"Enrico Fermi" Chair
2021/2022

Lectures at Sapienza Università di Roma,
January-May 2022

A History of the Science of Light
From Galileo's telescope to the laser and the quantum information technologies

Serge Haroche,
Ecole Normale Supérieure and
Collège de France, Paris

Lecture 10. March 31st 2022

Lecture 10: High resolution spectroscopy and Laser induced quantum interference phenomena

High resolution spectroscopy has been developped in the 1970's as soon as tuneable lasers whose frequencies could be continuously varied over a wide spectrum interval have been invented. This lecture is devoted to the study of the spectroscopic methods which take advantage of the high monochromaticity of laser light and also exploit non-linear saturation effects linked to the high intensities achievable with lasers.

The main limitation to the resolution of optical spectroscopy in the pre-laser era was the Doppler effect induced by the random motion of atoms in the samples irradiated by light. I describe in this lecture saturation spectroscopy and two-photon Doppler free spectroscopy, two laser methods which have been invented to get rid of the Doppler effect and to reduce the line width down to the natural one, equal to the inverse of the natural life time of the levels implied in the studied transition.

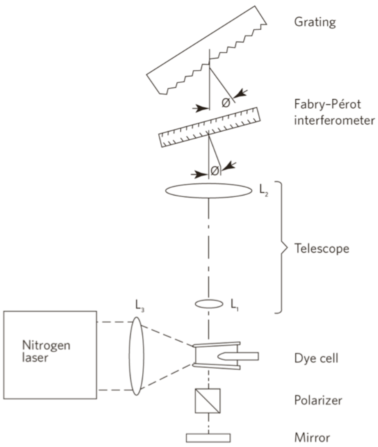
Another way to eliminate Doppler shifts is to perform spectroscopy on nearly motionless atoms. Methods to bring ions to rest in electromagnetic traps was developped since the 1970's and lasers were used to explore the spectrum of the trapped ions. I will describe these methods (laser cooling, state selective detection by observation of the ions quantum jumps). I will also show how the manipulation of ions with lasers makes it possible to employ them as qubits and to implement quantum gates for quantum information processing.

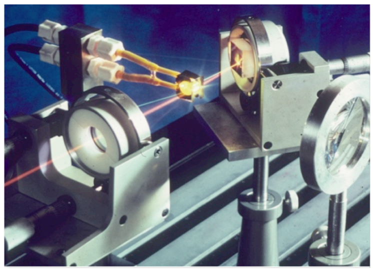
The experiments described in this lecture exploit quantum interference phenomena. I will briefly review in conclusion how these interferences are used in various ways in spectroscopy and in other kinds of quantum optics experiments.

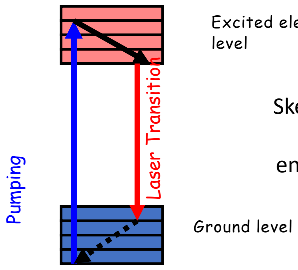
Theodor Hänsch

The 1970's: tuneable lasers open a new age for optical high resolution spectroscopy

Sketch of a pulsed dye laser (1971)



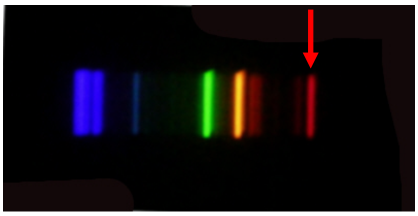




Sketch of dye molecule energy levels

Progress in spectroscopic resolution due to lasers (1960-2020)

Krypton lamp: red line at 605 nm defining the meter (1960)



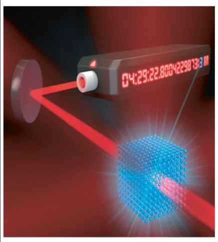
$\nu \sim 5.10^{14}\,Hz$; $\Delta\nu \sim 8.10^{8}\,Hz$ (*Doppler effect*)

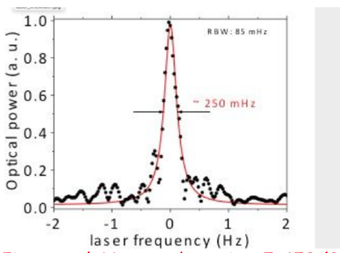
$\Delta\nu / \nu \sim 10^{-6}$

Coherence length (maximum path difference in interferometer):

$\sim 1\ meter$

Strontium and Ytterbium optical clocks probed by lasers (2020)





Y.Y Jiang et al, Nature photonics, 5, 158 (2011)

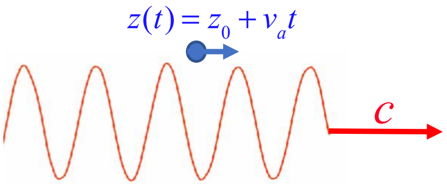
$\nu \sim 4.3\times10^{14}\,Hz\,;\,\Delta\nu \sim 0{,}25\,Hz$

$\Delta\nu / \nu \sim 6\times10^{-16}$

Coherence length: $\sim 10^{9}\ meters$

More than Earth-Moon round trip!

Doppler and recoil effect in absorption-emission by atom



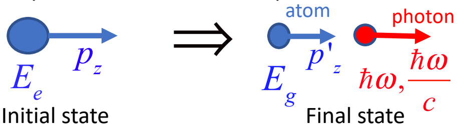
Atom moving with velocity v_a along a monochromatic light wave:

$$\cos\left[k.z(t)-\omega t\right]=\cos\left[k.\left(z_0+v_a t\right)-\omega t\right]=$$
$$\cos\left[kz_0-(\omega-kv_a)t\right]$$

In the atom rest frame, the light oscillation is seen at angular frequency $\omega_{v_a}=\omega-kv_a=\omega(1-\frac{v_a}{c})$

This is the Doppler effect For thermal velocities, v_a/c is ~ 10^{-5} 10^{-6} and the shift ~10^{9}-10^{10} Hz at optical frequencies

Interpretation in terms of photon (emission):



Momentum conservation

$$p_z=p'_z+\frac{\hbar\omega}{c}$$

Energy conservation

$$\frac{p_z^2}{2M}+E_e=\frac{p'^2_z}{2M}+E_g+\hbar\omega$$

and, eliminating p'_z between these two equations:

$$\frac{1}{2M}\left[p_z^2-(p_z-\hbar\omega/c)^2\right]=\hbar(\omega-\omega_{eg})$$

1st order Doppler shift — recoil energy / $\hbar$

$$\omega-\omega_{eg}=\frac{v_a}{c}\omega-\frac{\hbar\,\omega^2}{2Mc^2}$$

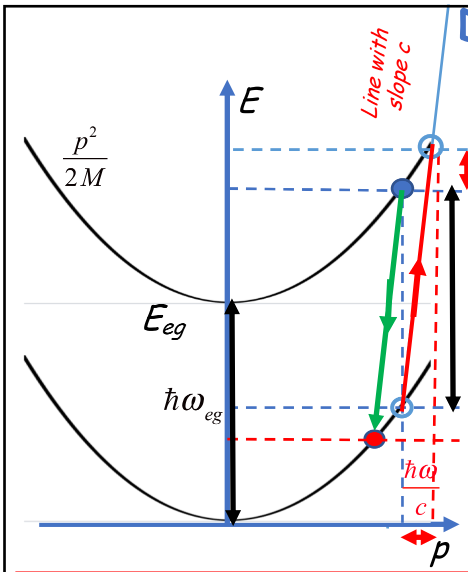
The emitted photon energy is increased by the Doppler effect (if atom moves in the direction of observer) and reduced by the kinetic energy of the recoiling atom. The recoil is a very small effect in optics, but large for γ ray emission (Mossbauer effect).

$$\Delta E_{recoil}=\frac{\hbar^2\omega^2}{2Mc^2}=\hbar\omega\times\frac{photon\ energy}{atom\ mass-energy}\sim10^{-10}-10^{-11}$$

The recoil shift changes sign in the case of absorption (see below).

Finally, there is a 2nd order small relativistic Doppler shift in v_a^2/c^2 due to time dilation in the atomic frame (see below).

Diagrammatic interpretation of Doppler and recoil effect



Parabolas representing variation with p of kinetic energy of atom

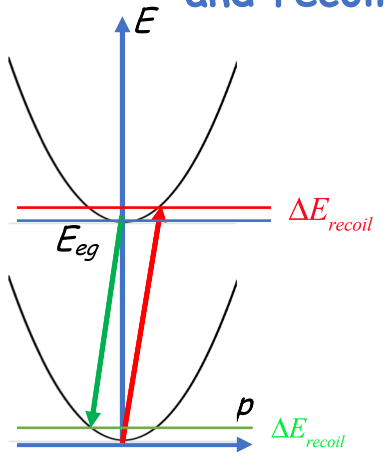
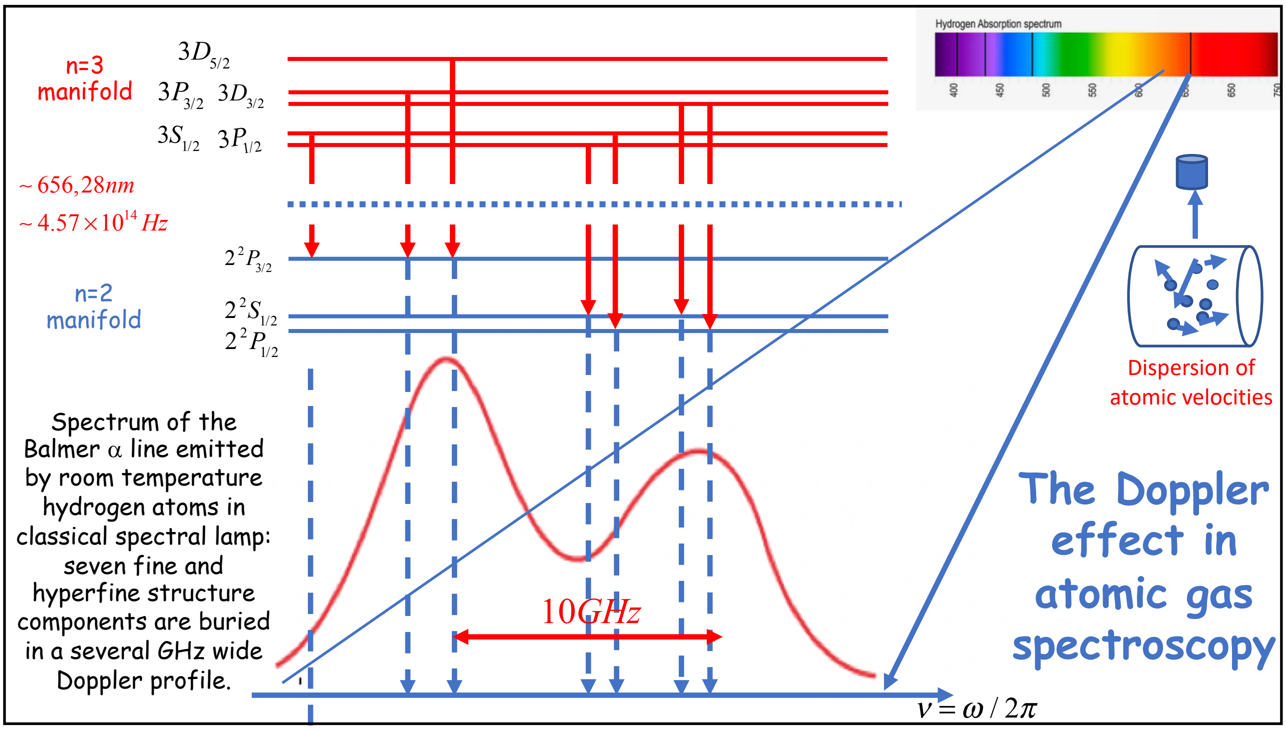


Diagram showing initial and final atomic energy and momentum for photon absorption (red line) and emission (green line) processes (second order recoil shift is dominated by 1st order Doppler effect and is not visible here)

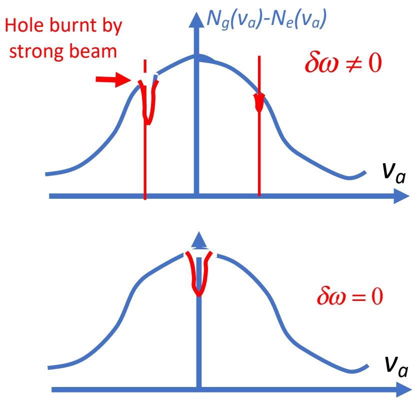
Diagram showing the recoil shift for an initially motionless atom: the photon frequency is increased by the recoil effect in absorption (red line) and decreased in emission (green arrow). To make effect more visible, the atom mass is smaller on this diagram than at left (lighter atom and steeper parabola)



Saturated absorption spectroscopy

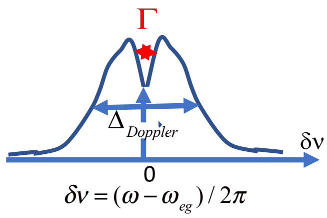
$$\omega=\omega_{eg}(1+v_a/c)\rightarrow v_a=c\frac{\omega-\omega_{eg}}{\omega_{eg}}\approx c\frac{\delta\omega}{\omega}\qquad\qquad \omega=\omega_{eg}(1-v_a/c)\rightarrow v_a=-c\frac{\omega-\omega_{eg}}{\omega_{eg}}\approx -c\frac{\delta\omega}{\omega}$$

If $|v_a|$(atom velocity along the light beam) is non-zero, light is resonant with two different classes of atoms propagating in opposite directions.

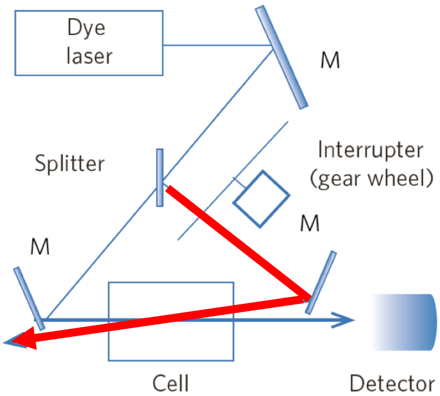


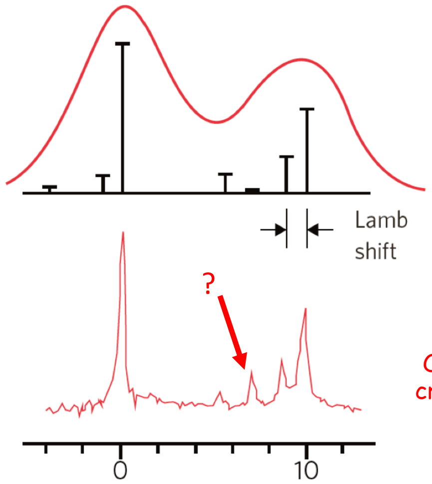
Distribution of atomic population difference between ground and excited state versus atomic velocity along Oz when a strong light beam propagates from right to left and a weak probe of same frequency from left to right: the strong beam produces a hole in the distribution for a negative velocity class (if δω is positive), while the weak probe interacts with atoms of opposite velocity. If δω is different from 0, the probe transmission is unaffected by the presence of the strong beam.

When δω is close to 0, the probe feels the hole burnt by the strong beam and its absorption is saturated (decreased as compared to what it is without strong beam): by scanning $\omega-\omega_{eg}$, one gets a narrow dip (called Lamb dip) in the center of the Doppler profile of the probe absorption.



Saturated absorption spectrum of Balmer α line of Hydrogen: first optical resolution of Lamb shift (Hänsch and Schawlow 1972)



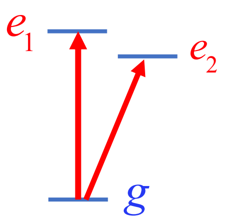


Strong beam is chopped and difference of probe absorption signal with strong beam on and off is recorded to eliminate the Doppler background

One extra-line: cross over signal

Cross-over resonances in saturation spectroscopy

Saturation spectroscopy in three level system



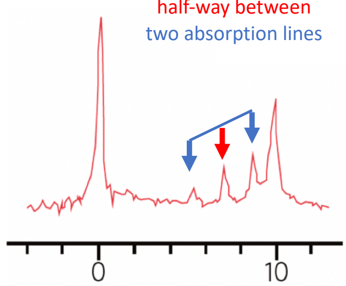
The population of ground state is depleted by the pump beam when it is resonant with the g to e_1 transition for atoms with velocity v_a such that:

$$v_a = -c\frac{\omega - \omega_{e_1g}}{\omega}$$

The same atoms will be resonant with the probe field on the g to e_2 transition provided:

$$v_a = c\frac{\omega - \omega_{e_2g}}{\omega}$$

Cross over resonance half-way between two absorption lines

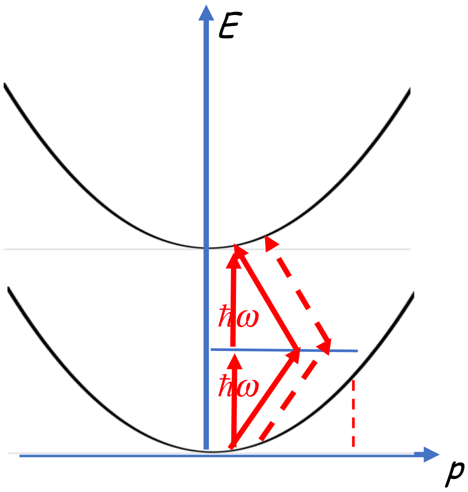


These two expressions of v_a are equal if:

$$\frac{\omega_{e_1g} + \omega_{e_2g}}{2} = \omega$$

When ω is the mean of ω_{eig} and ω_{e2g}, there is a class of atoms whose absorption on one transition is saturated by the pump beam resonant with the other transition. Via this class of atoms, the two transitions which share a common ground state « talk to each other », giving rise to a « *cross over* » peak in the signal.

2 photon Doppler free spectroscopy (Cagnac,Grynberg,Chebotaiev, Hänsch)



Absorption of two photons propagating in opposite directions: Doppler shift and recoil shift cancel: all atoms are resonant at same frequency whatever their velocities

$$E_e - E_g = \hbar\omega(1 - v_a / c) + \hbar\omega(1 + v_a / c) = 2\hbar\omega \; (\forall v_a)$$

$$p' = p + \frac{\hbar\omega}{c} - \frac{\hbar\omega}{c} = p \; (\forall v_a) \quad \textit{no recoil}$$

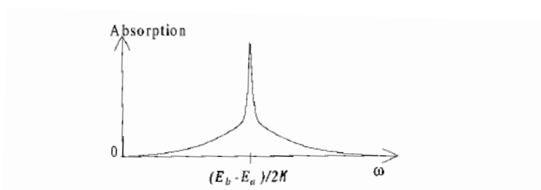
The only remaining velocity dependent shift is the second order Doppler shift (relativistic time dilation effect): the atom' « clock » is going slower than in the lab frame:

$$\omega = \omega_{eg}\left(1 - \frac{v_a^2}{2c^2}\right)$$

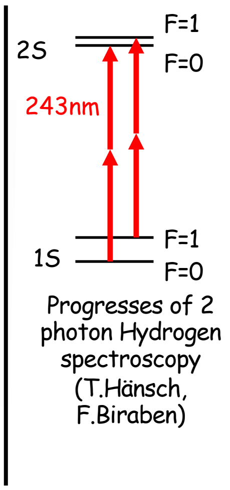
At thermal velocities, this is a 10^{-10} to 10^{-12} very small effect, which becomes negligible with cold atoms (10^{-20})

Two photon Doppler free spectroscopy of Hydrogen Lyman α line (n=1 to n=2) (Hänsch-1975-2000)

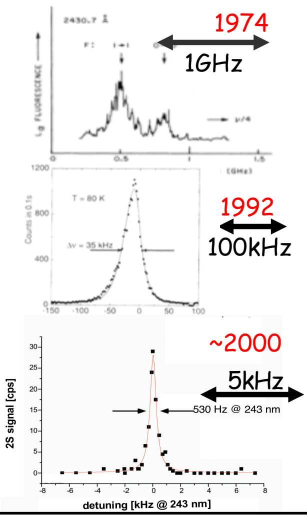
Non-linear absorption of two photons propagating in opposite directions: suppression of Doppler effect again



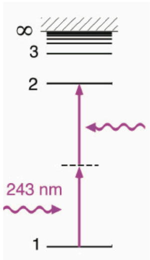
Applied to 1s-2s transition in H, with a frequency doubled dye laser. First with pulsed, then cw lasers…



Progresses of 2 photon Hydrogen spectroscopy (T.Hänsch, F.Biraben)

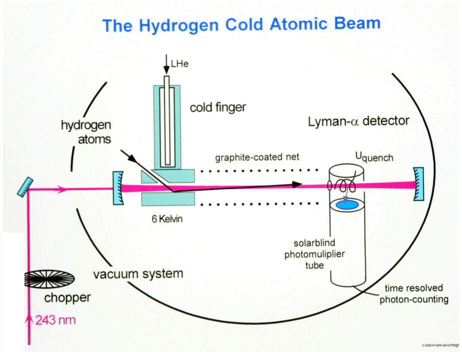


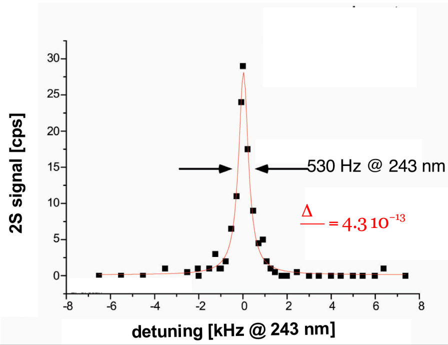
Two photon spectroscopy of the 1S-2S transition in H



(natural line width: 1.3 Hz)

(T.Hänsch group experiment in Garching, Germany)

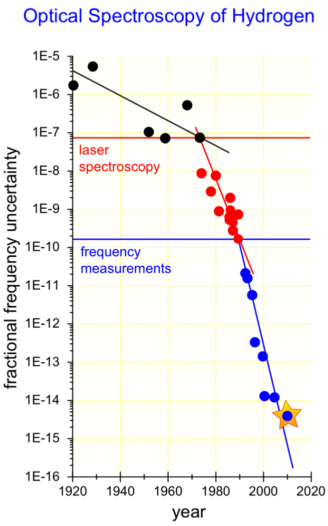




Doppler effect is suppressed. A factor of about 10^6 decrease of line width since the first experiments in the 1970's.

Progress of Spectroscopy of Hydrogen and resulting metrologic precision

C.G.Parthey et al, PRL 107, 203001 (2011)



Frequency of 1S-2S transition of H:
2 466 061 413 187 035 ± 10 Hz

$$\frac{\Delta\nu}{\nu} = 4.2\times10^{-15}$$

Rydberg constant:
R= 10 973 731, 568527 (73) m^{-1}

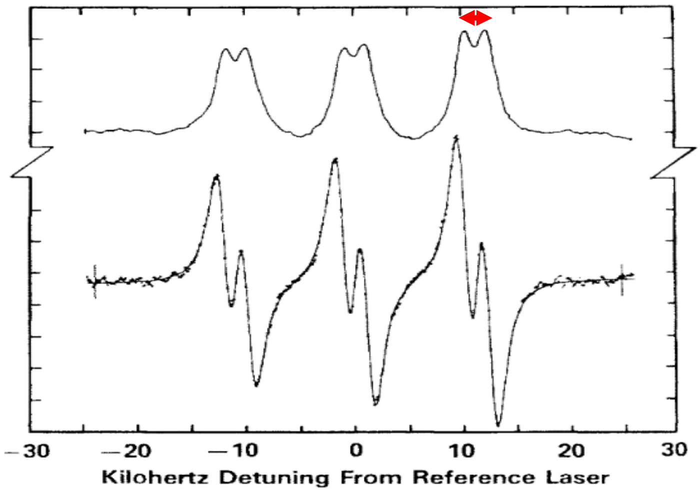
$$\frac{\Delta R}{R} = 6\,10^{-12}$$

Lamb-shift of 1S state:
8 172,876(29)MHz

To achieve these levels of precision, narrow optical lines are not sufficient…One must be able to count directly optical frequencies with frequency-comb lasers which have revolutionized time measurements (see Lecture 12).

Observation of the recoil doublet in saturation spectroscopy

Hyperfine spectrum of rotation-vibration infrared line of methane molecule



Hall, Bordé, Uehara (PRL, 1976)

$$\Delta\omega_{recoil} = \pm\frac{\hbar\omega^2}{2Mc^2}$$

Optical resonance of CH_4 molecule at 3.39 µm:

$$\nu = \frac{\omega}{2\pi} \sim 8{,}85\times10^{13}\,Hz$$

$$M_{CH_4} = 16\,a.u\,;\; M_{CH_4}c^2 = 16\times0{,}938\,GeV \sim 15\,GeV$$

$$\hbar\omega = h\nu \sim 58.4\times10^{-21}\,J = 0{,}365\,eV$$

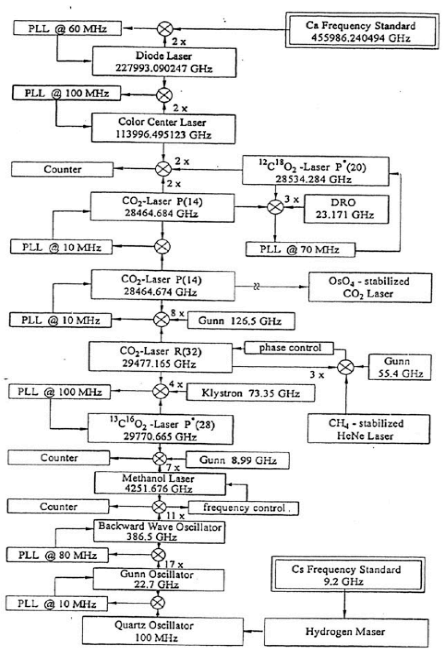
$$\frac{\hbar\omega}{2Mc^2} \approx 1.2\times10^{-11}$$

$$\Delta\nu_{recoil} = \pm1075\,Hz$$

Doublet splitting : 2150 *Hz*

A heroic endeavour: the microwave to optical frequency chain

Here the PTB frequency chain realized in the 1990's

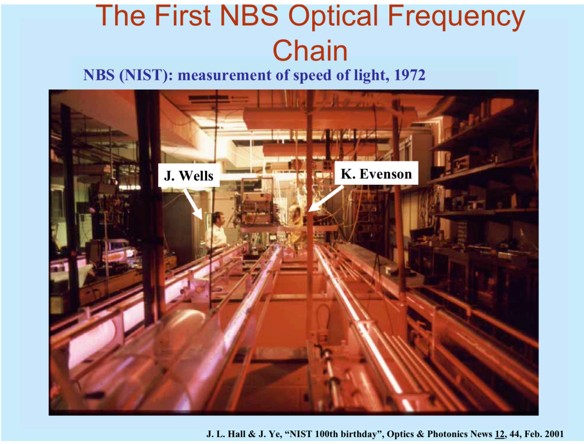


Laser locked on optical Ca line at 657 nm (~4.10^{14} Hz)

Comparing frequencies by harmonic generation and beat notes, amounting to a multiplication by ~ 5.10^4

Caesium Clock at $9.2\ 10^9$ Hz

Measurement of the speed of light and definition of the meter (Evenson, Hall et al, PRL 29, 1346 (1972)



Very delicate experiments performed in small number of metrology laboratories

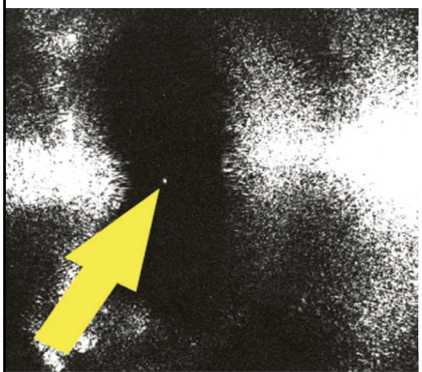
Interferometric measurement of the wave length λ of a laser locked on the 3,39 μm of CH_4 and measurement of its frequency ν with the first NIST frequency chain : product of λ and ν yields very precise value of c. This experiment led to fix c and to define meter from the measurement of time: meter is the distance travelled by light in 1/299792458 second (second defined by Cs clock)

$$\nu_{CH_4} = 88\,376\,181\,627 \pm 50\ kHz$$

$$\lambda_{CH_4} = 3\,392{,}231\,390 \pm 0.000\,01\ nm$$

$$c = \nu_{CH_4} \times \lambda_{CH_4} = 299\,792\,458\ m/s$$

Single ion spectroscopy: shelving method and quantum jump detection

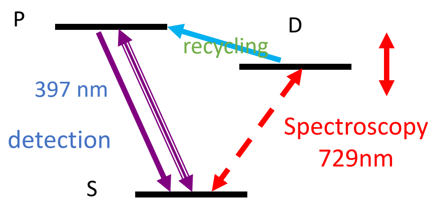


First detection of single ion in a trap by Toschek and Dehmelt(1978): excitation by a laser of a two-level transition induces a strong fluorescence visible through a microscope. For spectroscopy, the Doppler and recoil effects are suppressed. The last cause of line broadening is the excited state natural line-width Γ. The transition used to detect the ion must have a strong coupling to light, hence a large Γ, which is not good for spectroscopy. The solution is to take advantage of a three-level configuration and to perform spectroscopy on a transition towards a long lived metastable state while using a well allowed detection for detection. This is the *shelving method* which exploits the detection of *quantum jumps*.

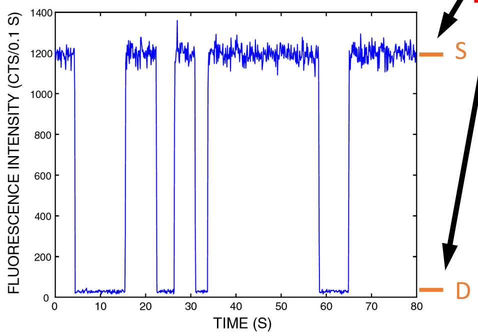
Quantum jump spectroscopy (Innsbruck group)

Simplified level scheme

Calcium ion

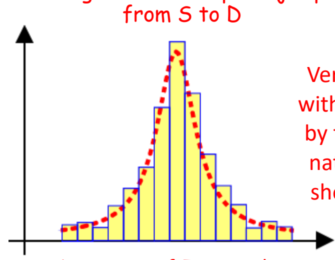


397 nm laser detects the atom in level S while it undergoes Rabi oscillation on S to D transition: the measurement induces the collapse of the state in S (fluorescence) or in D (no fluorescence)



Cycles of absorption and emission on S to P transition at 397 nm causes strong ion fluorescence which is interrupted when ion jumps from S to D (shelving state) under interaction with weak laser transition at 729 nm. Fluorescence reappears when atom falls back from D to S by spontaneous emission or is recyled by auxiliary laser excitation from D to P.

Histogram of absorption jumps from S to D

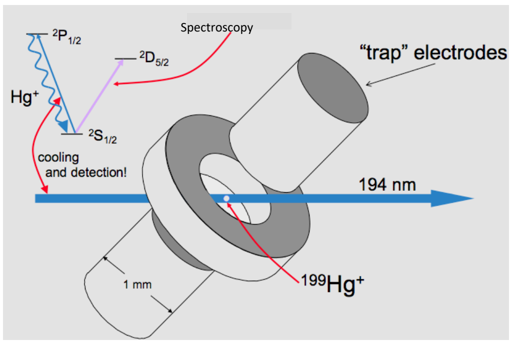


Very narrow line with width limited by the very small natural width of shelving D state

Detuning of 729 nm laser

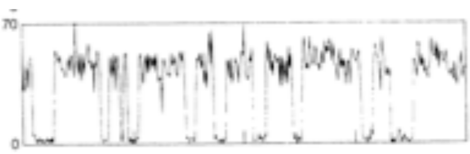
Blatt et al, Innsbruck

Other observation of single ion by quantum jumps



Nagourney, Sandberg et Dehmelt, Phys.Rev.Lett. 56, 2797 (1986)

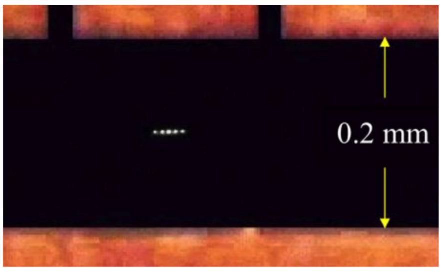
Sauter, Neuhauser, Blatt et Toschek, Phys.Rev.Lett. 57, 1696 (1986)

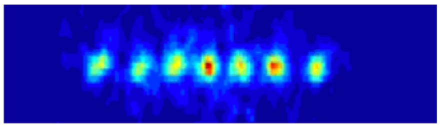


Bergquist, Hulet, Itano et Wineland, Phys.Rev.Lett. 57, 1699 (1986)

One dimension Ion crystals

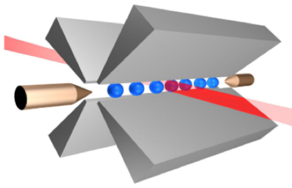
Beryllium (NIST)

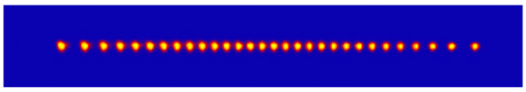




Calcium (Innsbruck)

Ions submitted to attractive linear potential and Coulomb repulsion. They vibrate around equilibrium position at different frequencies corresponding to different oscillation modes





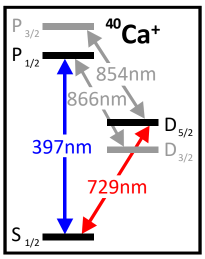
In center of mass translation mode, ions oscillate by global translation back and forth, keeping their mutual distance constant

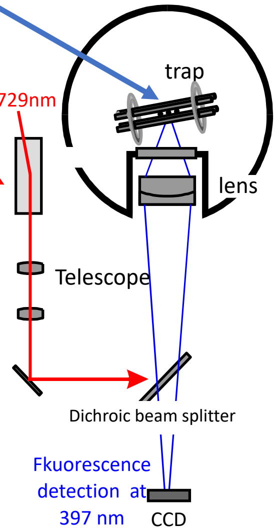
Selective addressing and detection of three trapped ions

Laser beam at 397 nm normal to figure plane excites the ions fluorescence on the S to P transition

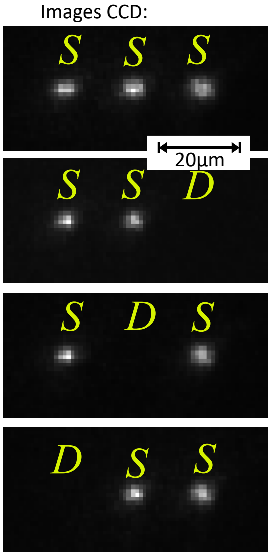
729 nm laser used to perform Rabi pulses on the S-D transition on selected ions

Optical fibre





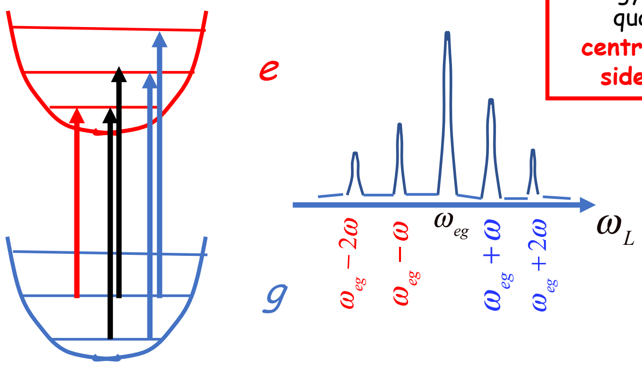
Images CCD:



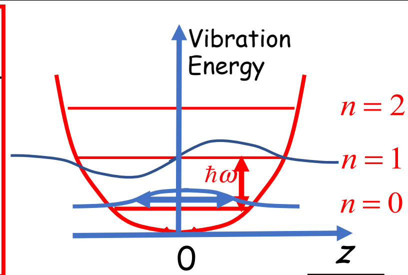
Ion tomography: ion visible in state S, invisible in state D

H.C. Nägerl et al., Phys. Rev. **A 60**, 145 (1999)

Laser spectrum of two-level ion in Harmonic trap: Lamb-Dicke regime



If η<1, ion moves over distances << λ: optical phase almost constant over ion's motion and recoil energy < vibration quantum : **central band > side bands.**



Zero-point fluctuations: $\Delta z_0 = \sqrt{\dfrac{\hbar}{2M\omega}}$

Lamb-Dicke parameter measures the localization of the ion within an optical wavelength :

$$\eta = k_L \Delta z_0 = 2\pi \Delta z_0 / \lambda = \sqrt{\frac{\hbar k_L^2}{2M\omega}} = \sqrt{\frac{Recoil\ energy}{\hbar\omega}}$$

Orders of magnitude:

$\omega / 2\pi \sim 5\times10^5\,Hz\,;\ \Delta z_0 \sim 0{,}03\mu m\,;$

$\lambda \sim 1\mu m \quad ; \quad \eta \sim 0.2$

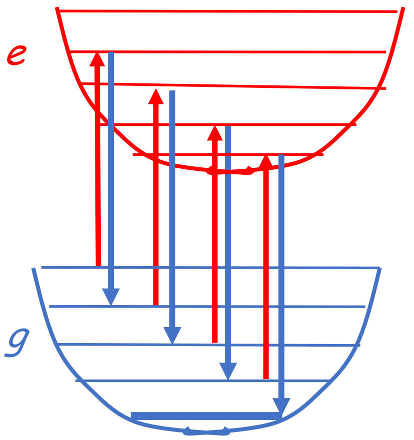
Doppler and recoil shift suppressed on central band: ideal condition for spectroscopy

Central band $g,n \to e,n \quad ; \quad \omega_L = \omega_{eg}$

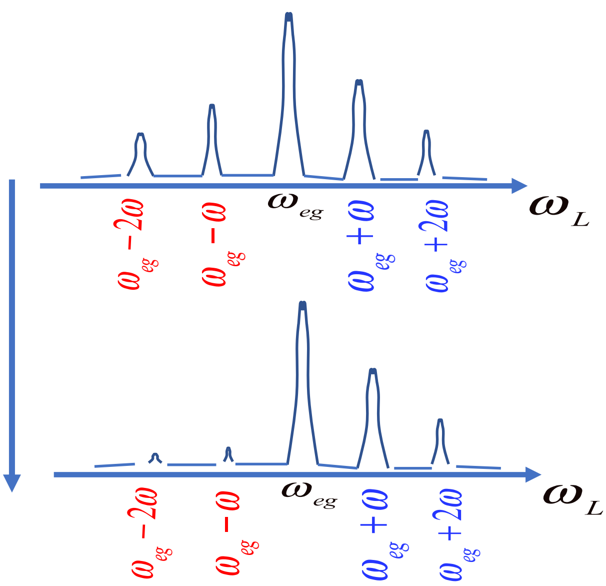
1st blue side-band $g,n \to e,n+1 \quad ; \quad \omega_L = \omega_{eg} + \omega$

1st red side-band $g,n \to e,n-1 \quad ; \quad \omega_L = \omega_{eg} - \omega$

Side-band cooling of trapped ion



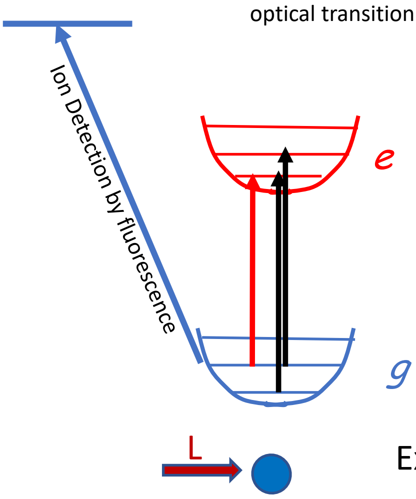
Ion excited on first red side-band fluorescesces mostly on carrier central band: optical pumping mechanism brings step by step ion to vibration ground state (one vibration quantum lost at each cycle)



As cooling progresses, the red side-band becomes weaker and finally disappears when ion is pumped in the vibration ground state

Two kinds of ion qubits

Qubit states:
ground state (g) and metastable state (e) linked by a one-photon « forbidden » optical transition



Carrier : $\omega_L=\omega_{eg}$

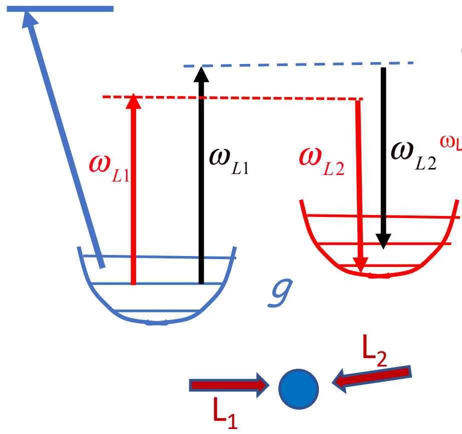
(One qubit gate)

Red side band: $\omega_L= \omega_{eg}-\omega$

(Two qubit gate)

Ex: Ca

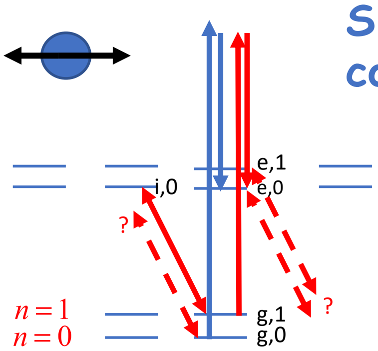
Qubit states:
Two hyperfine ground states g and e linked by a coherent two-photon Raman process



Carrier : $\omega_{L1}-\omega_{L2} = \omega_{eg}$

Red side band: $\omega_{L1} - \omega_{L2} = \omega_{eg} - \omega$

Ex: Be



Single ion in trap: internal states e and g coupled to vibration mode in n=0,1 states

The qubit levels are the hyperfine ground state levels F=1/2, m_F=1/2 state (g) and F=3/2, m_F=1/2 state (e). They are coupled by Raman processes induced by a couple of laser beams: a $\Delta m_F= 0$ carrier Raman pulse ($\omega_{L1}-\omega_{L2} = \omega_{eg}$) and a $\Delta m_F=0$ red side band Raman pulse ($\omega_{L1} - \omega_{L2} =\omega_{eg}-\omega$). A third auxiliary level i (F=3/2, m_F=-1/2) is coupled to g,1 only by a $\Delta m_F = -1$ red side band Raman pulse.

Action of a σ_- polarized 2π Raman pulse on red-side band on qubit in state g or e in the presence of 0 or 1 vibration quantum: the qubit + vibration mode undergoes a π-phase shift of its quantum state *if and only if* ion is in g and there is one vibration quantum.

$$|g,0\rangle \xrightarrow[2\pi\ pulse]{\sigma\_red\ side-band} |g,0\rangle \,;\, |e,0\rangle \xrightarrow[2\pi\ pulse]{\sigma\_red\ side-band} |e,0\rangle \,;\, |e,1\rangle \xrightarrow[2\pi\ pulse]{\sigma\_red\ side-band} |e,1\rangle$$

$$|g,1\rangle \xrightarrow[2\pi\ pulse]{\sigma\_red\ side-band} -|g,1\rangle$$

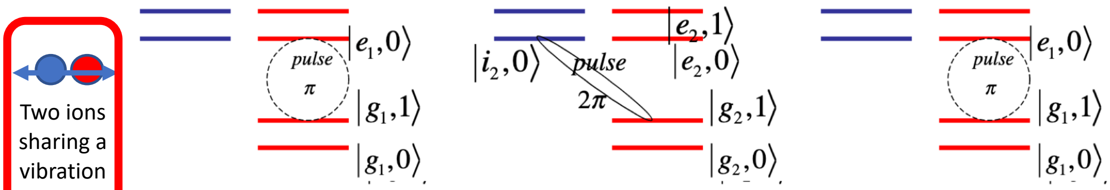
A useful gate operation

Cirac-Zoller quantum gate: 2 ions coupled to common vibration mode in n=0,1.

π Rabi-Raman pulse on red side-band on ion 1: $|g,0\rangle \to |g,0\rangle$, $|e,0\rangle \to |g,1\rangle$

2π Rabi-Raman pulse on red side band on ion 2: $|g,1\rangle \to -|g,1\rangle$

π Rabi-Raman pulse again on red side-band on ion 1



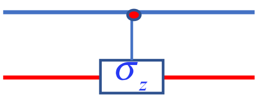
$$|g_1,(g_2);0\rangle \to |g_1,(g_2);0\rangle = |(g_1),g_2;0\rangle \to |(g_1),g_2;0\rangle = |g_1,(g_2);0\rangle \to |g_1,g_2;0\rangle$$

$$|g_1,(e_2);0\rangle \to |g_1,(e_2);0\rangle = |(g_1),e_2;0\rangle \to |(g_1),e_2;0\rangle = |g_1,(e_2);0\rangle \to |g_1,e_2;0\rangle$$

$$|e_1,(g_2);0\rangle \to |g_1,(g_2);1\rangle = |(g_1),g_2;1\rangle \to -|(g_1),g_2;1\rangle = -|g_1,(g_2);1\rangle \to |e_1,g_2;0\rangle$$

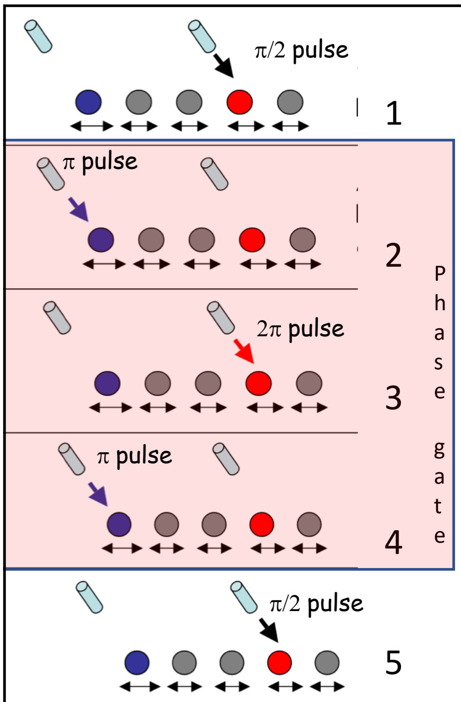
$$|e_1,(e_2);0\rangle \to |g_1,(e_2);1\rangle = |(g_1),e_2;1\rangle \to |(g_1),e_2;1\rangle = |g_1,(e_2);1\rangle \to -|e_1,e_2;0\rangle$$

Calling ion 1 control and ion 2 target with g = qubit state 0 and e= qubit state 1, the succession of 3 pulses realizes the phase gate:

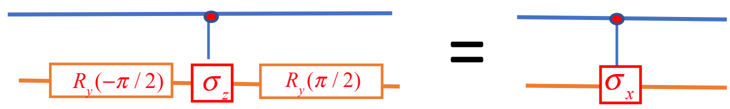


To turn this gate into a control-not, add two pulses performing rotations on target qubit before and after phase gate (carrier $\pi/2$ pulses)

$$R_y(\pm\pi/2) = \frac{1}{\sqrt{2}}\left(I \mp i\sigma_y\right)$$



Turning the phase gate into a C-Not Gate



Blue ion: control ; Red ion: target

1: $R_y(-\pi/2)$ pulse on red ion with carrier laser π-polarized ($\omega_L=\omega_{eg}$)

2: π Rabi pulse with π-polarized laser on red side-band of blue ion ($\omega_L=\omega_{eg}-\omega$)

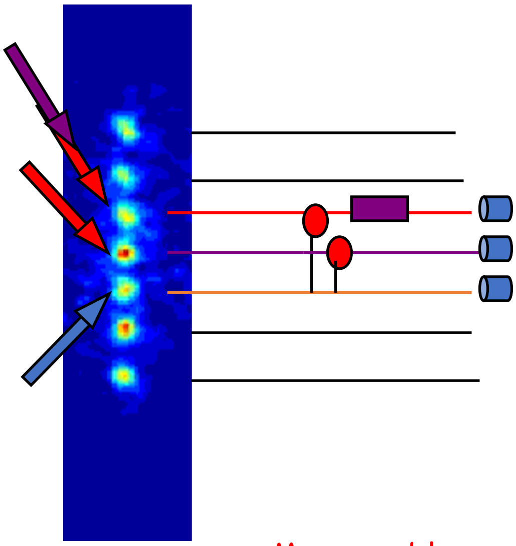
3. 2π Rabi pulse on red side band of red ion (σ_- polarization) ($\omega_L=\omega_{eg}-\omega$)

4. π Rabi pulse with π-polarized laser on red side-band of blue ion ($\omega_L=\omega_{eg}-\omega$)

5. $R_y(\pi/2)$ pulse on red ion with carrier laser π-polarized ($\omega_L=\omega_{eg}$)

5 Laser pulses can select any pair of ions in a linear ion cristal to realize this gate

Implementing quantum algorithm with a chain of ions

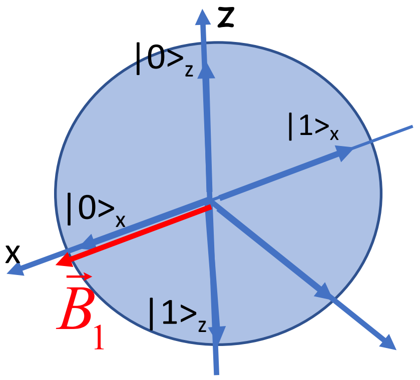


Successively applied laser pulses on ions of the chain realize one qubit rotations and two-qubit gates.

Fluorescence detection (after a possible one qubit rotation) extract information from system.

Many problems must be solved to build a useful quantum computer…..

Rabi oscillation interpreted as a quantum interference



At time t=0, the spin is pointing along the direction Oz of the static magnetic field B_0 (state $|0\rangle_z$) A resonant field B_1, rotating in the xOy plane at angular frequency ω is switched on. In the rotating frame, B_1 is aligned along Ox.

The initial $|0\rangle_z$ state is a linear superposition of the spin pointing in opposite directions along Ox:

$$|0\rangle_z = \frac{1}{\sqrt{2}}\left(|0\rangle_x + |1\rangle_x\right)$$

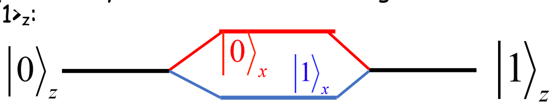
The two terms of this superposition evolve with opposite phases:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_1 t/2}|0\rangle_x + e^{+i\omega_1 t/2}|1\rangle_x\right)$$

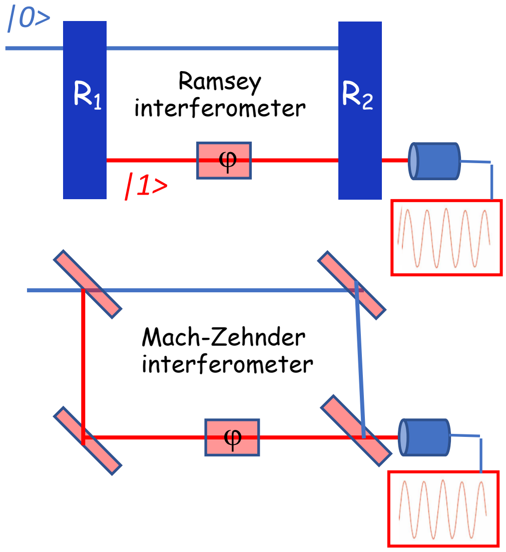
The probability amplitude to find spin in $|1\rangle_z$ state at time t is:

$${}_z\langle 1|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left({}_z\langle 1|0\rangle_x\, e^{-i\omega_1 t/2} + {}_z\langle 1|1\rangle_x\, e^{+i\omega_1 t/2}\right) = \frac{1}{2}\left(e^{-i\omega_1 t/2} - e^{+i\omega_1 t/2}\right)$$

The oscillation of the spin flipping probability can be seen as resulting from an interference between two pathes leading the spin from state $|0\rangle_z$ to $|1\rangle_z$:

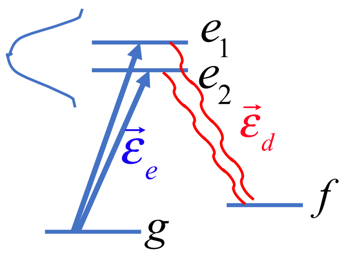


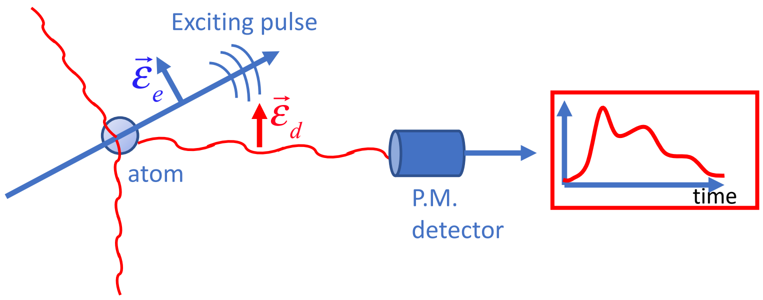
Ramsey fringes: another quantum interference effect



The Ramsey signal can be interpreted as an interference effect. The first pulse R_1 splits the two-level system state in two parts with probability amplitudes of same modulus. These two parts are recombined by the second pulse R_2, before detection of the system in |0> or in |1>. Depending upon the phase difference accumulated between the two pathes, the final probability exhibits positive interference (when the two parts are in phase), negative interference when they are π out of phase. The interference is observed because there is no way to know whether the system flipped in the first zone or in the second one. The figure shows the analogy with a Mach Zenhder interferometer splitting the path of a photon in two branches before recombining them. The sensitivity of this spectroscopic method is limited by the inverse of the pulse time separation $1/\tau$. The same sensitivity would be theoretically obtained by recording a Rabi spectroscopy spectrum with a unique pulse lasting during time τ. In practice, this would require to maintain a perfect homogeneity of the field during a long time, which is not easy (see later application to microwave atomic clocks).
Another advantage of Ramsey spectroscopy is to be sensitive to differential effects dephasing in different ways the two states (see also later).

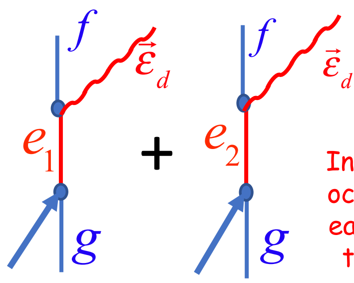
Quantum beats in fluorescence





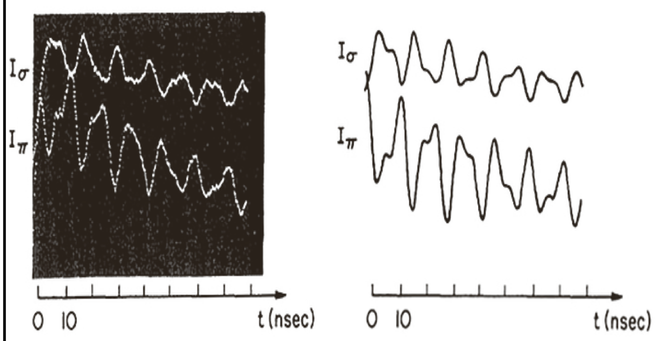
Exciting from ground state g a superposition of atomic states e_1 and e_2 with a short broad-band laser pulse and detecting the fluorescence light with polarization ε_d subsequently emitted on transition toward a final state f. The signal presents time modulation at the Bohr frequency $(E_1-E_2)/h$.

This is a quantum interference effect between two probability amplitudes leading from the same initial state g to the same final state f. Interference as long as there is no way to know through which state (e_1 or e_2) the atom has scattered the light.

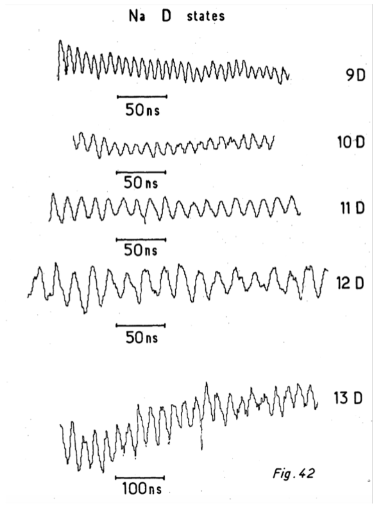


Quantum Interference occurs within each atom of the sample

Examples of quantum beats (1972-1976)

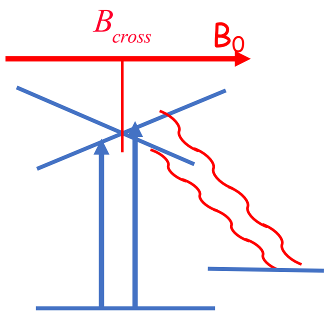


Hyperfine quantum beats of the $7^2P_{3/2}$ state of Cesium

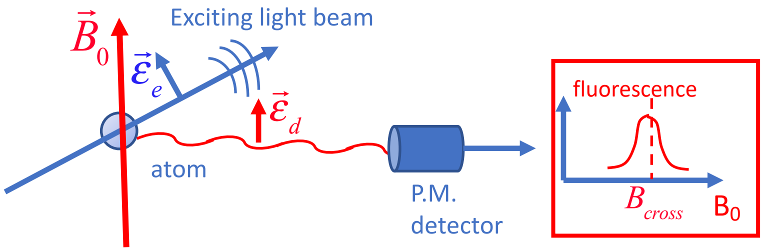


Fine structure quantum beats in the 9D to 13 D states of Sodium

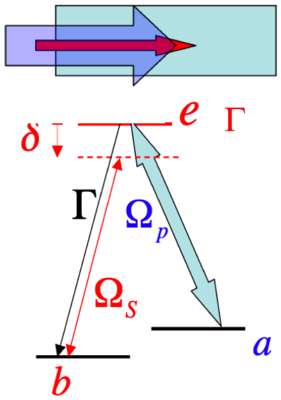
Level crossing spectroscopy



Broad band continuous excitation of a superposition of states in the vicinity of the static magnetic field B_{cross} for which the states cross (have same energy): the quantum coherence accumulates at the crossing point, resulting in a resonant variation of the fluorescence detected on a transition toward same final state. The exciting beam and the detected fluorescence light must have a polarization allowing the transition from the initial and final state to the two crossing excited levels. Analogy with the Hanle effect described above in ground state



Dark states in a three-level atomic gas



Two colinear laser beams propagate in a cold atom gas (negligible Doppler effect). The atoms have a Λ shaped three level energy diagram: two lower states b and a and one excited state e, of natural width Γ. A pump beam is resonant with the a-e transition with Rabi frequency Ω_p << Γ. A weaker probe beam has its frequency swept around the frequency of the b to e transition, with a Rabi frequency Ω_s<Ω_p. Consider at time t=0 the superpositions of the a and b states:

$$|\Psi_-\rangle = \frac{1}{\sqrt{\Omega_p^2+\Omega_s^2}}\left(\Omega_p|b\rangle - \Omega_s|a\rangle\right) ; \langle\Psi_-|D|e\rangle = 0 \qquad |\Psi_+\rangle = \frac{1}{\sqrt{\Omega_p^2+\Omega_s^2}}\left(\Omega_s|b\rangle + \Omega_p|a\rangle\right) ; \langle\Psi_+|D|e\rangle \neq 0$$

The superposition $|\Psi_->$ is uncoupled to the excited state e while the orthogonal state $|\Psi_+>$ has a strong coupling to e. $|\Psi_->$ is called a dark state. At a later time t, it becomes:

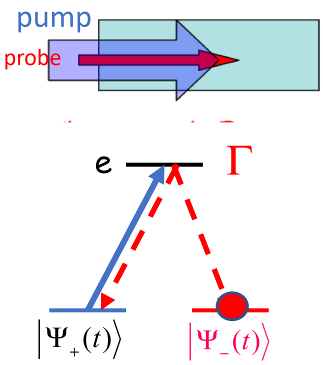
$$|\Psi_-(t)\rangle = \frac{1}{\sqrt{\Omega_p^2+\Omega_s^2}}\left(\Omega_p e^{-iE_b t/\hbar}|b\rangle - e^{-iE_a t/\hbar}\Omega_s|a\rangle\right)$$

It will stay uncoupled to light provided the destructive interference condition is satisfied:

$$E_b/\hbar + \omega_s = E_a/\hbar + \omega_p \rightarrow \boxed{\omega_s - \omega_p = (E_a - E_b)/\hbar}$$

This is a Raman resonance condition: the atom transits resonantly from a to b by absorbing a pump photon and emitting by stimulated emission a probe photon, if the difference of the photon energies is equal to that of the state energies. When this condition is satisfied, the state $|\Psi_->$ remains dark at all times, i.e. it stays uncoupled to the excited state e when the two light beams are on.

Dark state and Electromagnetically Induced Transparency (EIT)

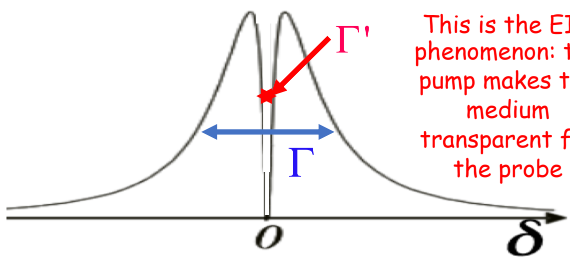


The three level atom has two states strongly coupled to light and one dark state uncoupled to the lasers and connected to the excited state only through spontaneous emission. Optical pumping quickly makes the atoms fall into the dark state and stay there.

The dark state condition is satisfied only for δ very small
The relaxation rate of the coherence between the a and b states is due to the optical pumping process which brings back to the ground state a fraction of the unstability of state e. The absorption and the fluorescence of the e state are cancelled provided:

$$\delta \leq \Gamma' = \frac{\Omega_p^2}{\Gamma} \ll \Gamma$$

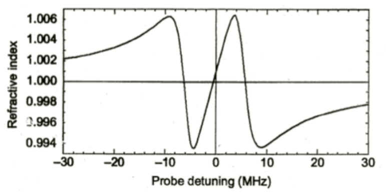
The absorption profile of the probe has a Lorentzian shape of width Γ, with a narrow dip around δ=0 of width Γ'<<Γ:

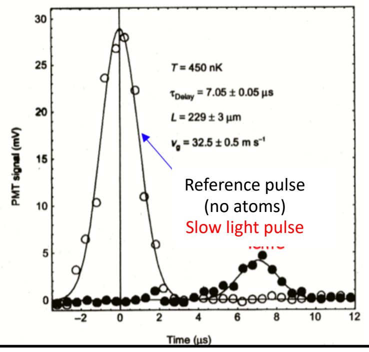


This is the EIT phenomenon: the pump makes the medium transparent for the probe

Application of EIT: slow light

L.Hau et al,
Harvard (1999)





In the EIT phenomenon, dispersion (variation of refractive index with frequency around resonance) presents also very rapid variation around the resonance condition. The index varies as the derivative of the absorption, over a frequency range of the order of Γ', typically a few Mhz in a cold atomic gas. The derivative of the index versus frequency (steep slope of $n(\omega)$ around δ =0) is huge, resulting in a strong slowing down of the light group velocity V_g in medium:

Profile of light pulse in medium (one dimension model):

$$f(z,t) = \int g(\omega) e^{in(\omega)kz - \omega t} \, d\omega$$

Peak of pulse z(t) obtained by stationary phase condition:

$$\frac{n(\omega)}{c} z + \frac{\omega}{c} \frac{dn}{d\omega} z - t = 0$$

which gives: $$V_g / c = \frac{z}{ct} = \frac{1}{n(\omega) + \omega \frac{dn}{d\omega}} \sim \frac{1}{\omega \frac{dn}{d\omega}}$$

The factor $\omega dn/d\omega$ reaches a value 10^6 to 10^7: The EIT pump beam can reduce the light group velocity to a few m/s !

Conclusion of Lecture 10

I have described in this lecture various methods of high resolution spectrscopy applied to atomic gases and to ions in traps. I have also shown how interactions of trapped ions with lasers can implement basic steps of quantum information processing (one and two qubit gates). Finally, I have analized various quantum interferencces phenomena exploited in spectroscopy and in quantum optics.

In Lectures 11 and 12, I will focus on laser cooling and trapping experiments performed on neutral atom and I will describe the application of these methods to ultra-high precision metrology (realization of optical atomic clocks and their applications).