Transport Properties in Multichannel Systems

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HTSC as Multichannel System

The high $T_c$ is only one of the intriguing aspects of HTSC!

- Several coexisting/competing orders near the superconducting one
- Anomalous normal state properties above $T_c$ (Focus of this thesis)
Open Issues: Cuprates

- PARTICLE-PARTICLE Interactions (superconducting fluctuations)
- PARTICLE-HOLE Interactions (density-density and current-current interactions)
Open Issues: Cuprates

- PARTICLE-PARTICLE Interactions
  (superconducting fluctuations)
- PARTICLE-HOLE Interactions
  (density-density and current-current interactions)

**Study the interplay between current-current interactions and superconducting fluctuations.**
Crucial to account how the interactions renormalize the current with respect to the velocity.

Current-current Fermi-liquid corrections to the superconducting fluctuations on conductivity and diamagnetism: LF et al. PRB 85, 024507 (2012)
Open Issues: Pnictides

- **MULTIBAND** Systems: Holes and Electrons

- **DOMINANT INTERBAND**
  Interactions: Exchange of AF fluctuation between the bands
Open Issues: Pnictides

- **MULTIBAND** Systems: Holes and Electrons
- **DOMINANT INTERBAND**
  Interactions: Exchange of AF fluctuation between the bands

**What happens when holes and electrons interact?**
Let’s look at the Hall effect

\[ R_H \equiv \frac{E_y}{J_x H_z} = \frac{1}{nq} \]

Semiclassical result
Unconventional features of the Hall Effect in pnictides

- Semiclassical Boltzmann-like multicarrier picture

\[
R_H = \frac{1}{e} \frac{(n_h \mu_h^2 - n_e \mu_e^2)}{(n_h \mu_h + n_e \mu_e)^2}, \quad \mu_\alpha = \frac{e^2 \tau_\alpha}{m_\alpha}.
\]

For a compensated semimetals \( n_e \approx n_h \), \( \Rightarrow \) \( R_H \sim 0 \)?
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\[ \mu_\alpha = e^2 \tau_\alpha / m_\alpha. \]

For a compensated semimetals \( n_e \approx n_h \), \( R_H \sim 0 \)?

- Prediction vs Experiments

  - Very large absolute value of \( R_H \) with \( R_H \leq 0 \) for electron/hole-doped systems.

  - Strong T-dependence

Experimental data on BaFe\(_2\)As\(_2\) 7%
Co-doped from Rullier-Albenque et al.
PRB 81, 224503 (2010).
Going beyond the Semiclassical Picture

We need to take into account the **interactions** between carriers
How to do that?
Going beyond the Semiclassical Picture

We need to take into account the \textbf{interactions} between carriers
How to do that?

\textbf{Full Gauge-Invariant Calculation that includes vertex correction:}
The interaction dresses the bare velocity

\[ J = v + \text{Again } J \neq v! \]
Going beyond the Semiclassical Picture

We need to take into account the **interactions** between carriers. How to do that?

**Full Gauge-Invariant Calculation that includes vertex correction:** The interaction dresses the bare velocity

\[
R_H = \frac{\sum_\alpha \sigma_{xy}^\alpha}{(\sum_\alpha \sigma_{xx}^\alpha)^2 H_z}
\]

\(\alpha\) band-index

\[
\sigma_{xx}^\alpha \sim \mathbf{J}^\alpha \cdot \mathbf{v}^\alpha \tau^\alpha,
\]

\[
\frac{\sigma_{xy}^\alpha}{H} \sim \mp |\mathbf{J}^\alpha|^2 \tau^2 \alpha^2
\]

\(\mp\) depends only on the band character (electron/hole)
Vertex correction in a nutshell

**QP lifetime** $\tau$

- e.g. ARPES measurements

![Graph 1](image1)

**QP scattering time** $\tau_{tr}$

- e.g. transport measurements

![Graph 2](image2)

**Effect of Vertex Correction** ⇒ $J \neq v$, $\tau_{tr} \neq \tau$
Vertex correction in a nutshell

- Single-band Systems: \( J = \Lambda v \):

  vertex corrections can be recast in a renormalizations of
  the transport scattering time with respect to the QP
  lifetime

  \[
  \Rightarrow \quad \sigma_{xx} \sim v \cdot J \tau = v \cdot (\Lambda v) \tau = v^2 \tau_{tr}, \quad \text{with} \quad \Lambda \tau = \tau_{tr}
  \]
Vertex correction in a nutshell

- **Single-band Systems**: $J = \Lambda v$:
  
  vertex corrections can be recast in a renormalizations of the transport scattering time with respect to the QP lifetime
  
  $$\Rightarrow \sigma_{xx} \sim v \cdot J\tau = v \cdot (\Lambda v)\tau = v^2\tau_{tr}, \quad \text{with} \quad \Lambda\tau = \tau_{tr}$$

- **Multiband Systems**: $J^\alpha = \Lambda_{\alpha\beta}v^\beta$
  
  $$\Rightarrow \sigma_{xx}^\alpha \sim v^\alpha \cdot J^\alpha \tau^\alpha$$

  Presence of hole and electron carriers:
  
  $J^e \sim \Lambda_{ee}v^e + \Lambda_{eh}v^h$ ($v^e > 0, v^h < 0$)

  J can even have **opposite direction** with respect to v
  
  vertex corrections cannot be simply recast in a renormalizations of the transport scattering time
  
  $$\Rightarrow \text{Failure of the Boltzmann paradigm}$$
Our minimal model: 2-band with interband interactions

- 1 electron band + 1 hole band with 2D parabolic dispersion
- carriers interact via the exchange of spin fluctuations
Our minimal model: 2-band with interband interactions

\[ \chi(q - Q, \omega) = \frac{\chi Q}{1 + \xi_{AF}^2 (q - Q)^2 + i\omega/\omega_{sf}} \]

- 1 electron band + 1 hole band with 2D parabolic dispersion
- carriers interact via the exchange of spin fluctuations

- the interaction is stronger the more similar is the size of the pockets
- vertex corrections vanish when \( \chi \) is momentum-independent i.e. when \( \xi_{AF} \xrightarrow{T \to \infty} 0 \)

Experimental \( \chi''(\omega) \) from Inosov et al. Nature Phys. 6, 178 (2010).
Bare velocities and renormalized currents

- $J$ can even change sign with respect to $v \Rightarrow \sigma_{xx}, \sigma_{xy}$ are no longer Boltzmann-like!

- strong T-dependence of both $J^e$ and $J^h$

- $|v^e| \gtrsim |v^h|$ but $|J^e| \gg |J^h|$
\( R_H(T) \)

\[ \sigma_{xy}^e \approx -|J^e|^2, \]
\[ \sigma_{xy}^h \approx |J^h|^2 \]
\[ |J^e| \gg |J^h| \Rightarrow R_H \ll 0 \]

LF et al. PRL 109, 096402 (2012).
A single mechanism for the whole phase diagram

- Strong T-dependence of $R_H$

- In e-doped $|J^e| \gg |J^h| \Rightarrow$ negative $R_H$
  In h-doped $|J^h| \gg |J^e| \Rightarrow$ positive $R_H$

- What happens by **doping** the system?
  Spin fluctuations are less effective, $\mathbf{J} \simeq \mathbf{v}$ and one approaches the Boltzmann result
Theoretical and experimental studies of the hydrodynamic resistance $R_H$ and Fe content $n_{Fe}$ as functions of temperature $T$ for different compositions $x$. The data is plotted over a range of temperatures from 0 to 300 K.

**Theory**

**Experiments**

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LF et al. PRL 109, 096402 (2012).
Conclusions

- Failure of the Boltzmann paradigm in multiband systems with interband interactions between carriers having different character (hole vs electron)

- Good agreement with the experimental findings in the normal state of pnictides:

  Anomalous Hall effect due to large vertex correction
Ph.D Work Take-Home Message

- Many of the anomalous transport properties of HTSC can be ascribed to the multichannel character of these classes of materials

**HTSC PROTOTYPE OF MULTICHANNEL SYSTEMS**

- The multichannel character of the interactions forces the revision of standard paradigm

**MORE IS DIFFERENT**

- Dealing with many relevant degrees of freedom is not always an easy task: Full Gauge-Invariant calculation must be carried on

**RELEVANCE OF VERTEX CORRECTIONS**