"Enrico Fermi" Chair 2021/2022

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A History of the Science of Light From Galileo's telescope to the laser and the quantum information technologies

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Lecture 12:

A passion for precision: the measurement of time

The progresses of high resolution spectroscopy in the microwave and optical domains have been correlated during the last 60 years with the advances of time metrology. Clocks have gained nine to ten orders of magnitude in precision, notably due to the development of the direct measurement of optical frequencies made possible by the invention of laser frequency combs. We analyse in this lecture these spectacular achievements and consider possible applications of this metrology of extreme precision.

Outline of lecture A reminder about Hydrogen spectroscopy as a test bench to demonstrate high precision spectroscopy methods A brief history of time measurement from pendulums to atomic clocks The frequency comb revolution Trapped ions optical clocks Neutral atoms optical clocks and applications

A test bench on precise measurements: a reminder about Hydrogen

The simplest of atoms has served as a test bench for high resolution laser spectroscopy, by using more and more precise methods. The comparison with theory gave new values of fundamental constants and made possible precise tests of Quantum Electrodynamics. Theodor Hänsch (Stanford, then Max Planck Institute in Garching, Germany) has been for over 50 years a pioneer in this domain of research.



























The frequency offset f_0 and the phase shift drift of the frequency comb

The phase shift drift is related to the fact that in general the mode frequency spacing is not an exact divider of the frequency of each mode. The comb frequencies are thus expressed as multiples of f_r plus a non-integer number f_o .

Consider for simplicity that the broad band laser spectrum is flat (all modes with same amplitude E_0). The field of the phase-locked superposition is a geometric series:

$$E(t) = \sum_{p=0}^{N-1} E_0 e^{2i\pi (f_0 + pf_r)t} = E_0 e^{2i\pi f_0 t} \times \sum_{p=0}^{N-1} e^{2ip\pi f_r t}$$
$$= E_0 e^{2i\pi f_0 t} \frac{e^{2iN\pi f_r t} - 1}{e^{2i\pi f_r t} - 1} = E_0 e^{2i\pi f_0 t} e^{i(N-1)\pi f_r t} \frac{\sin N\pi f_r t}{\sin \pi f t}$$

The light field is non-zero when the $sin(\pi f_rt)$ denominator is equal to zero, hence, for all k integers:

$$\pi f_r t = k\pi \rightarrow t = k / f_r = kT_r$$

The laser output is a sequence of pulses separated by $T_r=1/f_r$. The field phase is shifted from pulse to pulse by: $\Delta \varphi = 2\pi f_0 T_r = 2\pi f_0 / f_r$ To suppress this phase drift, the frequency comb can be adjusted so that f_0 is an integer multiple of f_r and all modes are harmonics of the fundamental frequency f_r . The comb parameters f_0 and f_r vary with the length L of the cavity and with the light intensity I via the non-linear medium index $n_i(I)$. L and I can be tuned independently to fix f_0 as multiple of f_r .

The time variation of the kth pulse then writes: $r_{r} = i\omega_{\tau} \sin N\pi f_{r} \tau$

$$E_{o}e^{i\omega_{c}\tau}\frac{\mathrm{SIII}N\pi f_{r}\tau}{\pi f_{r}\tau};$$

 $\tau = t - kT_r$ and $\omega_c / 2\pi = f_0 + (N-1)f_r / 2$ Frequency $\omega_c/2\pi$ of carrier is the average of the N modes frequencies

Mechanisms of Mode-Locking

The laser operation depends on non-linear optical effects. When several modes compete to extract energy from a large spectral band inverted medium, these non-linear effects favor regimes in which the modes cooperate to increase by positive interference the peak intensity of the field.

Active mode locking: insert in cavity an electro-optical crystal which modulates the medium transmission at a frequency f equal to the mode spacing of the laser. Initially the modes of the laser oscillate independently, with random phases. The modulation acting on a mode of frequency v generates side bands at frequencies v+f and v-f, very close to those of adjacent modes. Nearly degenerate oscillators (one mode and the side band of the adjacent one) are coupled because they interact with the same amplifying medium. Being very close in frequency, they have a tendency to lock in phase and to beat in unison. This locking extends, via side band generation, to many adjacent modes. This synchronization mechanism was first observed by Huygens: two pendulums of nearly same frequency and suspended to the same wall lock in phase after a few osciliations. Analyzed in the time domain, the modulation of the light transmission acts as a gate which periodically opens and favors the transmission of pulses of light synchroneous with the round trip time between the mirrors.

Passive mode locking: insert in cavity a saturable medium which absorbs in proportion more low intensity than high intensity one. Natural fluctuations of the phase between modes, corresponding to a temporary increase of the light intensity, will be preferably amplified, leading to a natural mode locking favoring the propagation of bursts of light making round trips between the mirrors. A particular case is the Kerr mode locking (KLM) effect already mentioned in lecture 9: a crystal with intensity dependent index acts as a lens focalizing intense light more than low intensity beams. A diaphragm placed behind the crystal will block a fraction of low intensity light but let all the intense light go through, thus favoring mode cooperation and the build up of a train of intense pulses.











Optical frequency clocks

The precision of the frequency determination of atomic and molecular lines is now limited by the definition of the second based on the microwave Cs fountain clock. Its uncertainty - about $10^{-16}-10^{-17}$ per day of averaging - is the present limit, while the stability of frequency comb lasers locked on an ultra-narrow atomic optical transition is two to three orders of magnitude better. Metrologists are thus considering to redefine the second based on such an optical transition in Strontium or Ytterbium atoms. Instead of using the FC as multipliers of radiofrequencies allowing to relate optical frequencies to the Cs standard, an optical clocks will employ them as frequency dividers playing the role of an escape mechanism referred to an ultra-stable optical standard . The extremely high quality factor of optical transitions towards metastable states gives to these clocks a stability and a precision far superior than that of microwave clocks. The principle of the optical clock operation is sketched below.









Comparing two Al⁺ optical clocks

Comparing an Al-Be and an Al-Mg clock. The same fibre-laser at 1070 nm, whose second harmonic is locked on an ultrastable cavity, probes by its 4th harmonic (at 267.4 nm) the two Al⁺ ions. An acousto-optic modulator (AOM) is used to lock the fibre-laser on the Al-Mg clock with an offset fixed by the acousto-optic modulator AOM1. The probe frequency of the Al-Be clock is locked by the modulator AOM2. The difference between the frequencies of the AOM1 and AOM2 modulators indicates the mismatch between the two clocks.





Neutral atoms optical clocks

Comparison between the performance of the Al⁺ clock with a fountain microwave Cesium clock: The Al⁺ clock has a quality factor $Q = v/\Delta v \sim 10^{14}$, \sim about 10⁴ times larger than the Q factor of the standard Cs clock, but it operates with a single ion (recycled about every second) while each cycle of the Cs clock (which lasts also one second) implies about N= 10⁶ atoms, which gives an S/N advantage $\sim 10^3$ to the Cs clock. *Finally, the optical ion clock decreases the uncertainty over the microwave clock by a factor of the order of 10.*

In order to exploit better the potential precision advantage of optical transitions with a large Q factor, it is necessary to probe an ensemble of N atoms with N \gg 1. This is impossible with trapped ions which perturb each other. The solution is to build clocks with neutral atoms trapped in an optical lattice with one atom per lattice site. These clocks are now being developed in many metrology labs (Tokyo, Boulder, Paris, Braunschweig..). The atoms employed have two outer electrons (Sr, Yb..) and have clock transitions with a very large Q factor (${}^{1}S_{0}$ to ${}^{3}P_{0}$ transition).

The atoms experience energy light shifts in the optical potential wells in which they are trapped. These shifts depend on the polarizability of the levels which are in general different for the two levels involved in the clock transition. There are however « magic » wavelengths for which the two polarizabilities are equal. The transition frequency is then independent of the light intensity. (*The idea of the magic wavelength was given by Katori et al, PRL, 103, 153004 (2009)).*













The JILA Strontium Clock (Jun Ye et al)

> Uncertainty < 10⁻¹⁹ per day

1/20 second time mismatch between two clocks synchronized at the origin of the Universe

Gravitational red shift observed for altitude difference of about 1mm (Nature volume 602, pages 420–424 (2022))

Conclusion of Lecture 12

Time can now be measured with a precision such that two optical clocks do not deviate by more than a fraction of a second over the age of the universe. The definition of the second, based today on the Cs microwave clock standard, will probably soon be redefined with improved precision as a multiple of the period of an Ytterbium or Strontium optical clock. The time defined by these clocks will have to be accompanied by a precise specification of their position with respect to the gravitational field they experience.

These clocks, on ground or embarked in satellites, will allow physicists to test with increased precision the special and general relativity theories. They will also test cosmological theories predicting variations over time of fundamental constants (fine structure constant and ratio of electron over proton mass). With still increased sensitivity, they could be used to detect the tiny change in space-time curvature produced by gravitational waves. At a more practical level, they could also be used to improve navigation systems or to develop probes sensitive to small variations of the gravitational field due to spatial inhomogeneities or temporal variations of mass densities below ground (applications to geodesy and geophysics or to the detection of small motions of earth plaques announcing earthquakes or volcano activity).

Another prospective improvement could come from the development of clocks based on transitions between energy levels in nuclei, falling in the γ ray domain. This will require research to identify narrow γ ray nuclear lines and to extend the frequency comb technology. to the XUV and γ ray domains.