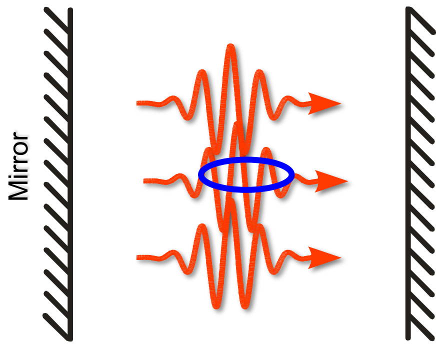
Resonant CQED experiments:
atom-photon and atom-atom entanglement, atom-photon quantum gates

Non-resonant CQED light shifts:
Quantum non demolition photon counting and observation of field quantum jumps.

CQED light shifts observed by microwave spectroscopy:
Deterministic projection of a coherent state on photon number state

Trapping one or a few photons in a box

Testing fundamental quantum laws…

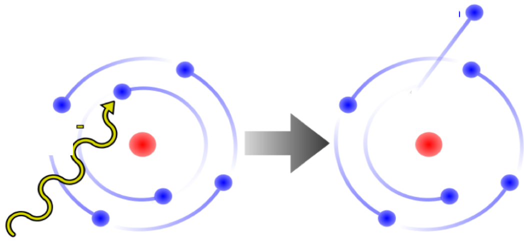


..and demonstrating elementary steps of quantum information processing

..and observe their interaction with one atom

The most fundamental process of light-matter coupling

Usual light detection by photoelectric effect destroys photon (Einstein, 1905):



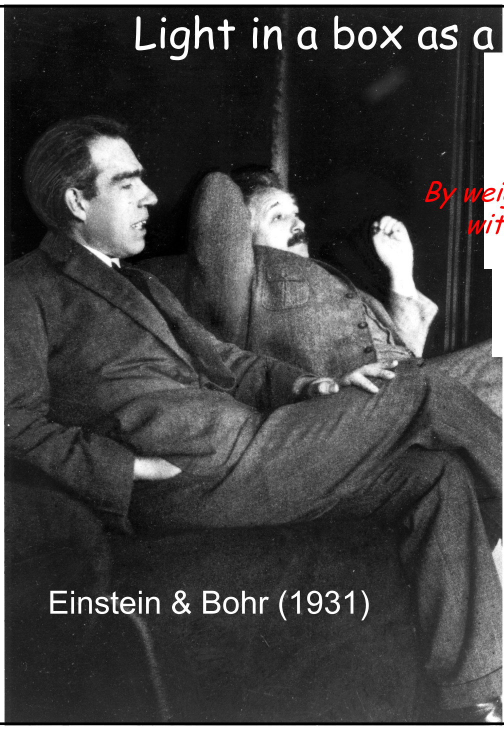
Photon annihilated while electron jumps in state continuum:

Photon observed by detecting the escaping electron

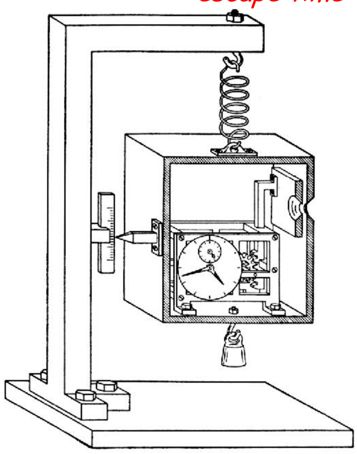
Similar annihilation process in ordinary vision

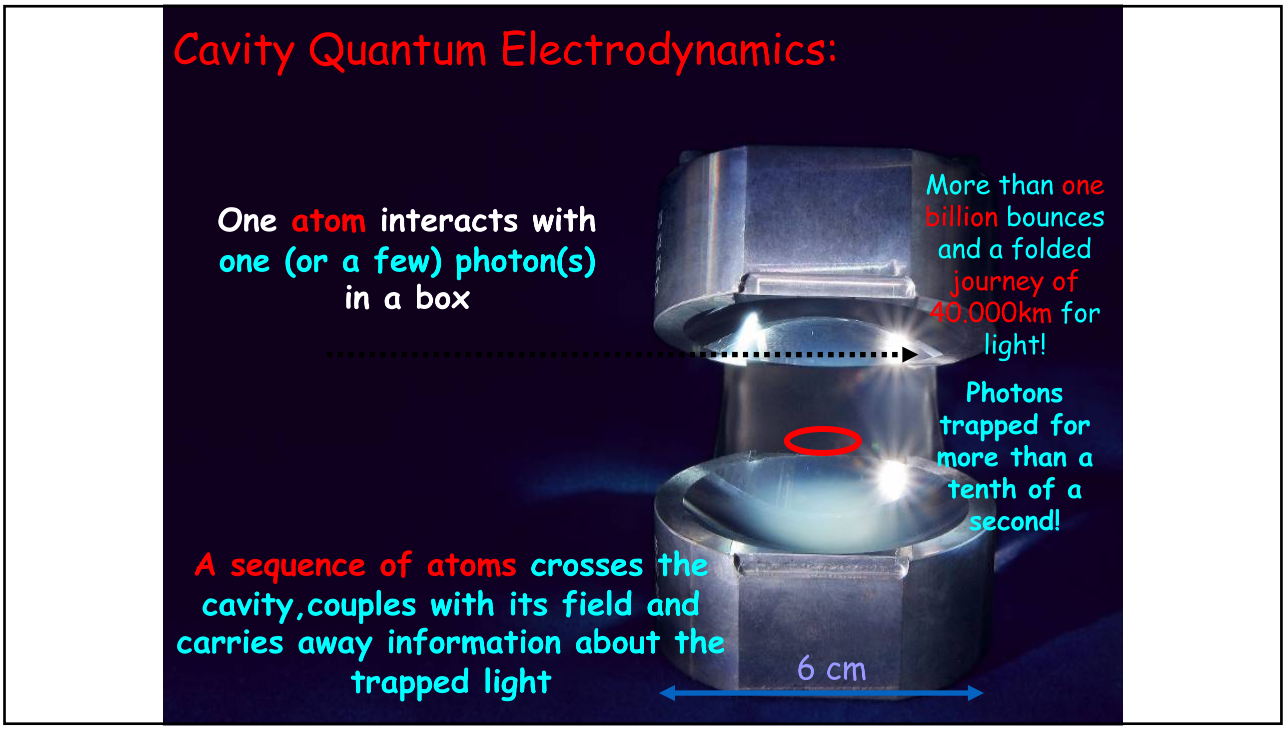
Can one detect photons differently, without destoying them?

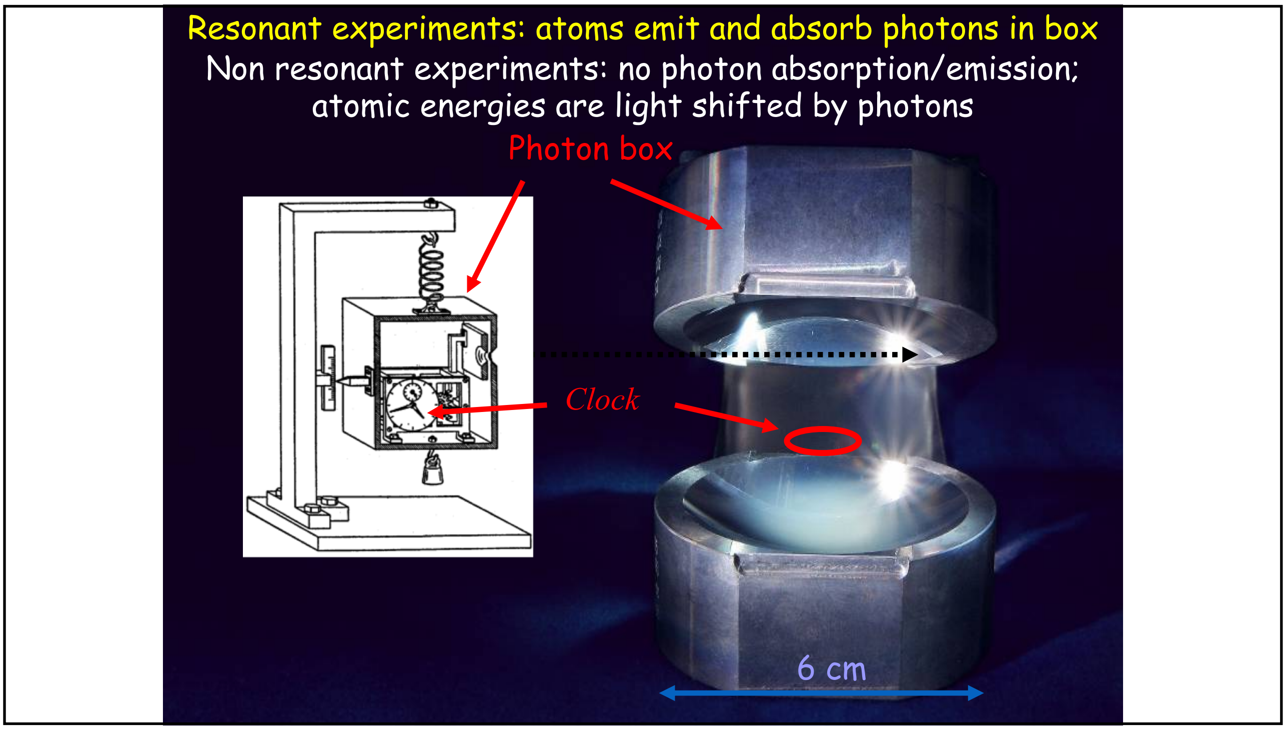
Light in a box as a thought experiment


Einstein & Bohr (1931)

By weighing a photon, one could detect it without destroying it and measure its escape time




Cavity Quantum Electrodynamics:
One atom interacts with one (or a few) photon(s) in a box
More than one billion bounces and a folded journey of 40.000km for light!
Photons trapped for more than a tenth of a second!
A sequence of atoms crosses the cavity,couples with its field and carries away information about the trapped light
6 cm


Resonant experiments: atoms emit and absorb photons in box
Non resonant experiments: no photon absorption/emission; atomic energies are light shifted by photons
Photon box
Clock
6 cm

Useful vacuum Rabi pulses (quantum « knitting »)

$|e,0\rangle \xrightarrow[\pi/2 \text{ Rabi pulse}]{} |e,0\rangle + |g,1\rangle$

Atom-field entanglement

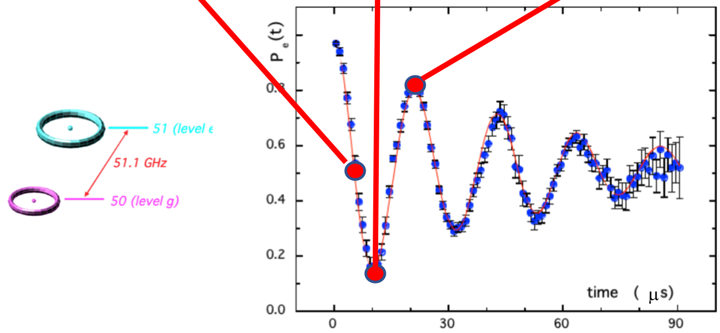
$|e,0\rangle \xrightarrow[\pi \text{ Rabi pulse}]{} |g,1\rangle$

Single photon preparation

$|e,0\rangle \xrightarrow[2\pi \text{ Rabi pulse}]{} -|e,0\rangle$

$|g,1\rangle \xrightarrow[2\pi \text{ Rabi pulse}]{} -|g,1\rangle$

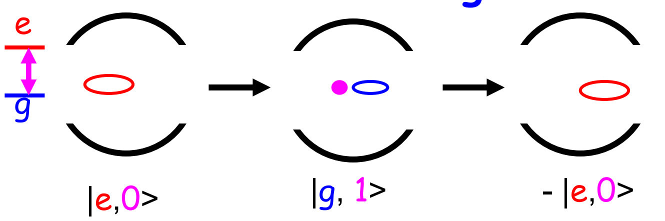
Phase shift of 0,1 photon state



Brune et al, PRL **76,** 1800 (96)

Microscopic entanglement

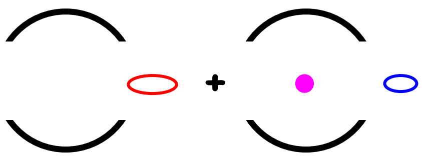
The resonant regime in a very good cavity: atom-field state superpositions and entanglement



Vacuum Rabi oscillation

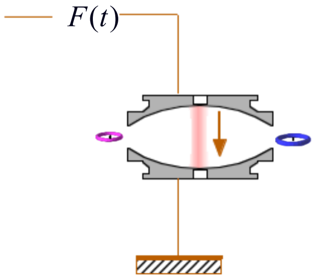
Reversible spontaneous emission and superpositions of combined atom-field states

a |e,0> + b |g,1>



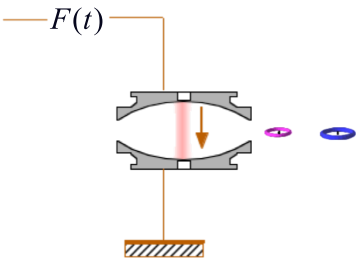
If atom emerges from cavity when a=b, there is a non-local maximum entanglement between atom and photon

Entangling two atoms by resonant photon exchange



$$|e_1,0\rangle\otimes|g_2\rangle \xrightarrow{\pi/2 \text{ vacuum Rabi pulse on atom1}} \frac{1}{\sqrt{2}}\left(|e_1,0\rangle+|g_1,1\rangle\right)\otimes|g_2\rangle$$

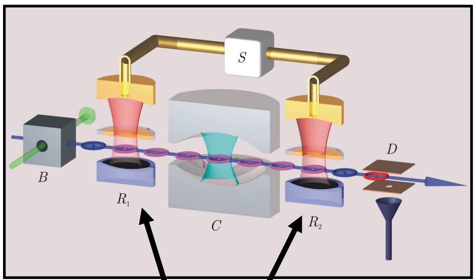
Atoms are entangled via their sucessive interactions with the cavity field which catalyses entanglement, remaining in vacuum at the end of process

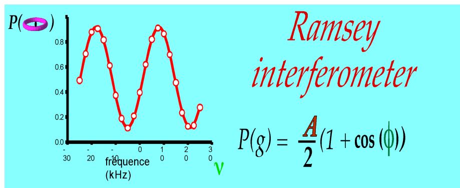


$$\xrightarrow{\pi \text{ Rabi pulse on atom2}} \frac{1}{\sqrt{2}}\left(|e_1,g_2\rangle+|g_1,e_2\rangle\right)\otimes|0\rangle$$

Electric field F(t) used to tune atoms 1 and 2 in resonance with C for times t corresponding to $\pi/2$ or π Rabi pulses

Ramsey interferometer with phase controled by field in the cavity



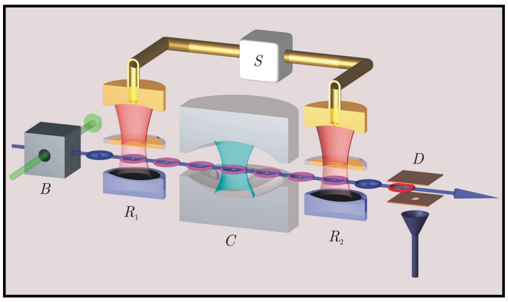


Probabilities P_e (or $P_g=1-P_e$) for finding atom in e or g oscillate versus ϕ .

Resonant classical $\pi/2$ pulses in auxiliary cavities R_1-R_2 (with adjustable phase offset ϕ between the two) prepare and analyse atom state superpositions

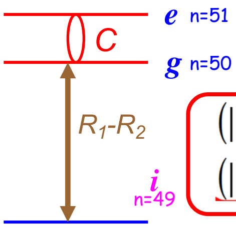
The phase of the atomic fringes and their amplitude depend upon the state of field in C, which affect in different ways the probability amplitudes associated to states e and g

Effect of 2π Rabi flopping on Ramsey signal



Cavity C resonant with e-g (51-50) transition.

Ramsey R_1-R_2 interferometer resonant with g-i (50-49) transition.



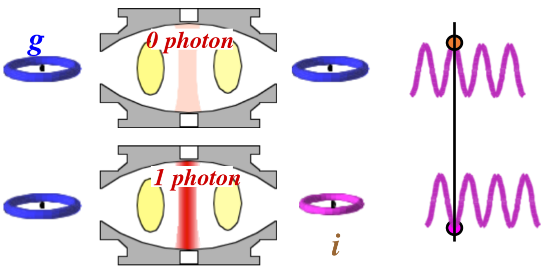
2π Rabi flopping on transition e-g in 1 photon field induces a π phase shift between the g and i amplitudes

$$(|i\rangle+|g\rangle)|1\rangle \rightarrow (|i\rangle+e^{i\pi}|g\rangle)|1\rangle$$

$$(|i\rangle+|g\rangle)|0\rangle \rightarrow (|i\rangle+\ \ |g\rangle)|0\rangle$$

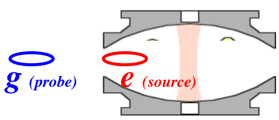
g-i fringes are inverted when photon number in C increases from 0 to 1

Ramsey fringes conditioned to one photon in C

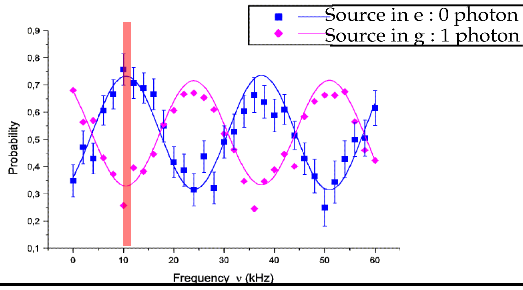


With proper phase choice, atom is detected in g if $n = 0$, in i if $n = 1$: quantum gate with *photon (0/1)* as control qubit and *atom (i/g)* as target qubit

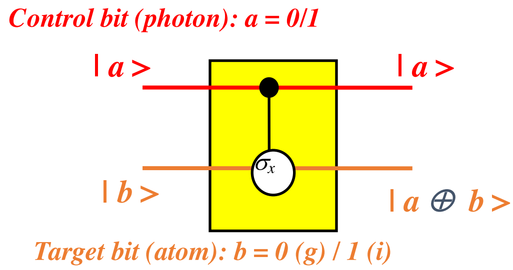
Correlation between 1st and 2nd atom detections



Experiment with 1st atom acting as source emitting 1 photon with probability 0.5 ($\pi/2$ pulse on e-g transition) and 2nd probe atom undergoing Ramsey interference on g-i transition



The quantum gate with photon as control bit realises a quantum non-demolition measurement (QND) of field



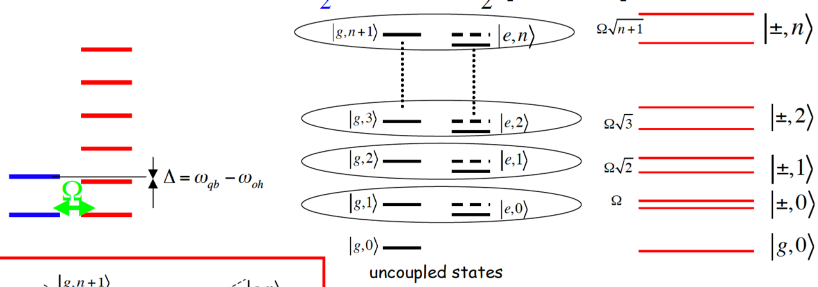
The atom carries away information about field energy without altering the photon number (2π Rabi pulse). This is very different from usual photon detection, which is destructive

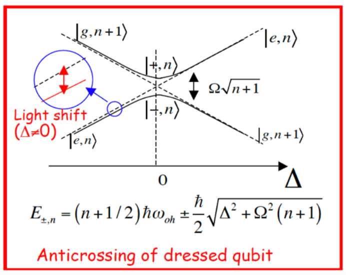
M.Brune et al, Phys.Rev.Lett. 65, 976 (1990)
G.Nogues et al, Nature 400, 239 (1999)
A.Rauschenbeutel et al, Phys.Rev.Lett. 83, 5166 (1999)
S.Gleyzes et al, Nature, 446, 297 (2007)
C.Guerlin et al, Nature, 448, 889 (2007)

See later how photon number can be measured with non-resonant method

Qubit-oscillateur coupling: the Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{qb}\frac{\sigma_z}{2} + \hbar\omega_{oh}a^{\dagger}a - i\frac{\hbar\Omega}{2}\left[\sigma_+ a - \sigma_- a^{\dagger}\right]$$





Rabi oscillation

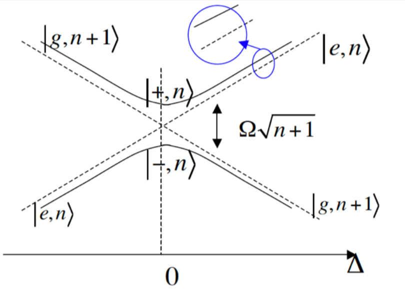
at resonance ($\Delta=0$)

$$|\pm,n\rangle = \frac{1}{\sqrt{2}}\left(|e,n\rangle \pm i|g,n+1\rangle\right)$$

$$|e,n\rangle \longrightarrow \cos\frac{\Omega\sqrt{n+1}t}{2}|e,n\rangle + \sin\frac{\Omega\sqrt{n+1}t}{2}|g,n+1\rangle$$

$$|g,n+1\rangle \longrightarrow -\sin\frac{\Omega\sqrt{n+1}t}{2}|e,n\rangle + \cos\frac{\Omega\sqrt{n+1}t}{2}|g,n+1\rangle$$

Non-Resonant coupling: light shifts in CQED



$$E_{\pm,n} = (n+1/2)\hbar\omega_{oh} \pm \frac{\hbar}{2}\sqrt{\Delta^2+\Omega^2(n+1)}$$

Anticrossing of dressed qubit

Second order perturbation theory:

$$E_{\pm,n} \approx (n+1/2)\hbar\omega_C \pm \hbar\left(\frac{\Delta}{2}+\frac{\Omega^2(n+1)}{4\Delta}\right)$$

Energy shift due to off-resonant coupling (distance between energy level and asymptot):

$$E_{+,n} - E_{e,n} \approx \hbar\frac{\Omega^2(n+1)}{4\Delta} + \ldots \quad ;$$

$$E_{-,n} - E_{g,n+1} \approx -\hbar\frac{\Omega^2(n+1)}{4\Delta} + \ldots$$

Vacuum shift (Lamb-shift):

$$\delta_{g,0} = 0 \quad ; \quad \delta_{e,0} = E_{+,0} - E_{e,0} \approx \hbar\frac{\Omega^2}{4\Delta}$$

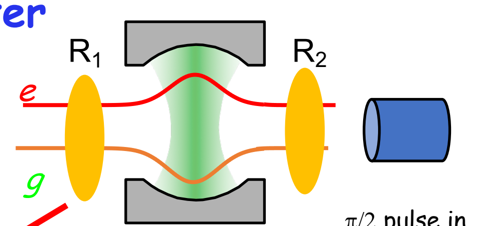
Light shift induced on eg transition by n-photons:

$$E_{+,n} - E_{-,n-1} = \hbar\omega_{eg} + \hbar\frac{\Omega^2}{2\Delta}n + \ldots \rightarrow \delta(\omega_{eg}) = \frac{\Omega^2}{2\Delta}n \quad ; \quad \varphi_0 = \frac{\Omega^2 t}{2\Delta}$$

φ_0=Phase shift per photon accumulated during time t

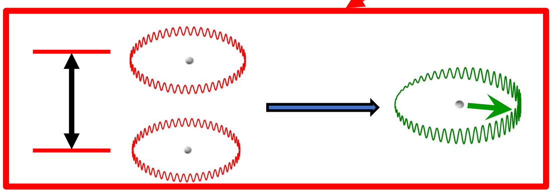
Principle of QND photon counting with Ramsey interferometer

A non-resonant atom undergoes a light-shift proportional to the photon number N, with opposite signs for levels e and g



The rotating dipole undergoes a photon number dependent phase shift in cavity

$\pi/2$ pulse in R_2 followed by e/g state detection measures direction of atomic dipole at cavity exit



$\pi/2$ pulse in R_1 prepares atom state superposition: De Broglie waves with n and n+1 oscillations (n=50) interfere positively on one side of orbit and destructively on other side, realize a wave packet rotating at 51 GHz in circular orbit plane

The circular Rydberg atom dipole is like the hand of a clock whose rotation is perturbed by the light shift produced by the photons in C

A non-resonant atom undergoes a light-shift proportional to the photon number N, with opposite signs for levels e and g

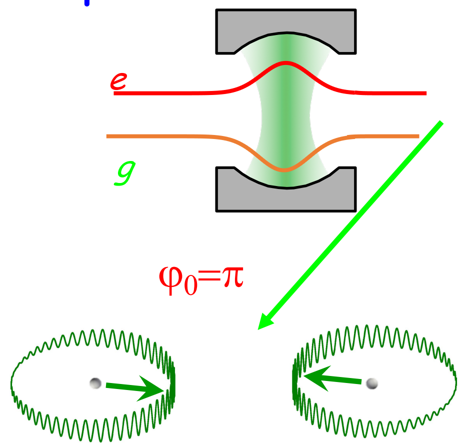
This effect produces a phase shift of the atomic dipole when the atom crosses the cavity:

$$\Delta\Phi(N) = N\varphi_0$$

φ_0 : phase shift per photon can exceed the value π

$$\Phi_0 = \frac{\Omega_0^2}{2\Delta} t_{\mathrm{int}} \sim \pi - 10\pi$$

$$(\Omega_0 / 2\pi = 50kHz\ ;\ \Delta \approx 2\Omega_0\ ;$$
$$t_{\mathrm{int}} \sim 5.10^{-5}s - 5.10^{-4}s)$$

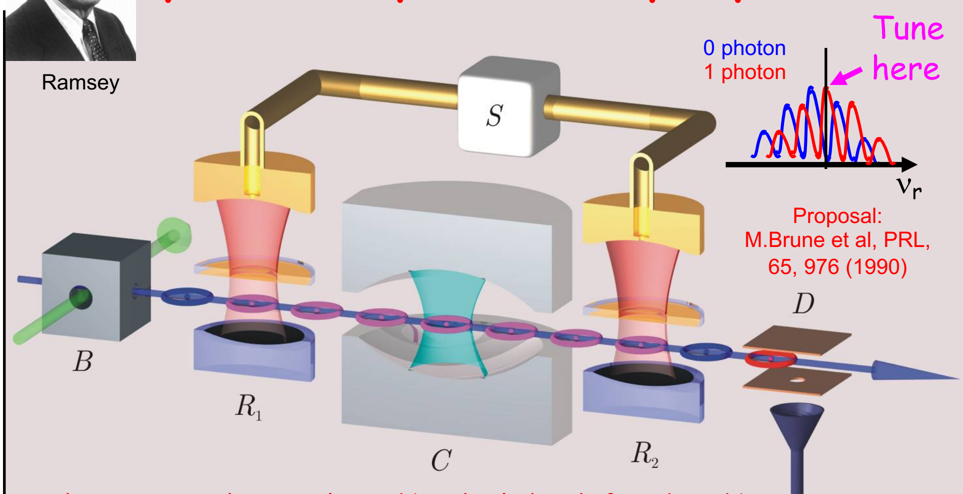


0 photon   1 photon

Measuring ΔΦ amounts to a non destructive photon counting

Atomic clock delayed by trapped photons (π phase shift per photon)

Ramsey

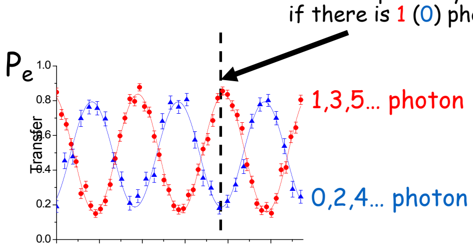


The atomic ticking is altered by the light shift induced by a microwave field in the slightly non-resonant cavity C. A single photon can delay clock by 1s/month (a "huge" 10^{-7} effect corresponding to half a fringe shift)

A π phase shift per photon measures photon number parity

Photon number (0/1) controls the atom state (e/g): a quantum logical gate

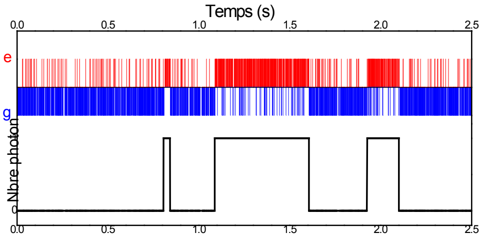
Atom most probably in e (g) if there is 1 (0) photon



If probability to have more than 1 photon is negligible, photon number (0 or 1) is counted with single atoms

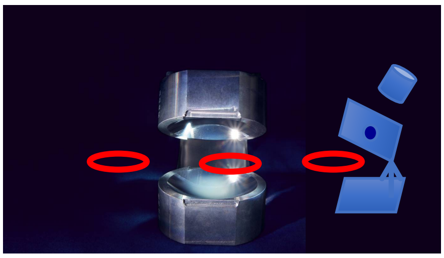
$T = 0.8K$
$P(0) = 0.95$
$P(1) = 0.05$
$P(n > 1)$ *negligible*

A quantum-non destructive (QND) measurement



Small thermal field continuously monitored with atoms undergoing π phase shift per photon

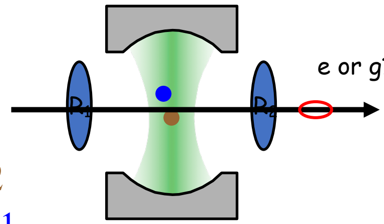
How to realize non-destructive photon counts in cavity QED?

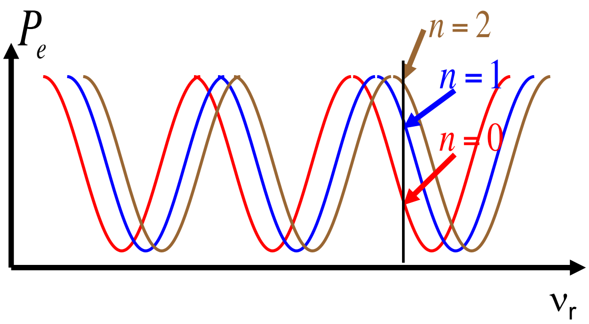


Use information carried by slighly off-resonant Rydberg atoms to measure the light-shifts induced by photons. The atoms are destroyed when detected, but the photons are not (non-resonant interaction). The process is the counter part of the non-destructive detection of atoms by photon counting in ion trap or cold atom physics. Quantum jumps of photons, analogous to those of ions or atoms, are observed in the sequence of atomic events when field suddenly changes due to external processes.

How the atomic clock counts several photons

Probability to detect atom in state e or g is given by a « Ramsey fringe signal » oscillating with a phase depending upon photon number n:



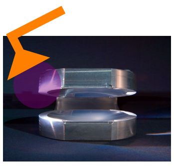


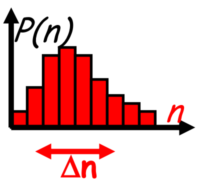
For a given setting ν_r, the probability for finding the atom in e (or g) takes different n-dependent values.

To pin-down n, enough statistics must be accumulated in time short compared to T_c!

The fringe phase shift measures the photon number **in non-destructive way (QND).**

QND measurement of arbitrary photon numbers: progressive collapse of field state



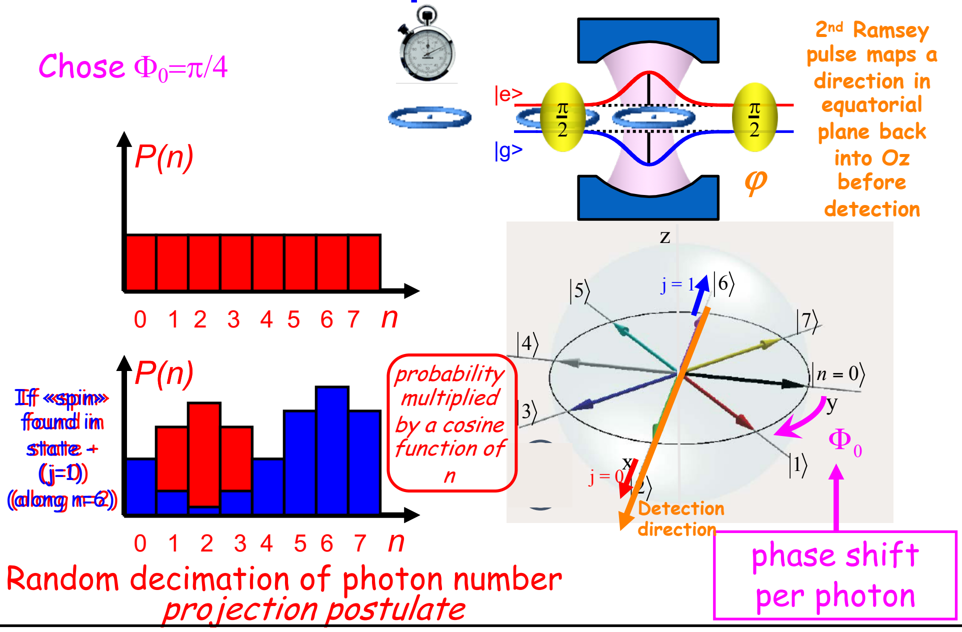


A coherent field (Glauber state) has uncertain photon number:
$\Delta n \Delta \phi \geq 1/2$
Heisenberg relation

A small coherent state with Poissonian uncertainty and $0 \leq n \leq 7$ is initially injected in the cavity and its photon number is progressively pinned-down by QND atoms

Experiment illustrates on light quanta the three postulates of measurement: state collapse, statistics of results, repeatability.

Counting larger photon numbers: 1st atom effect on inferred photon distribution



Photon decimation process explained by measurement projection postulate

Atom prepared by $\pi/2$ pulse in R_1 enters cavity with field in photon number superposition state:

$$|\psi\rangle=\sum_n c_n|e,n\rangle \xrightarrow{\pi/2\ R_1\ pulse} \frac{1}{\sqrt{2}}\sum_n c_n\left(|e,n\rangle+|g,n\rangle\right)$$

Photon number dependent phase shift in cavity (Φ_0: phase shift per photon):

$$\rightarrow \frac{1}{\sqrt{2}}\sum_n c_n\left(|e\rangle+e^{in\Phi_0}|g\rangle\right)\otimes|n\rangle$$

Atom is detected after $\pi/2$ pulse in R_2 with adjustable phase realizing a rotation bringing transverse spin direction φ back to energy basis. This amounts to measuring the transverse spin state:

$$|\pm\rangle_\varphi=\frac{1}{\sqrt{2}}\left(|e\rangle\pm e^{i\varphi}|g\rangle\right)$$

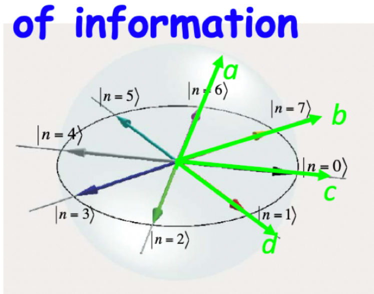
$$\xrightarrow[\text{spin state (direction }\varphi)]{\text{projection along }\pm\text{ transverse}} \frac{1}{\sqrt{Z_\pm}}\sum_n c_n\left(1\pm e^{i(n\Phi_0-\varphi)}\right)|\pm\rangle_\varphi\otimes|n\rangle$$

Probability that field contains n photons after +/- measurement is x by:

$$\frac{1}{2}\left|\left(1\pm e^{i(n\Phi_0-\varphi)}\right)\right|^2=1\pm\cos(n\Phi_0-\varphi)$$

Random +/- result

A step-by-step acquisition of information



To pin down photon number, send a sequence of atoms one by one….

…and change direction of spin detection to decimate ***different*** numbers

$$P^{(N)}(n) = \frac{P^{(0)}(n)}{2Z} \prod_{k=1}^{N} \left[1+\cos\left(n\Phi_0 - \phi(k) - j(k)\pi\right)\right]/2$$

$\phi(k)$: a/b/c/d $j(k)$: 0/1

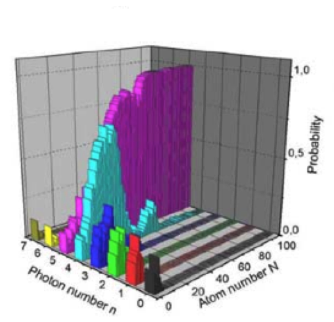
Spin reading $000101101010001011001\cdots$

Direction $abdcadb\,cbadcaa\,bcbacd\,b\cdots$

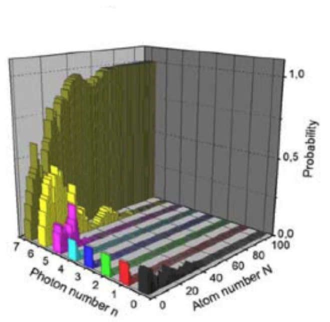
$$P^{(N)}(n) \longrightarrow \delta(n-n_0)$$

Progressive collapse!

Progressive field projection on Fock state

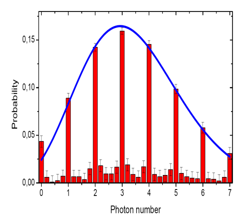


Convergence towards n=5



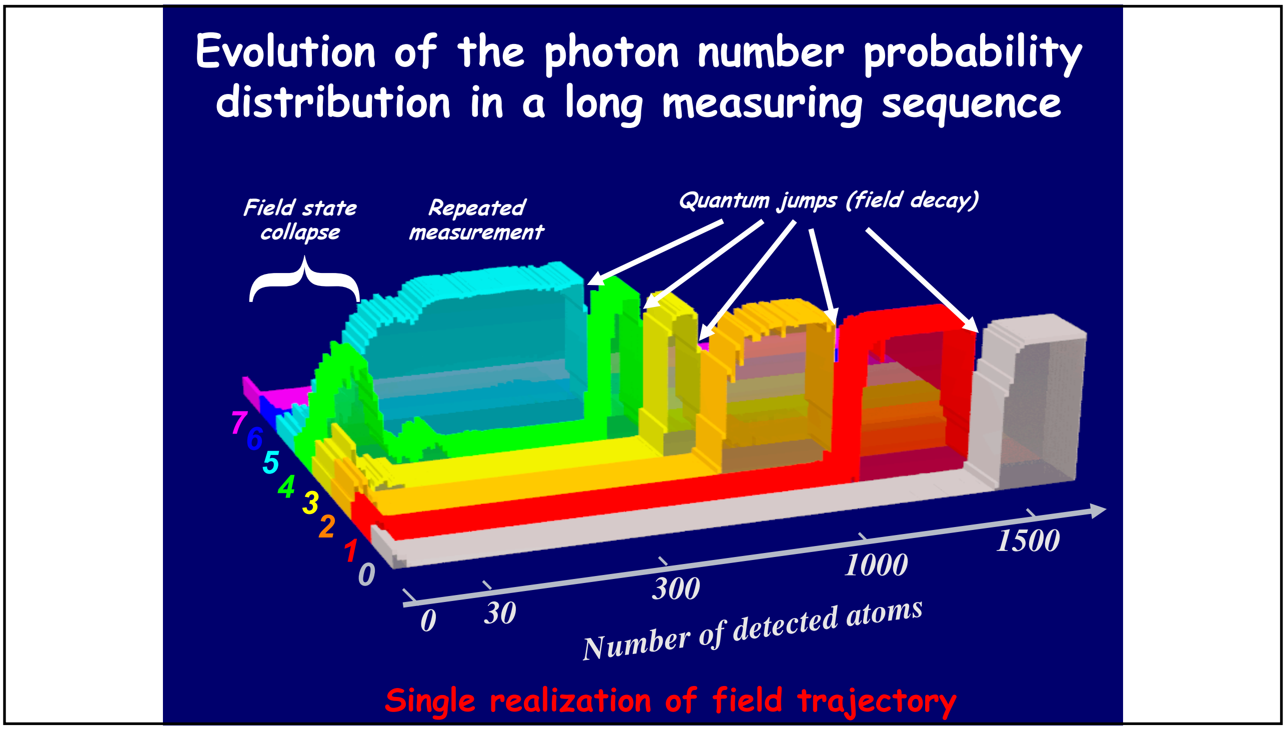
Convergence towards n=7

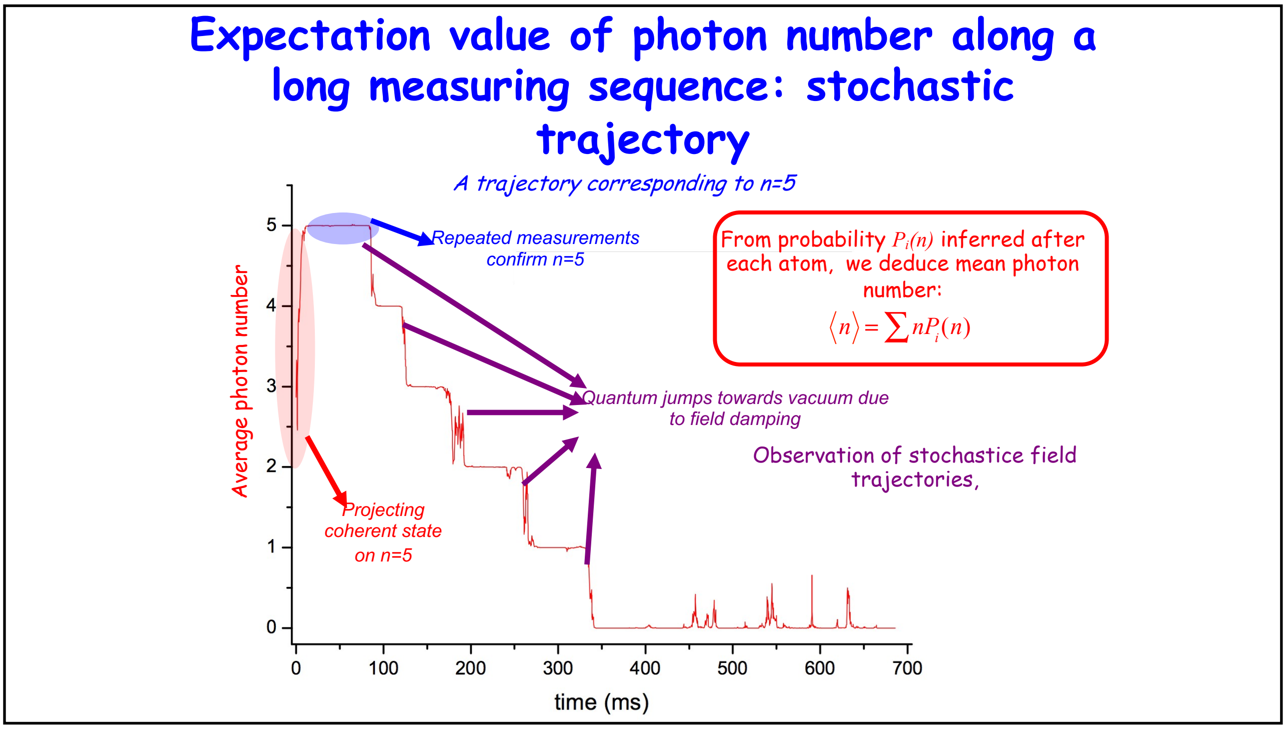
Histogram of results reconstructed from 2000 measurements of coherent state; the Poisson law is recovered.



Evolution of the inferred photon number distribution along independent sequences measuring an initial coherent state with photon numbers comprised between 0 and 7:
« God plays dice »

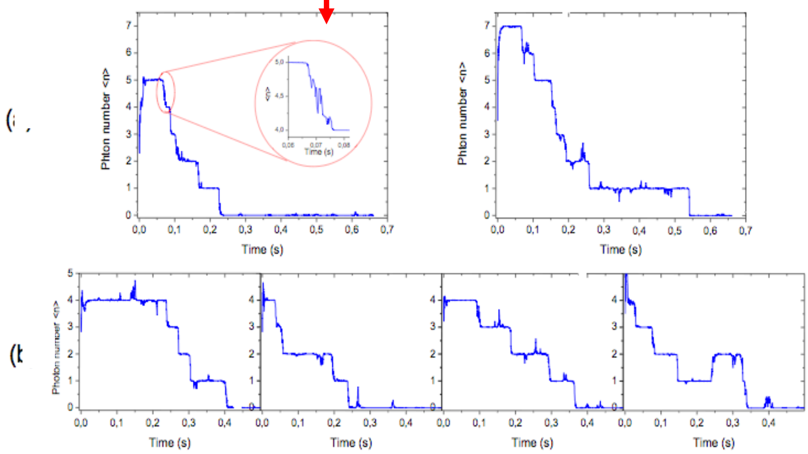
C.Guerlin et al, Nature, 448, 889 (2007)


Evolution of the photon number probability distribution in a long measuring sequence
Field state collapse
Repeated measurement
Quantum jumps (field decay)
7
6
5
4
3
2
1
0
0
30
300
1000
1500
Number of detected atoms
Single realization of field trajectory


Expectation value of photon number along a long measuring sequence: stochastic trajectory
A trajectory corresponding to n=5
Repeated measurements confirm n=5
From probability $P_i(n)$ inferred after each atom, we deduce mean photon number:
$\langle n \rangle = \sum nP_i(n)$
Quantum jumps towards vacuum due to field damping
Observation of stochastice field trajectories,
Projecting coherent state on n=5
Average photon number
time (ms)
0
1
2
3
4
5
0
100
200
300
400
500
600
700

Other trajectories

It takes some time for atoms to recognize that a jump has occured.

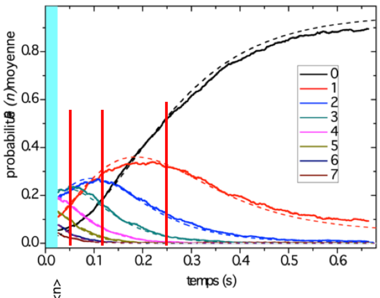


Four trajectories following a projection in n=4

A fundamentally random process (step durations fluctuate from one realization to the next one and only the statistics can be computed)

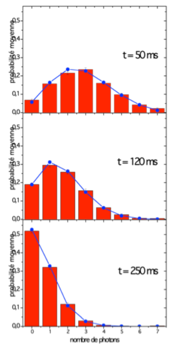
A statistical analyzis of trajectories: averaged evolution recovers classical field decay

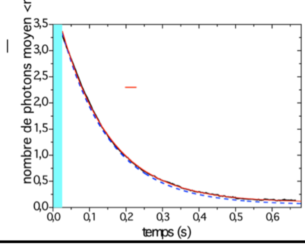
Analyzis of an *ensemble of trajectories* starting from the same initial coherent state: Probability $\Pi(n,t)$ for finding n photons at time t obtained by averaging thousands of measurements



Left: $\Pi(n)$ (n=0 to 7) versus t for an initial coherent state with <n>=3.5. full lines: experiment, dotted lines: theory. The blue bar at t=0 indicates duration of initial QND measurement.

Right: Histograms $\Pi(n)$ at times indicated by the 3 vertical lines of left figure. Blue line: theory. The photon number distribution remains Poissoniian during field damping. Experiment shows that photon number state |n> has a life-time T_c/n.

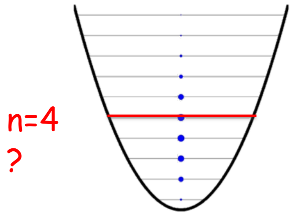




Left: Evolution of the mean photon number: the average of the « stair case » trajectories yields a smooth classical exponential field decay, without visible quantum jumps.

(Brune et al. PRL 101, 240402 (2008))

Use light shifts to project photon number in cavity



Does cavity contain n_0 photons or not?

Amounts to measuring the projector on $|n_0>$

Observable with eigenvalue 1 if n_0 photons, 0 otherwise

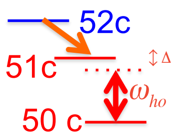
How to do it with a single atom?

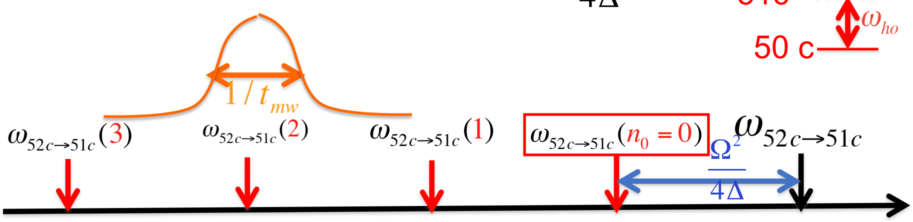
Perform high resolution spectroscopy of atom-cavity system resolving in one shot single photon light shifts

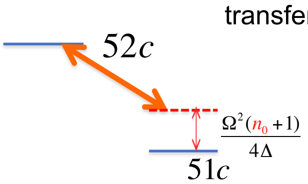
Atom-cavity spectrum on the 51c -52c transition (Cavity detuned by Δ from 51c-50c transition)

Cavity photons shift level 51c but not 52c

$$\omega_{52c\to 51c}(n_0) = \omega_{52c\to 51c} - \frac{\Omega^2(n_0+1)}{4\Delta}$$







If microwave is properly tuned, atom prepared in 52c is transferred by microwave to 51c only if n_0 photon in cavity.

Requires high resolution, hence long interrogation time t_{mw} (cold atoms)

$$\frac{\Omega^2 t_{mw}}{4\Delta} > 1$$

Probing system with this precision requires long time and slow atoms….

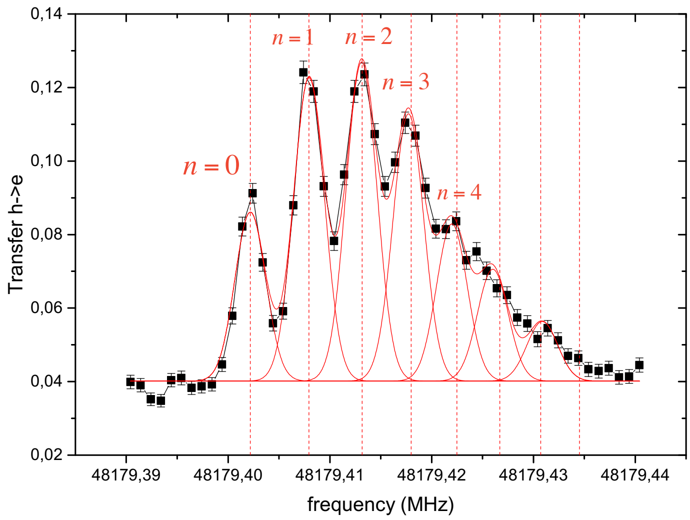
Electrodes to generate circularly polarized rf

Cavity QED set-up with a vertical atomic beam

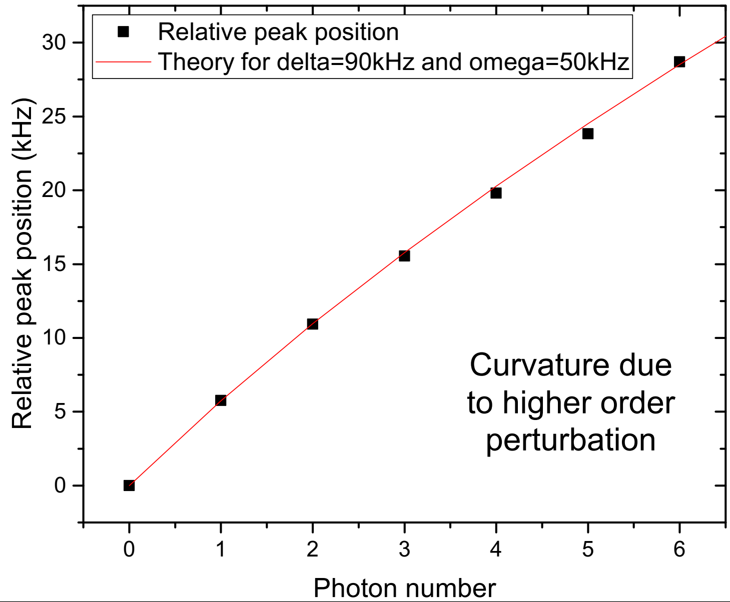
An atomic fountain fed by atoms cooled in a MOT. Atoms spend several milliseconds in cavity at top of parabolic trajectory

Circular Rydberg states are prepared in cavity by circularly polarized radiofrequency photons

Transfer 52c to 51c versus microwave frequency exhibits resolved photon numbers from 0 to 6 (coherent field in cavity)



Energy shift versus photon number

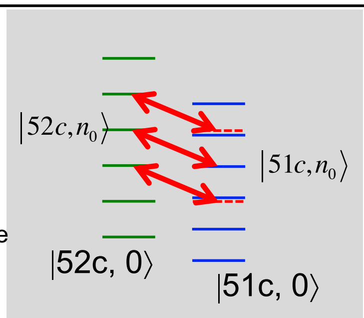


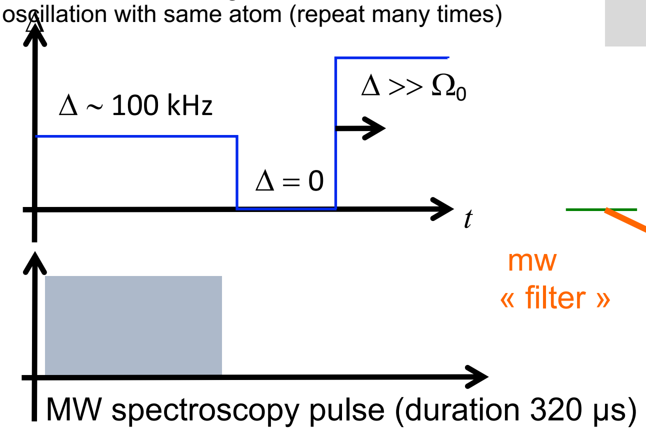
Photon number filter

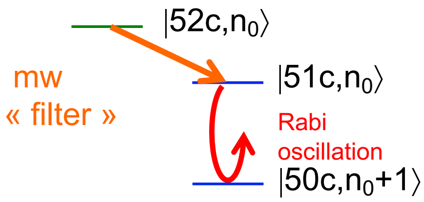
Atom, initially in 52c with cavity containing coherent field and detuned by Δ from 51c-50c frequency is irradiated during 0.3 ms by mw at $\omega_{52c\text{-}51c}$ (n_0) frequency.

If atom detected in 51c, n_0 photon are selected in cavity

To prove it, set cavity to resonance (Δ=0) during variable time t before detecting atom in 51c and record Rabi oscillation with same atom (repeat many times)

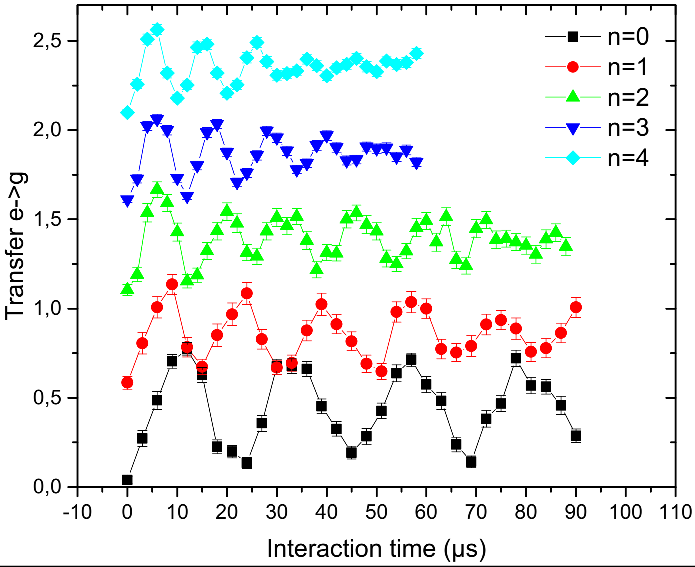






Rabi oscillations after selection of n_0 photons by same atom (n_0=0 to 4)

(PhD of Frederic Assemat)



Conclusion of Lecture 14

In this lecture, we have described resonant and non-resonant interaction processes between atoms and photons in the strong coupling regime of CQED

Using resonant coupling, we have seen how to entangle atoms via their successive coupling with the cavity field containing 0 and 1 photon and how to realize a quantum gate with the photon as a control and the atom as a target bit.

In the non-resonant case, we exploit photon number dependent light shifts to count photons in the cavity without destroying them (Quantum non demolition detection), to observe field quantum jumps and to filter pure photon number states out of coherent fields.

In the last lecture, we will see how to use the non-resonant atom-field interaction to generate and study superpositions of coherent fields with different phases (so called « Schrödinger cat states ») and we will study their decoherence. We will finally describe briefly how CQED has been generalized in circuit-QED to the study of the coupling of superconducting qubits to microwave fields, a very active domain of research today in quantum information science.