PhD Research Project

A systematic study of phenomenological implications of dim-6 operators

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1 Introduction and Motivations

The Standard Model (SM) of particle physics has been producing reliable estimates to a high precision level in the last decades. Moreover, the discovery of a scalar particle with properties consistent with the SM Higgs boson, together with the fact that no Beyond Standard Model (BSM) particle has been detected so far, suggests that the New Physics (NP) scale Λ can be placed above the electro-weak symmetry breaking (EWSB) scale.
Therefore, an effective field theory built solely using the SM fields, called

Standard Model Effective Field Theory (SMEFT), can be used to describe the low energy limit of BSM physics. This theory should be written adding to the renormalizable SM interactions further terms of higher dimensions, suppressed by suitable powers of the scale Λ and invariant under the SM gauge group. The schematic Lagrangian is

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}_5 + \mathcal{L}_6 + \dots . \tag{1}$$

In $\mathcal{L}_5$ there is only the Weinberg operator, which can be used to provide masses to neutrinos. Assuming the conservation of baryon number, in $\mathcal{L}_6$ there are 59 operators, many of which have flavor indices (explicitly taking them into account, the number of operators grows up to 2499); a complete list can be found in [1]. The full one-loop anomalous dimension (AD) matrix needed for renormalization group evolution (RGE) of the dimension 6 operators has been recently computed [2, 3, 4, 5]; several entries have been found to be of order 1, therefore suggesting a relevant mixing between some of these operators.

The full one-loop RGE of the independent set of dimension 6 operators can be used to interpret any pattern of deviations in SM processes. The BSM fields can in fact be integrated out of the theory at the NP scale, in such a way that there is no evident sign of their presence at the EWSB scale, but their effects can be mimicked by means of dimension 6 operators; moreover, if an operator develops an anomalous dimension, it will induce the presence of logarithmically enhanced terms at the low scale. In order to take these terms into account, it is necessary to compute the correspondent Wilson coefficients (WC) at the NP scale and evolve them at the EWSB scale using the RGE equations. Hence, any pattern of deviations in SM processes can be interpeted as an effect of the WC associated with dimension 6 operators, properly evolved at the low scale.

In my PhD thesis I will study phenomenological implications due to the effect of dimension 6 operators. I will put particular care in the study of flavour observables: while there have already been some studies in this field (namely concerning the $\mu \to e\gamma$ decay [6] and the $b \to s$ and $b \to c$ transitions [7]), a systematic approach is still lacking.

As a first step of my project, I'm recomputing the RGE from [2, 3, 4, 5], in order to check the correctness of their results. Moreover, at the low scale there are also contributions stemming from the finite parts of the one-loop diagrams involving dimension 6 operators that are yet to be computed: my next task will be the computation of such contributions. Once this contributions will be known, I'll start developing a systematic way to take into account the effects of dimension 6 operators in phenomenology.

2 The Operators Basis

As stated in the introduction, there are 59 independent operators in the dimesion 6 Lagrangian $\mathcal{L}_6$ which conserve the baryon number [1]. Moreover, all the operators which contain two or more fermion fields carry flavour indices; hence, if one wants to differentiate operators with the same field structure but with different flavour indices, the total number of operators grows up to 2499.
This plethora of operators can be sub-divided in 8 classes, in terms of the fields content and the number of covariant derivatives contained in each of them. Denoting with X a Gauge field strength tensor, with ψ a fermion field, with H a Higgs field and with D a covariant derivative, the 8 classes (and the number of operators which populate each class) are listed in the following table.

X^3	4
H^6	1
H^4D^2	2
X^2H^2	8

ψ^2H^3	3
ψ^2XH	8
ψ^2H^2D	8
ψ^4	25

The most populated class (especially if one takes flavour indices into account) is the ψ^4 one; however, this class is further divided into 5 subclasses, according to the chirality of the fermion fields: $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$, populated by 5, 7, 8, 1 and 4 operators respectively.
It is important to take in consideration the fact that it is possible to build more than 59 dimension 6 operators containing solely SM fields and invariant under the SM gauge group (e.g. the dimension 6 operators basis developed in [8] contained 80 operators). However, some of these operators are redundant due to SM equations of motion (EOM): it is indeed possible to use the EOM to redifine the redundant operators (called EOM operators) in terms of other dimension 4 and dimension 6 operators, so that the actual number of independent operators is indeed 59. Nevertheless, EOM operators do contribute to the calculation of the AD and of the finite terms, hence it is mandatory to properly take these operators into account.

3 The Anomalous Dimension

The first part of my PhD project is devoted to the re-computation of the AD matrix of the dimension 6 operators, needed to perform the RGE of the respective Wilson coefficients. The complete computation of this AD has been performed only recently [2, 3, 4, 5]. However, only a small subset of the entries have been cross-checked in the literature so far, hence an independent cross-check of all the entries of the matrix might yield interesting results.
The computation of the AD of a set of operators is closely connected to the ultraviolet (UV) divergences renormalization of such operators, due to the following relation between the AD matrix $\hat{\gamma}$ and the renormalization matrix $\hat{Z}$:

$$\hat{\gamma} = \hat{Z}^{-1} \frac{d\hat{Z}}{d \ln \mu} \, . \tag{2}$$

Bearing this in mind, the calculation of the AD is carried out after performing the following preliminary operations:

- the Feynman rules for the dimension 6 Lagrangian $\mathcal{L}_6$ are obtained using the FeynRules package [9], after creating a suitable model file;

- all relevant one-loop diagrams are written employing the FeynArts package [10].

Once these operations are carried out, the actual computation can be performed. The employed algorithm is the Passarino-Veltman reduction [11]: this algorithm allows to rewrite all tensor integrals in terms of well-known scalar ones. A recent review can be found in Appendix A of [12], while here I will cover only the main features of the method.
The computation of a generic one loop diagram requires calculating integrals of the following form:

$$I_N \sim \int \frac{d^4 l}{(2\pi)^4} \frac{\mathcal{N}(l)}{(l^2 - m_1^2)((l+q_1)^2 - m_2^2) \ldots ((l+q_{N-1})^2 - m_N^2)} \, , \tag{3}$$

where N is the number of external particles (with momentum p_i) and $q_i = \sum_{j=1}^{i} p_j$. The numerator $\mathcal{N}(l)$ is a polynomial function of the loop momentum l, and a scalar integral is the special case where $\mathcal{N}(l) = 1$. It is important to stress the fact that power counting implies that a UV divergence appears in I_N only if $\mathcal{N}(l)$ contains a tensor of rank $r \geq 2N - 4$; hence, only one-point and two-point scalar integrals are UV divergent. In the presence of UV divergences, the integrals require regularization; a conventional scheme is dimensional regularization [13], where the number of dimensions of the space-time is set to $D = 4 - 2\epsilon$, and the limit $\epsilon \to 0$ is performed at the end of the computation. As a result, the integration measure in eq. (3) is changed to

$$\frac{d^4 l}{(2\pi)^4} \to \frac{d^D l}{(2\pi)^D} \,. \tag{4}$$

The Passarino-Veltman reduction scheme makes use of Lorentz invariance in order to reduce the tensor integrals to scalar ones. Focusing for example on $N = 1, 2$, the one-point and two-point integrals can be written as

$$\begin{aligned} A_0(m) &= \frac{1}{i\pi^{D/2}} \int d^D l \frac{1}{(l^2 - m^2)} \,, & (5) \\ B_0, B^\mu, B^{\mu\nu}(p_1, m_1, m_2) &= \frac{1}{i\pi^{D/2}} \int d^D l \frac{1, l^\mu, l^{\mu\nu}}{(l^2 - m_1^2)\left((l+p_1)^2 - m_2^2\right)} \,; & (6) \end{aligned}$$

hence, it is possible to expand in terms of so-called form factors the two-point tensor integrals:

$$\begin{aligned} B^\mu &= p_1^\mu B_1 \,, & (7) \\ B^{\mu\nu} &= g^{\mu\nu} B_{00} + p_1^\mu p_1^\nu B_{11} \,. & (8) \end{aligned}$$

By contracting this equations with p_1 and $g^{\mu\nu}$, all form factors can be expressed in terms of the well-known scalar integrals A_0 and B_0.
The renormalization matrix (and hence the AD matrix) can be finally obtained by taking in consideration the coefficients of the divergent parts of the diagrams (i.e. the coefficients of the ϵ^{-1} terms of the scalar integrals); these coefficients can be extracted employing the following relations:

$$\mathrm{Div}[A_0(m)] = m^2 \,, \qquad \mathrm{Div}[B_0(p_1, m_1, m_2)] = 1 \,, \tag{9}$$

and remembering that the one-point and two-point scalar integrals are the only scalar integrals to develop a UV divergence.

4 Finite Terms

The second part of my PhD project is devoted to the computation of finite terms stemming from one-loop diagrams which involve operators of the dimension 6 Lagrangian $\mathcal{L}_6$. The calculation of such contributions has not been carried out so far in the literature, and the result is renormalization-scheme dependent; however, these contributions might yield non-negligible effects at the low scale, especially when there are no logarithmically enhanced terms due to vanishing entries of the AD. Hence, the study of such terms is crucial if one wants to study in a reliable way BSM physics by the means of a

SMEFT.
The computation of these contributions is strictly entangled to the calculation of the AD, performed as explained in the previous section. The employed algorithm is again the Passarino-Veltman reduction [11], by means of which the diagrams will be written in terms of scalar integrals. However, the bare operators under study in the previous section will be substituted with renormalized ones, which will display a regular UV behaviour. Therefore, to compute the finite terms, it is not enough to focus only on the one-point and two-point scalar integrals A_0 and B_0 as done before (which was enough for the computation of the AD, since these are the only scalar integrals to develop a UV divergence), but all n-point integrals encountered in the calculation will have to be computed. This computation will be performed employing the well-known results for all one-loop scalar integrals [14, 15, 16, 17].

5 Phenomenology

The final stage of my PhD project is devoted to develop a systematic way to take into account the effects of dimension 6 operators in phenomenology. A particular care will be put in the study of flavour phenomenology, since most of the analyses done so far in the literature have been focusing on electroweek and Higgs phenomenology.
The main goals of this final part are two: on one side, I'll obtain bounds for (and possible correlations within) the dimension 6 WC starting from current data. On the other side, I'll look for which decay channels are more sensitive to the contributions stemming from dimension 6 operators; a byproduct of this study will be the identification of the channels better suited to impose constrains on each dimension 6 WC, hence pointing out those decays whose theoretical study could benefict from better results on the experimental side. This analysis will be performed in a Bayesian framework using the HEPfit tool, which is a general code developed to study the combination of indirect and direct constraints on High Energy Physics models[1].

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