

Classical and quantum nonlinear waves

Ph.D. course AA 2020 21

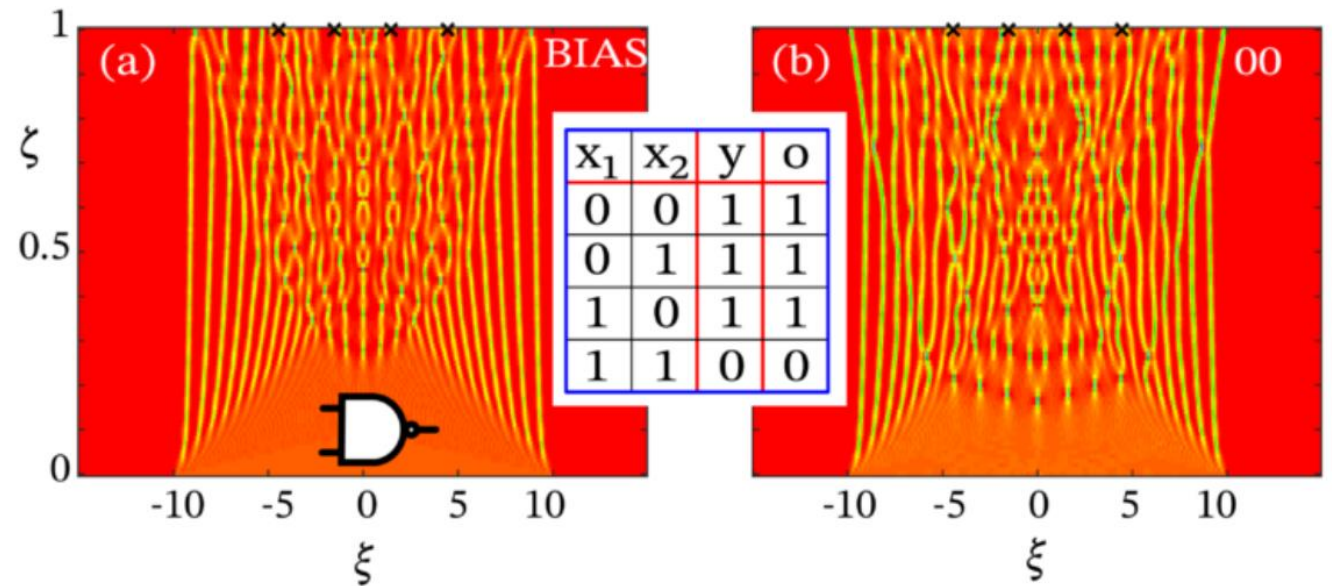
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Modern nonlinear waves

- Quantum fluids
- Quantum simulations
 - Analog gravity
 - Fundamental physics
- Neuromorphic computing
 - Machine learning
 - Data driven physics

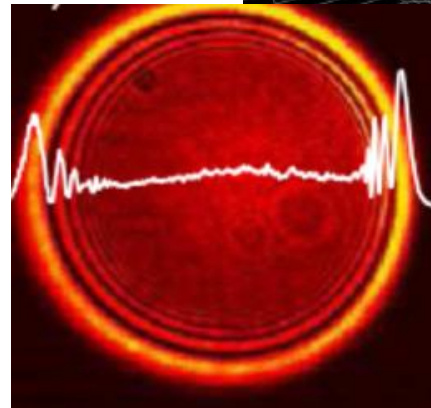
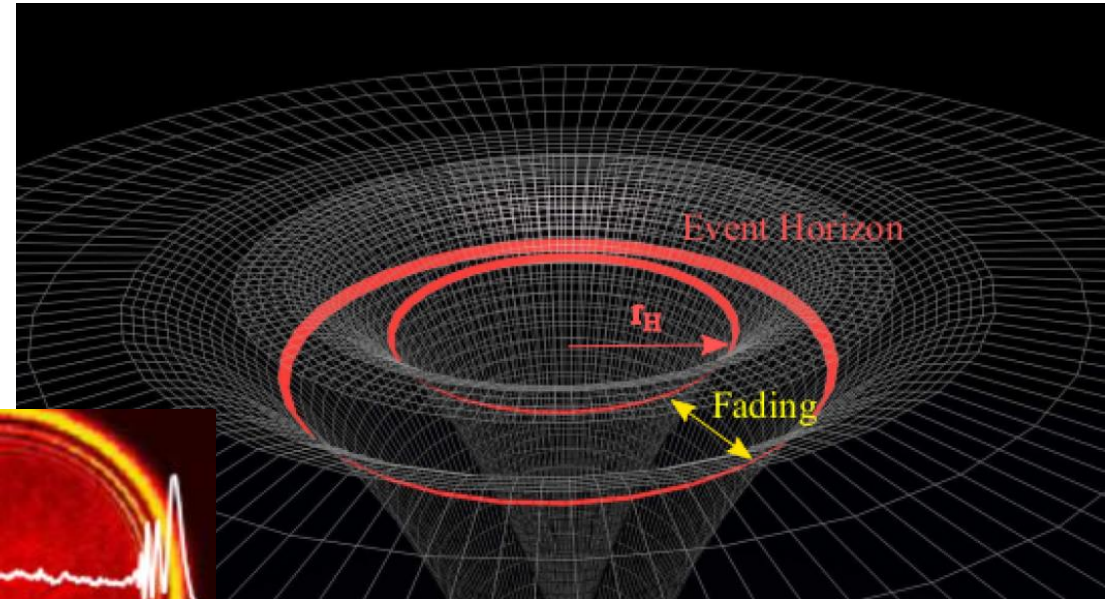


Objective 1: Solitons and analog gravity

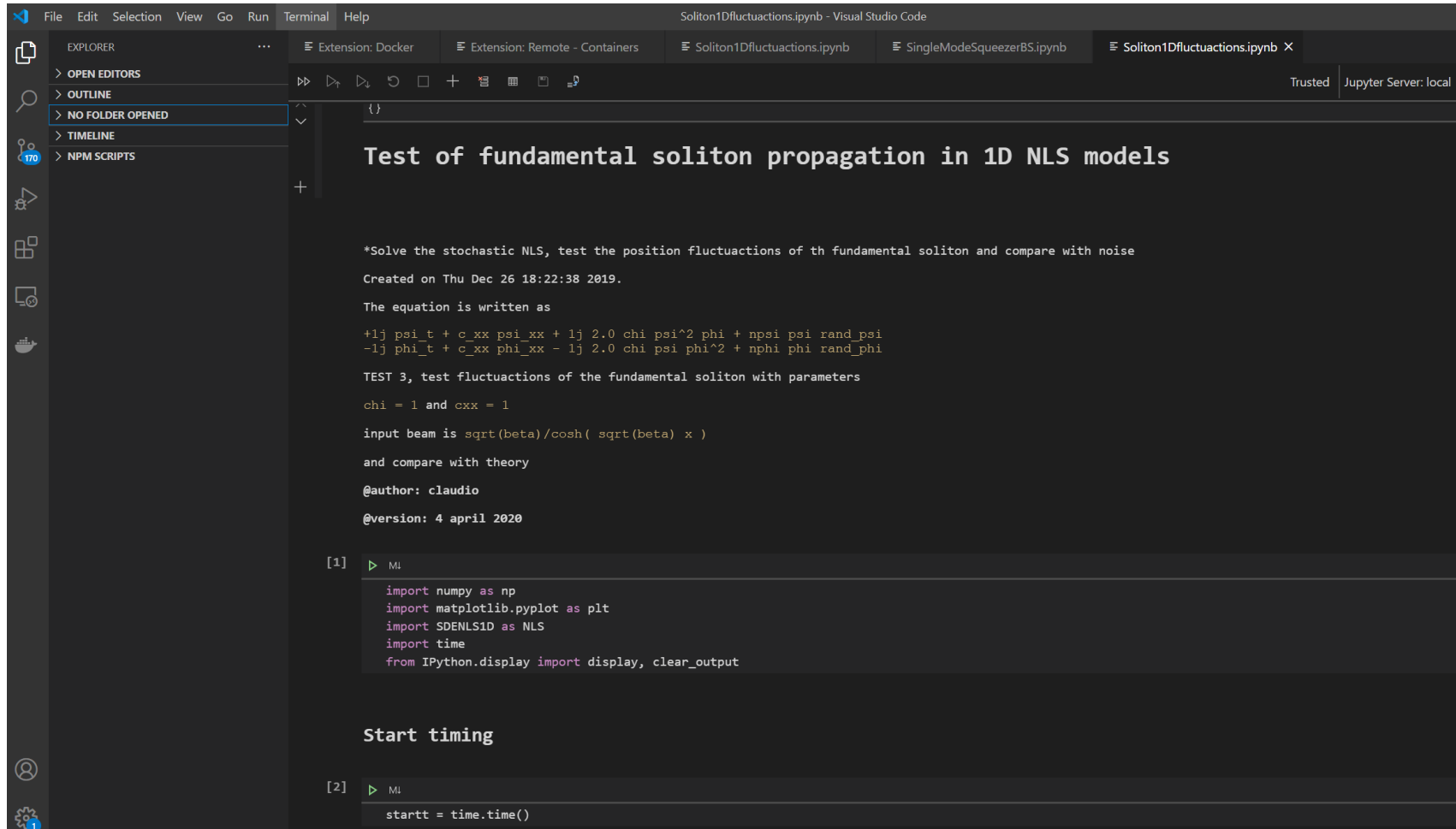
Solitons and NLS (theory)

$$i\partial_t\psi + \partial_x^2\psi + 2|\psi|^2\psi = 0$$

Black hole analogy (phenomenology)



Objective 2: Numerical methods



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File Edit Selection View Go Run Terminal Help
Soliton1Dfluctuations.ipynb - Visual Studio Code
Extension: Docker Extension: Remote - Containers Soliton1Dfluctuations.ipynb SingleModeSqueezerBS.ipynb Soliton1Dfluctuations.ipynb X
Trusted Jupyter Server: local c
EXPLORER
OPEN EDITORS
OUTLINE
NO FOLDER OPENED
TIMELINE
NPM SCRIPTS
+
Test of fundamental soliton propagation in 1D NLS models
*Solve the stochastic NLS, test the position fluctuations of th fundamental soliton and compare with noise
Created on Thu Dec 26 18:22:38 2019.
The equation is written as
+1j psi_t + c_xx psi_xx + 1j 2.0 chi psi^2 phi + npsi psi rand_psi
-1j phi_t + c_xx phi_xx - 1j 2.0 chi psi phi^2 + nphi phi rand_phi
TEST 3, test fluctuations of the fundamental soliton with parameters
chi = 1 and cxx = 1
input beam is sqrt(beta)/cosh( sqrt(beta) x )
and compare with theory
@author: claudio
@version: 4 april 2020
[1] ▶ MI
import numpy as np
import matplotlib.pyplot as plt
import SDENLS1D as NLS
import time
from IPython.display import display, clear_output
Start timing
[2] ▶ MI
startt = time.time()
```



Obj 3: Second quantization and phase space

Theoretical methods

- Positive P representation
- Stochastic equations

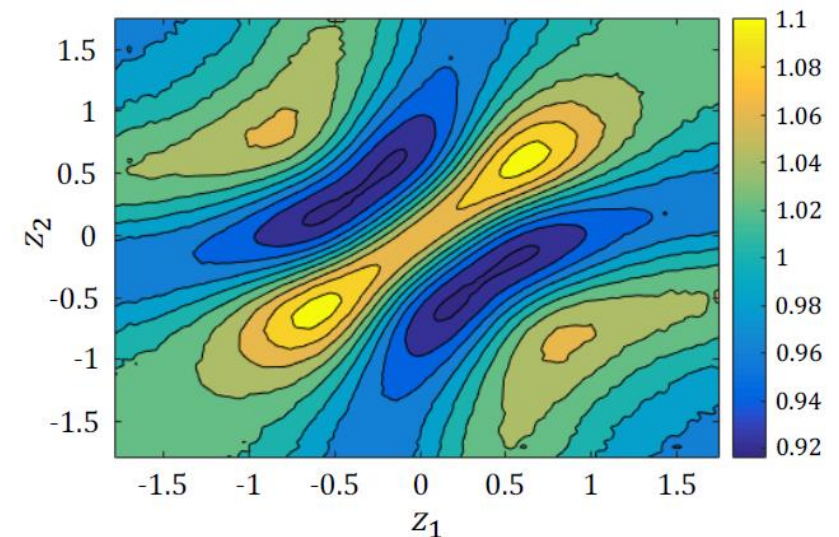
$$\hat{H}_{1D} = \int dr \left[\frac{-\hbar^2}{2m} \hat{\Psi}^\dagger \frac{\partial^2}{\partial r^2} \hat{\Psi} + \frac{g}{2} \hat{\Psi}^\dagger{}^2 \hat{\Psi}^2 \right],$$

$$\frac{d\psi}{d\tau} = i\nabla_z^2 \psi - 2iC\psi^+ \psi^2 - i\sqrt{2iC}\psi\eta(\tau, z)$$

$$\frac{d\psi^+}{d\tau} = -i\nabla_z^2 \psi^+ + 2iC\psi^+{}^2 \psi - \sqrt{2iC}\psi^+ \eta^+(\tau, z),$$

Phenomenology

- Squeezing and entanglement in nonlinear waves



Program (20 hours, 3 cfu)

- Solitons and nonlinear Schroedinger equation
 - Derivation of NLS
 - Solitons solutions
- Multidimensional nonlinear waves
 - Hydrodynamic limit
- Analog gravity
 - Metric and black hole analogy
 - Experiments
- Numerical methods
 - Nonlinear partial differential equations
 - Python and jupyter
- Second quantization
 - Phase space methods
 - Stochastic differential equations

THEORY

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{\beta \hbar^4}{m} \nabla^4 \psi + V(\mathbf{r})\psi \quad \left\langle \right\rangle \quad \frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = 0$$

EXPERIMENTS



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