Multiphoton hybrid systems and their applications in quantum information and quantum communication

PhD. Research Project

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I. INTRODUCTION

The basic unit of information in classical systems is the bit or binary digit: it can assume a value of either 0 or 1. There is an analogous quantity in the quantum context which is called "qubit" or quantum bit. Instead of two defined values, the qubit is represented by unit vectors in a two dimensional complex Hilbert space allowing the possibility to encode and transfer a higher amount of information respect to the classical counterpart. The representation of the qubit in the computational basis ($|0\rangle$,$|1\rangle$) is given by:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

where the complex coefficients α and β satisfy the normalization relation $|\alpha|^2 + |\beta|^2 = 1$[1].
Qubits are experimentally implemented by exploiting different systems such as atoms, semiconductor quantum dots or photons. Each of these systems has advantages and disadvantages associated, so choosing one over another depends primarily on the protocol to be implemented. For instance, photons are more suitable for a communication protocol [2] while atoms are a better choice for qubits storing [3]. An important advantage of using photons is that they can be easily produced and the manipulation/detection can be performed with standar optical techniques, which exploit polarizing beamsplitter (PBS), beamsplitters (BS), single photon counter detector, half and quarter waveplates and so on. Single photons are routinely produced by using spontaneous parametric down conversion (SPDC) technique. Photonic qubits are generally encoded in the polarization degree of freedom.

Entanglement theory is the basis of many key discoveries: quantum dense coding [4], quantum cryptography [5], quantum teleportation [6], entanglement swapping [7]. All the previous works have been experimentally demonstrated [8–12]. Entanglement is not only a subject of philosophical debates but also a way to develop technologies that are superior or simply impossible to generate with classical physics. Unfortunately, entanglement has also a very complex structure which is sensitive to environment. A significant advance has been achieved in the last few years but some fundamental issues need still a complete answer (i) which are the optimal methods to detect theoretically and experimentally entanglement; (ii) how to control, characterize and quantify entanglement for different systems.

In order to understand better the nature of entanglement and also improve protocols is mandatory to explore higher-dimensional Hilbert spaces. A wide spectrum of experiments has been done until now, but many fundamental questions are still open. One way to explore a higher-dimensional Hilbert space is through the orbital angular momentum of light (OAM) which is related to the photon's transverse-mode spatial structure [13]. OAM allow us to implement a qudit encoded in a single photon [14]. It's even possible to combine different degrees of freedom as an alternative to encoded a d-dimensional Hilbert space, for instance OAM with path or polarization degree of freedom, such approach is known as "hybrid encoding". These hybrid qudits can be generated by using a "q-plate" [15], a birefringent phase plate whose optical axis orientation angle is not uniform, generating in this way a difference of phase in the transverse plane. The optical axis orientation at each point is given by:

$$\alpha(r,\phi) = q\phi + \alpha_0 \tag{1}$$

where α is the angle between the optical axis and a reference axis "x" in the transverse plane "xy", ϕ is the azimuthal angle's coordinate in the same plane, "q" and "α_0" are parameters for the topological charge and a constant respectively. The q-plate entangles or disentangles the OAM with the polarization for each photon, for example the action of a q-plate over a horizontally polarized photon (H) is given by:

$$QP\,|H\rangle_\pi\,|0\rangle_{oam} = \frac{1}{\sqrt{2}}(|L\rangle_\pi\,|{-2q}\rangle_{oam} + |R\rangle_\pi\,|2q\rangle_{oam}) \tag{2}$$

which is a hybrid entangled state between polarization and OAM.

II. PROJECT OF PHD. RESEARCH

The framework of my PhD. studies is related with the entanglement of d-dimensional photonic systems achieved by exploiting the orbital angular momentum of light. In particular, my research project aims to explore the different properties and applications of photons under the balanced non-separable superposition of polarization-OAM eigenmodes. The first part of my research will consider a type-II spontaneous parametric down conversion source able of generate the following state:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|R,0\rangle_a |L,0\rangle_b - |L,0\rangle_a |R,0\rangle_b) \tag{3}$$

where $|R,l\rangle$ ($|L,l\rangle$) denotes a photon with circular right (left) polarization carrying an amount $l\hbar$ of OAM and the subscripts "a,b" refers to the two different photons. A general scheme is shown in Fig.1 where all the required components for generation and analysis are presented.

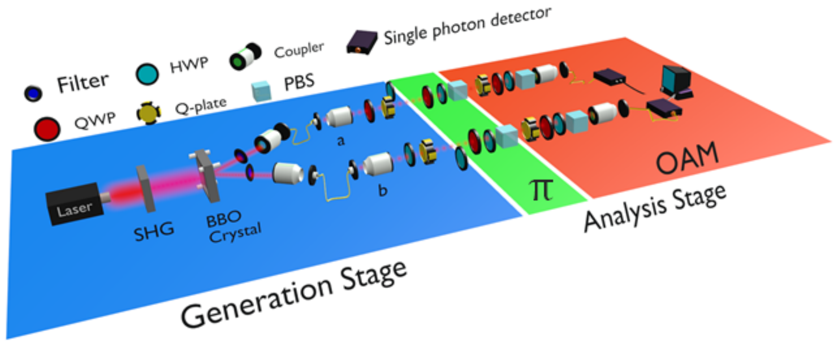

FIG. 1. Experimental setup and generated states. In the generation stage the state of each of two entangled photons (a and b) is locally manipulated via QWP, HWP and q-plates with settings according to the particular state to be prepared. The analysis stage is divided in two sections one for the polarisation analysis π and the other for OAM analysis. The polarisation analysis is performed by using a stage composed of QWP, HWP and PBS. The OAM analysis requires a q-plate to transfer the information encoded in the OAM space to the polarisation degree of freedom which can be then analysed by means of the same kit used in the π-section. After the analysis both photons are sent to single mode fibers connected to single photon detectors

A. Entanglement in vector vortex beams

Light beams having a vectorial field structure or polarization that varies over the transverse profile and a central singularity are known as vector-vortex (VV) beams and exhibit specific properties [16], such as focusing into "light needles" [17] or rotation invariance [18], with applications ranging from microscopy and light trapping to communication and metrology. Part of my research project has been the experimental realization of a the single-particle quantum entanglement between different degrees of freedom, in particular polarization and OAM. By using different topological charges in the q-plates, we generated diverse VV mode orders, corresponding to distinctive polarization patterns for the two beams.

A VV beam of order "m" is defined in the two dimensional Hilbert space spanned by $(|R,m\rangle, |L,m\rangle)$. In particular, we consider the two balanced superpositions $|\hat{r}_m\rangle = \frac{1}{\sqrt{2}}(|R,m\rangle + |L,-m\rangle)$, $|\hat{\theta}_m\rangle = \frac{1}{\sqrt{2}}(|R,m\rangle - |L,-m\rangle)$ representing the radial and azimuthal contributions respectively. For $m \neq 0$, all these modes show a polarization singularity and a null intensity in the center and hence exhibit the so-called doughnut profile. When $m = 1$, the states $|\hat{r}_1\rangle$ and $|\hat{\theta}_1\rangle$, correspond to the well-known radially and azimuthally polarized beams. By noticing that the complete polarization-OAM Hilbert space of order m is spanned by the four states $(|R,m\rangle, |L,-m\rangle, |L,m\rangle, |R,-m\rangle)$, we can also define π-modes of order m as balanced superposition in the space spanned by $(|R,-m\rangle, |L,m\rangle)$: $|\hat{\pi}^+_m\rangle = \frac{1}{\sqrt{2}}(|R,-m\rangle + |L,m\rangle)$ and $|\hat{\pi}^-_m\rangle = \frac{1}{\sqrt{2}}(|R,-m\rangle - |L,m\rangle)$.

The experimental scheme for the implementation of VVB is shown in the Fig.1. In the generation stage the entangled photon pairs are manipulated locally with HWP, QWP and q-plates in order to produce a particular VVB (equation 2), then the photons are sent in the analysis stage. By using another q-plate is possible to measure a VV beams. Indeed, a radial (azimuthal) VV beam of order m is transformed into a linear horizontal (vertical) uniformly polarized beam by a q-plate with $2q = m$. In this way, the measurement of a complex polarization pattern such as of a VV beam is reduced to a much simpler polarization measurement. A more general and complete approach to analyse VV beams is by measuring separately the polarization and OAM, for instance by performing a quantum state tomography in the complete polarization-OAM Hilbert space.

The main goal was to prove the existence of both the intrasystem entanglement between polarization and OAM within each photon and the intersystem entanglement between the two photon states: the former is related to the structure of VV states, the latter corresponds to entanglement between two complex vectorial fields, that has to our knowledge, never been reported before. After a polarization analysis stage (two waveplates and a polarizer), each photon is sent to a q-plate and a second polarization analysis stage. In this configuration, the q-plate transfer the information initially written in the OAM subspace into the polarization state of the photon that can be then analysed with standard techniques. An hyper-complete set of measurements of polarization and OAM for both photons (overall 1296 settings) has been performed to fully reconstruct the density matrix of the entangled photon pair in the sixteen dimensional Hilbert space. The average fidelity of the generated states was: $F = 0.97 \pm 0.01$. In order to quantify the intrasystem entanglement of VV states, the concurrence C for the single photon reduced density matrix in the polarization-OAM space after any projective measurement performed on the other photon. As expected, the distribution is divided in two regions $C = 1$ and $C = 0$. On the other hand, the intersystem entanglement between two VV beams can be quantified by directly calculating the concurrence C for the corresponding density matrix in the VV space. The average value for these concurrences over the five VV couples was $C = 0.91 \pm 0.03$.

One of the most common ways to certify the entanglement is through a violation of a Bell inequality. We performed a violation of CHSH inequality with a mean value of $S = 2.5906 \pm 0.016$ a result over the classical limit $S \leq 2$.

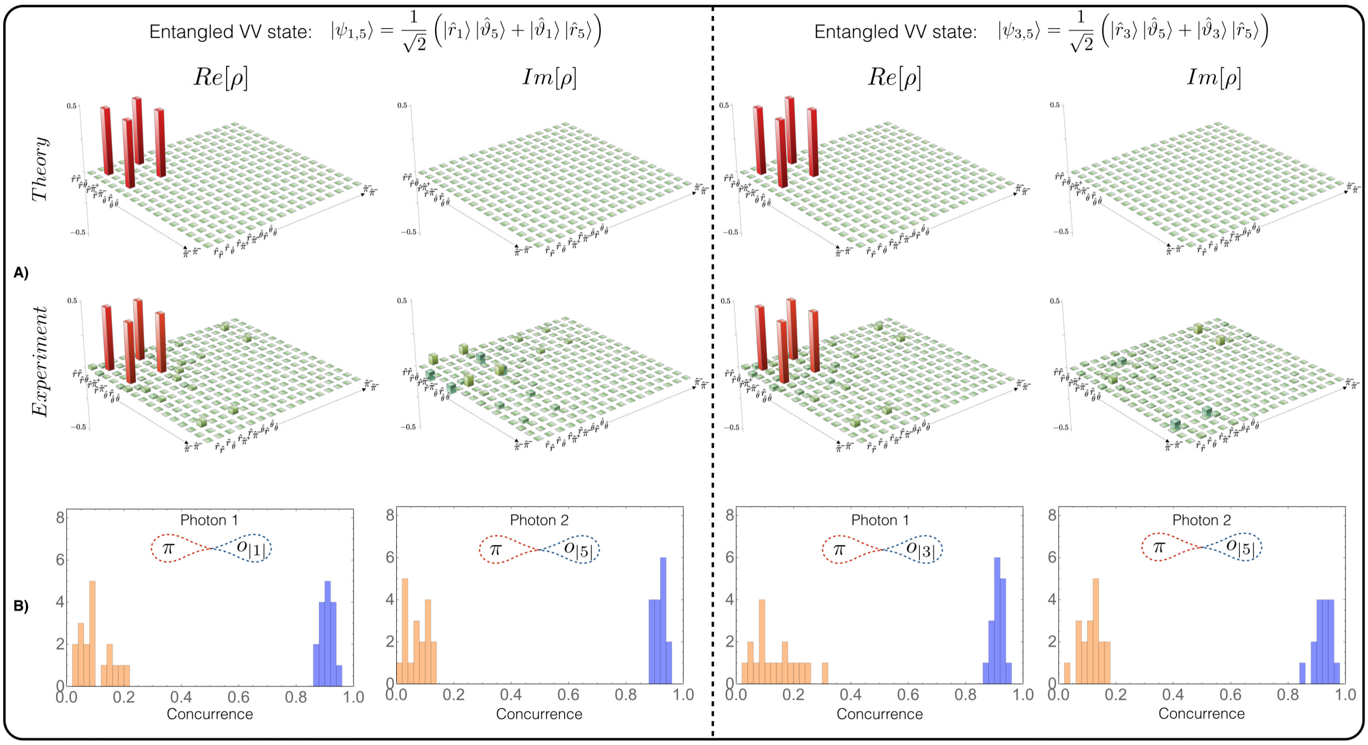

FIG. 2. **Experimental results.** A) Experimental and theoretical density matrices for entangled VV beams of orders ($m_1 = 1, m_2 = 5$) (on the left) and ($m_1 = 3, m_2 = 5$) (on the right). B) Polarization-OAM concurrence distributions. To quantify the intrasystem entanglement we calculated the concurrence distributions for the single photon reduced density matrix after 34 different projective measurements performed on the other photon. All the distributions are divided in two regions corresponding to entangled (blue) and separable (orange) states.

In this work we have fully investigated the entanglement properties of a photonic system composed of two entangled VV beams. We investigated the structure of our system by performing a full state tomography and quantifying both types of entanglement. Moreover, a non-locality test was performed in order to prove that entanglement between complex vectorial fields can be used as a resource in quantum protocols.

B. Geometry of GHZ-states

In 1989 Greenberger, Horne and Zeilinger found that entanglement is not restricted to a pair of particles and that is possible to access to new interesting properties when more particles are involved, these kind of states are called Greenberger-Horne-Zeilinger (GHZ) states [20]. Initially they were part of studies related with fundamental physics but now contribute in a large amount of experiments ranging from quantum simulations, topological error correction to quantum communication protocols. Genuine multipartite states are of special interest since they are the extreme version of entanglement, it means that all the parts contribute to the shared entanglement feature [19]. GHZ-states and their mixtures exhibit fascinating properties and are an example of genuine multipartite entanglement. Considering the setup of Fig. 1 a complete basis of GHZ-states can be constructed by properly choosing local basis rotations. The two physical photons allow us to explore a 16 dimensional Hilbert space with the structure $C_2 \otimes C_2 \otimes C_2 \otimes C_2$, the first and third qubit are encoded in polarization while the second and fourth are encoded in OAM.
The previous setup configuration is able to generate four-qubit GHZ-states having the form:

$$|GHZ_{0000}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle). \tag{4}$$

By changing the relative sign in the previous superposition, we obtain an orthonormal state. In order to construct other basis states orthogonal to these two, one can apply shift operators ($|0\rangle \to |1\rangle$ and vice versa) or/and phase operators on one or more subsystems. However, not all possibilities are successful: there exists a certain local substructure based on entangled entanglement. We will use the unitary Weyl operators (shift and phase operations) which correspond in our cases to the Pauli matrices ($W_{0,0} = 1, W_{0,1} = X, W_{1,0} = Y, W_{1,1} = Z$).
Naively, mixing any two GHZ-states of the 16 one expects that the resulting states have the same entanglement features. Indeed that is not the case, the local information makes the difference!. For instance, the mixture of any two GHZ-states does not destroy the property of genuine multipartite entanglement except when they are equally mixed (as we will prove in this part of the research project). However, for equal mixtures the entanglement properties differ considerably and can be divided in two groups:
Type I (twin GHZ-states): The resulting mixed state is fully separable.
Type II (un-twin GHZ-states): The resulting mixed state is entangled, though no longer genuine multipartite entangled, but still tri-partite entangled.
Type I states occur only for a single mixture, namely if one has chosen one GHZ-state in the set there exists exactly one which erases the entanglement property, a *twin* GHZ-state. This is immediately clear when considering the state defined in the Eq. (4) and the one with the relative minus sign in the superposition. An equal mixture leads to zero off diagonal elements and, consequently, to a product state. In all the other cases we have four non-zero off-diagonal elements for which it is not straightforward to detect their separability properties.

The goal of this part of my project will be to unmask different entanglement features based on their particular local geometrical connectedness for GHZ-states and mixtures. In particular, a specific GHZ-state in a complete orthonormal basis has a "twin" GHZ-state for which equally mixing leads to full separability in opposition to any other basis-state. To detect the separability properties we will exploit the HMGH-framework [21] which provide us a set of nonlinear witnesses for detecting k-separability. The criterion turns out to be optimal for GHZ-states when $k = 2$. We can write the criterion in Pauli's operators (for the linearised version) in order to detect genuine multipartite entanglement and the form is given as follow:

$$\begin{aligned}\tilde{I}_2(\rho) = &\frac{1}{8}\langle XXXX - YYXX - YXYX - XYYX - XXYY - XYXY - YXXY + YYYY\rangle_\rho \\ &-\frac{1}{8}\langle 7\mathbb{1}\mathbb{1}\mathbb{1}\mathbb{1} - ZZ\mathbb{1}\mathbb{1} - Z\mathbb{1}\mathbb{1}Z - Z\mathbb{1}Z\mathbb{1} - \mathbb{1}\mathbb{1}ZZ - \mathbb{1}Z\mathbb{1}Z - \mathbb{1}ZZ\mathbb{1} - \mathbb{1}\mathbb{1}\mathbb{1}\mathbb{1}\rangle_\rho\end{aligned} \tag{5}$$

where we used the abbreviation $XXXX$ for $X \otimes X \otimes X \otimes X$ and so on. $\tilde{I}_2(\rho)$ detects genuine multipartite entanglement if it is greater than zero and gives the maximal value (equal to one) only for the GHZ-state in the representation of Eq.(4). This kind of experiments can pave the way towards improvements in secret sharing protocols based on mixtures of GHZ-states [22] and for quantum algorithms exploring different types of multipartite entanglement [23]. Certainly, this local information between orthogonal basis states is relevant for any experimental setup and can be exploited to generate particular types of entanglement.

C. Alignment free quantum teleportation

Photonic free-space quantum communication has been demonstrated for very high distances [24, 25]. Standard quantum protocols such as entanglement swapping, quantum teleportation, quantum cryptography and so on, where the encoding is based on the polarization of the involved photons, require a "shared reference frame" (SRF). By using hybrid qudits we can improve this protocols since an opportune combination of polarization and OAM allow us to generate alignment-free scenarios [18].
The goal will be carry out an experiment of alignment-free quantum teleportation with vector vortex beams between two distant parties usually denominated (Alice and Bob). If Alice and Bob are movement stations or is impossible to stablish/maintain a fixed reference frame because they are out of sight, we have to circumvent the alignment problem.

The basis that will be employed to write the logical qubits (and solve the SRF problem) is defined as the hybrid single-photon state between polarization and OAM as follows:

$$|0\rangle_L = |L\rangle_p \otimes |r\rangle_o \qquad |1\rangle_L = |R\rangle_p \otimes |l\rangle_o \tag{6}$$

The subscript "p" and "o" indicates the Hilbert space of polarization and OAM respectively ($|l| = |r| = 1$) associated to the eigenvalue $\hbar$. For a given physical rotation of any angle θ with respect to the axis of beam's propagation, the eigenstates of circular polarization and OAM acquire equivalent phase factors:

$$\begin{aligned} |R\rangle_p \to e^{i\theta}\,|R\rangle_p \qquad |r\rangle_o \to e^{i\theta}\,|r\rangle_o \\ |L\rangle_p \to e^{-i\theta}\,|L\rangle_p \qquad |l\rangle_o \to e^{-i\theta}\,|l\rangle_o \end{aligned} \tag{7}$$

Hence by considering the hybrid-photon (combination of polarization and OAM) the phase factors cancel each other and the final state remains immune to all possible reference misalignments.

For this experiment we need to enable a second source of SPDC, since teleportation protocols require four photons. Three of them will be used by Alice and Bob, the fourth photon will act as a trigger.

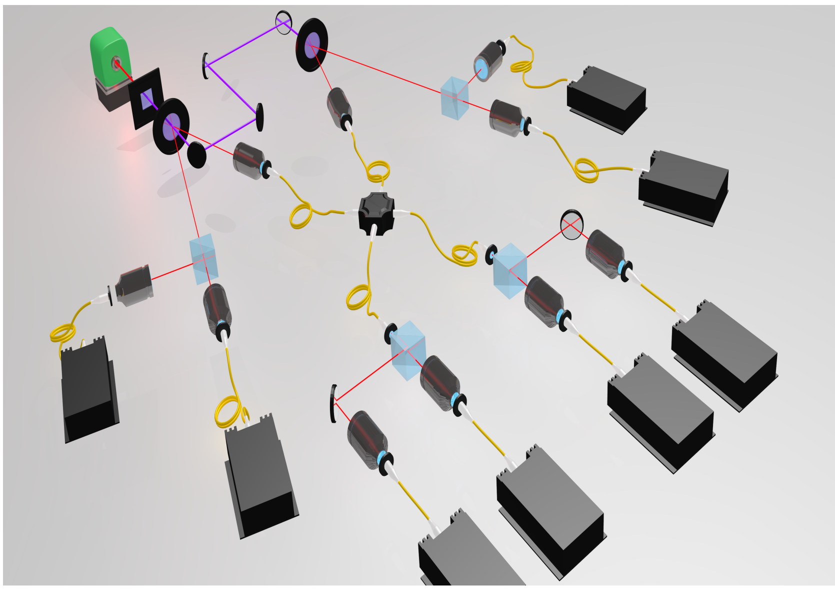

FIG. 3. Tentative version of the setup to be implemented in order to carry out multiphoton experiments with/without hybrid-coding

In order to perform an alignment-free teleportation protocol the photons of Alice and Bob will be generated by different SPDC sources which doesn't share a common reference frame (experimentally this will be executed by physically rotating at least one photon before the Bell's measurement). The scheme of teleportation says that Alice posses an entangled photon pair (first SPDC source), Bob has a third photon with the encoded information to be teleported (second SPDC) that will send to Alice who will perform a Bell's measurement on both photons (one of her pair and the photon received from Bob). Finally, the information encoded on Bob's photon will be teleported to the other photon not used by Alice in the Bell's measurement (with an unitary transformation associated that can be recovered by classical communication between the parties). This protocol will consider hybrid-photons in a vector vortex beam state, so the problem of the reference frame will be solved.

At this point we will count with two SPDC sources (see FIG. 3), so multiphoton experiments with hybrid-coding will be possible. Other experiments such as entanglement swapping, dense coding, quantum complementarity, multiphoton interference, non-local correlations and so on could be implemented. Following the concept underlying the working principle of rotational invariant qubits, it's possible to perform experiments with applications in metrology [26] by generating for example, NOON-like photonics states.

III. SCHOOLS AND CONFERENCES

1) PICQUE Roma Scientific School Scientific School in integrated quantum photonics applications, 6-10 July 2015
2) V Quantum Information School and Workshop - Paraty 2015 Paraty, Rio de Janeiro, Brazil, 04-15 Aug 2015 (Poster: Experimental entanglement in vector vortex beams)
3) 101 Congresso Nazionale della Societ Italiana di Fisica Roma 21-25 settembre 2015
4) 606.Wilhelm und Else Heraeus-Seminar on Nanophotonics and Complex Spatial Modes of Light, Bad Honnef 24-29 January 2016, Germany(Poster: Hybrid entangled entanglement in vector vortex beams).

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