Classical and quantum nonlinear waves

Ph.D. course AA 2020 21

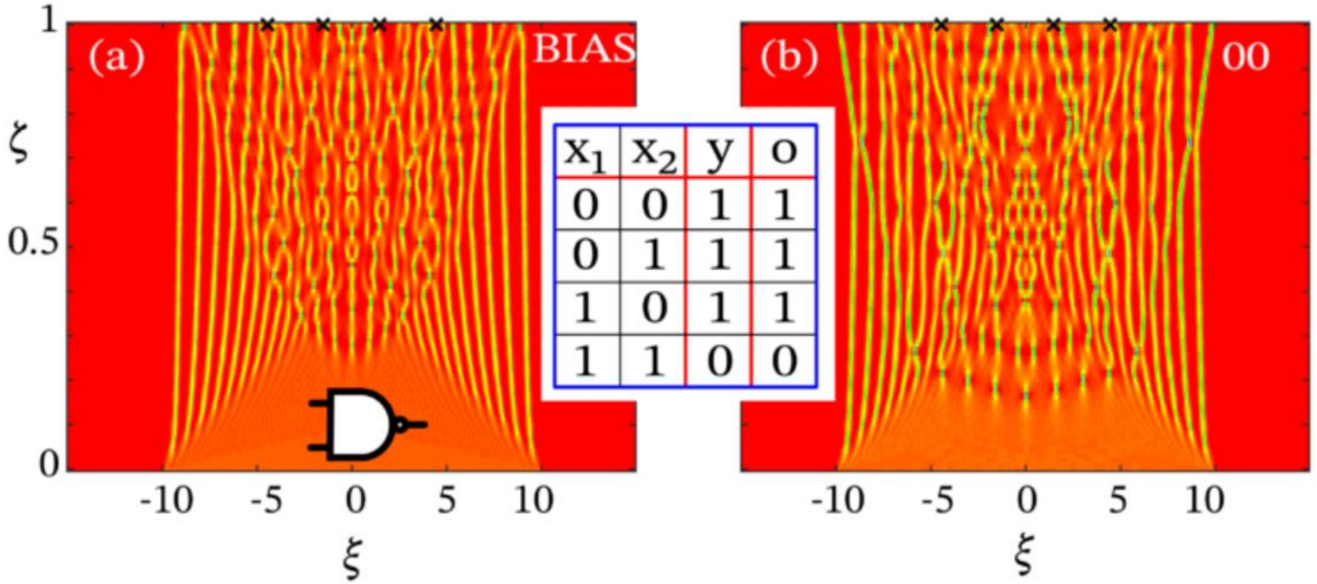
Claudio Conti

claudio.conti@uniroma1.it

newcomplexlight.org

Modern nonlinear waves

- Quantum fluids
- Quantum simulations
  - Analog gravity
  - Fundamental physics
- Neuromorphic computing
  - Machine learning
  - Data driven physics



x_1	x_2	y	o
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

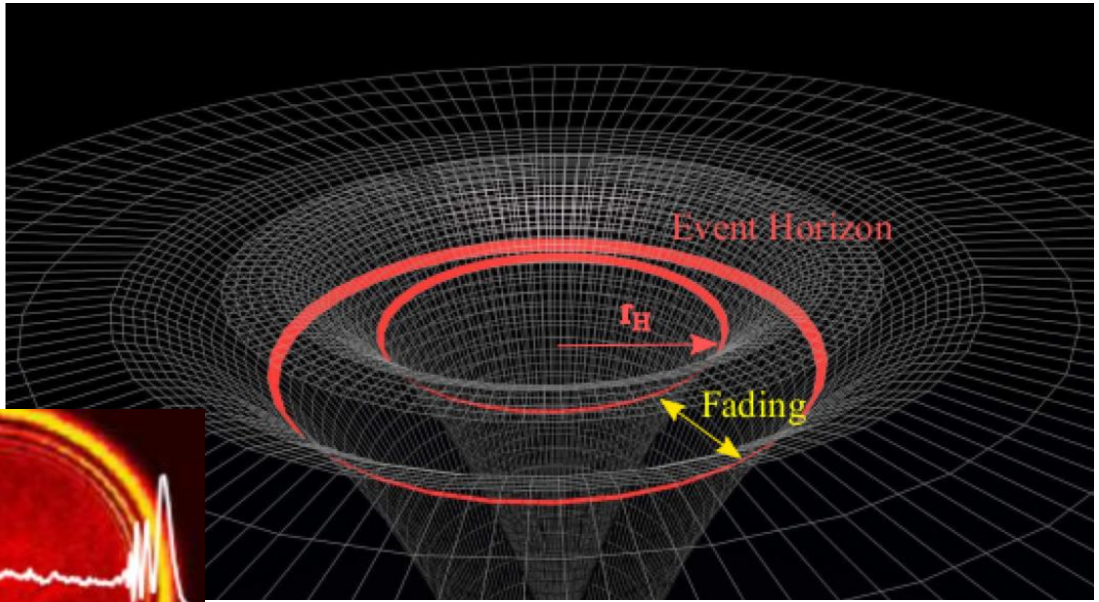
newcomplexlight.org

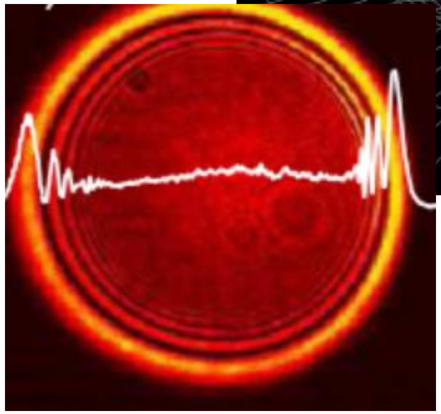
Objective 1: Solitons and analog gravity

Solitons and NLS (theory)

$$\imath\partial_t\psi + \partial_x^2\psi + 2|\psi|^2\psi = 0$$

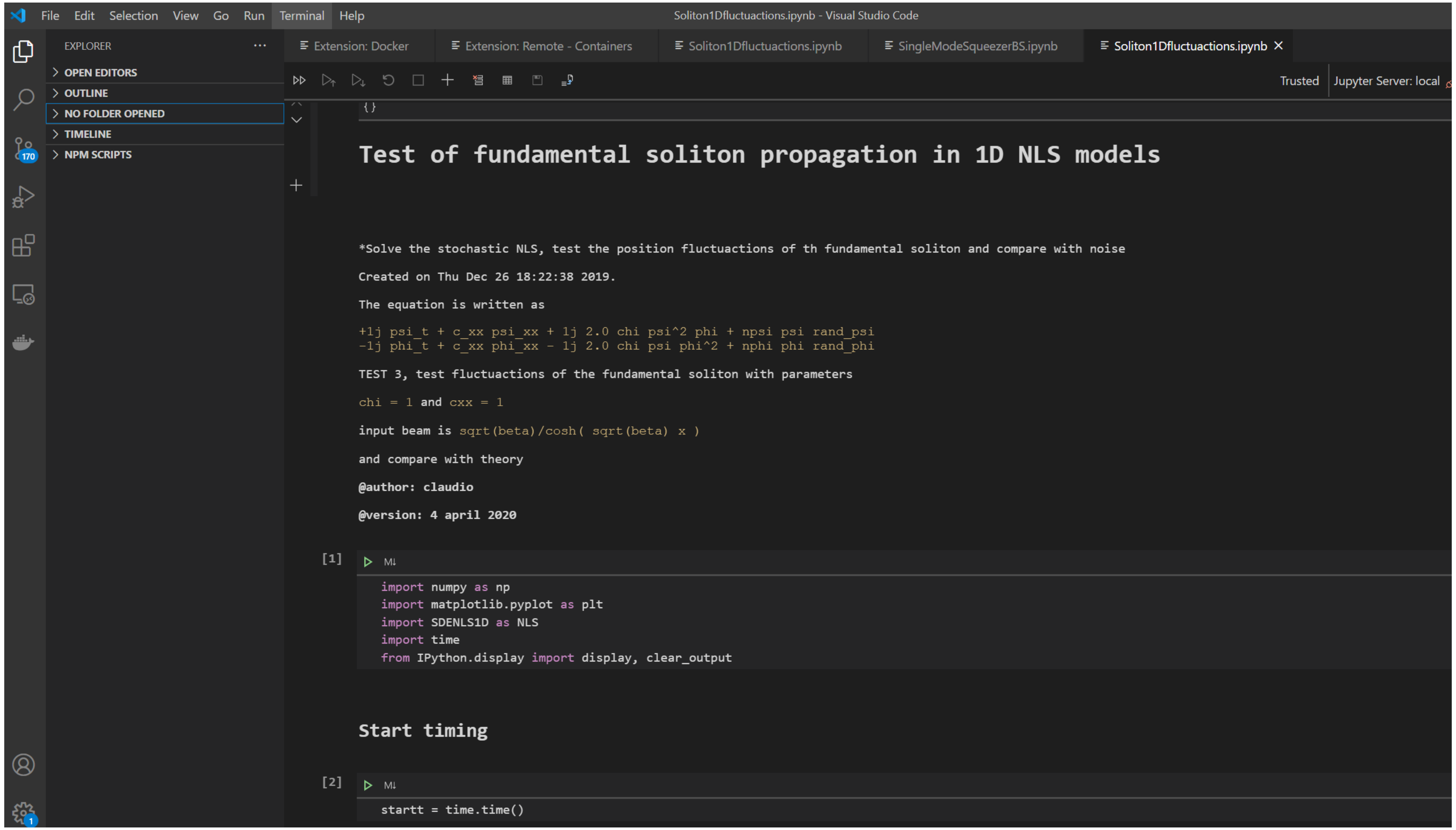
Black hole analogy (phenomenology)





newcomplexlight.org

Objective 2: Numerical methods



newcomplexlight.org

Obj 3: Second quantization and phase space

Theoretical methods

- Positive P rapresentation
- Stochastic equations

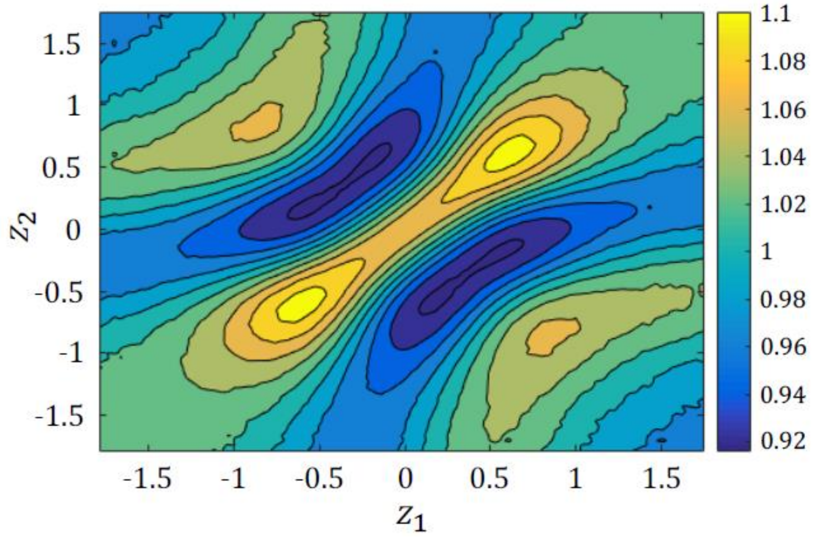
$$\hat{H}_{1D} = \int dr \left[\frac{-\hbar^2}{2m} \hat{\Psi}^\dagger \frac{\partial^2}{\partial r^2} \hat{\Psi} + \frac{g}{2} \Psi^{\dagger 2} \hat{\Psi}^2 \right],$$

$$\frac{d\psi}{d\tau} = i\nabla_z^2 \psi - 2iC\psi^+\psi^2 - i\sqrt{2iC}\psi\eta(\tau, z)$$

$$\frac{d\psi^+}{d\tau} = -i\nabla_z^2 \psi + 2iC\psi^{+2}\psi - \sqrt{2iC}\psi^+\eta^+(\tau, z),$$

Phenomenology

- Squeezing and entanglement in nonlinear waves



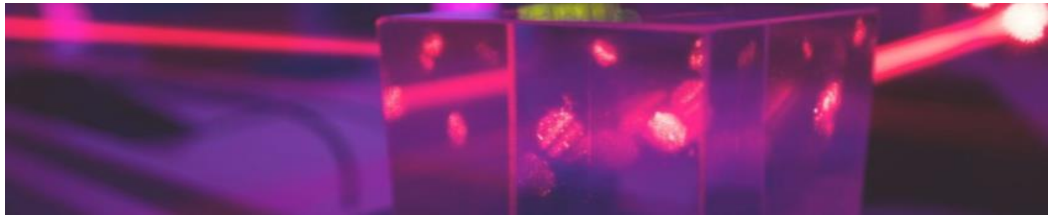
Program (20 hours, 3 cfu)

- Solitons and nonlinear Schroedinger equation
  - Derivation of NLS
  - Solitons solutions
- Multidimensional nonlinear waves
  - Hydrodynamic limit
- Analog gravity
  - Metric and black hole analogy
  - Experiments
- Numerical methods
  - Nonlinear partial differential equations
  - Python and jupyter
- Second quantization
  - Phase space methods
  - Stochastic differential equations

THEORY

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \frac{\beta\hbar^4}{m}\nabla^4\psi + V(\mathbf{r})\psi \iff \frac{\mathrm{d}^2x^\alpha}{\mathrm{d}\tau^2} + \Gamma^\alpha_{\beta\gamma}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\frac{\mathrm{d}x^\gamma}{\mathrm{d}\tau} = 0$$

EXPERIMENTS



claudio.conti@uniroma1.it

newcomplexlight.org