Precision flavor physics from Lattice QCD

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Outline

Introduction
- Open questions in Standard Model
- Flavour physics
- Lattice Gauge theories

Flavor physics on lattice
- Quark masses
- Decay constants
- Form factors
The Standard Model and the need for New Physics

**Standard Model of electroweak interactions**
- Present theory for fundamental interactions among elementary particles
- Unification of Electromagnetism with Weak Interactions
- Tested with success up to the explored scale of energy \( O(100 \text{ GeV}) \)

**Open questions**
- Hierarchy problem (higgs mass instability)
- Gravity problems
- Non-unification of the other three interactions
- Flavour unexplained
- No candidate for Dark Matter
- CP asymmetry too weak to explain baryogenesis
- Etc.

→ new physics (NP) beyond the Standard Model is expected at TeV scale
Where to search for new physics?

Directly
By discovering new particles
- Need to explore the TeV scale of energies
- Need very high energies accelerators (TeVatron, and mainly LHC)
Where to search for new physics?

Indirectly

Small corrections to the Standard Model predictions

- Possible still below $TeV$ scale
- Need high luminosity accelerators (B-factories BaBar, Belle, and the proposed SuperB)

Check of the experimental observables with SM prediction

- Comparison can give remarkable information
- Dominant role is played by flavor physics
- Quark sector: Cabibbo-Kobayashi-Maskawa matrix.
Unitarity triangle analysis

- CKM matrix elements from different experimental inputs
- Need different theoretical calculations
- Tightest test to the SM in the flavour sector

Difficulties caused by $QCD$

Calculations often involve QCD

- At distance $\gtrsim 1 \, fm$ (energy $\lesssim 1 \, GeV$) theory nonperturbative
- Low energy calculation needs nonperturbative treatment
- For instance to determine:
  - decay constants
  - form factors
  - matrix elements involved in meson mixings
  - etc
Lattice Gauge Theories

Lattice as regularizator for Quantum Field Theories

- Interacting Quantum Field Theory are $UV$ and $IR$ divergent, so need to:
  - regularize: put an ultraviolet cut-off on momenta
  - renormalize: fix bare parameters to reproduce some physical quantity in presence of cut-off
  - remove cut-off while tuning bare parameter, keeping physical quantities finite.

Possible regularization:

- momenta cut-off
- dimensional regularization
- spatial cut-off: lattice

Lattice is convenient when dealing with non perturbative problems
Interacting theory in the continuum (euclidean)

\[ S = \int d^4x \mathcal{L}(x), \quad \mathcal{L}(x) = \bar{\psi}(x)(\mathcal{D} - m)\psi(x) + F_{\mu\nu}(x)F_{\mu\nu}(x) \]

Observables:

\[ \langle O \rangle = \frac{\int D\psi D\bar{\psi} D A_\mu O e^{-S}}{\int D\psi D\bar{\psi} D A_\mu e^{-S}} \quad \text{average over field configurations} \]

Very badly defined integral

Discretization

- Fields on grid \( x_i \) of size \( L \) and spacing \( \alpha \)
- Discretized lagrangian \( \hat{\mathcal{L}} \)
  - Has \( \mathcal{L} \) as limit when \( \alpha \to 0 \)
  - Gauge invariance: \( U \equiv e^{i\int A_\mu} \)
- Lattice will:
  - put cut-off of \( 1/\alpha \) in \( UV \), \( 1/L \) in \( IR \)
  - regularize functional integral:
    \[ \int D[\psi(x)] \to \prod_i \int d\psi_i \]

\[ \langle O \rangle = \frac{\prod_i \int d\psi_i d\bar{\psi}_i dU_i O e^{-S}}{\prod_i \int d\psi d\bar{\psi} dU e^{-S}} \quad \text{well defined integral} \]
**Evaluation of the functional integral**

To evaluate $\langle O \rangle = \frac{\prod_i \int d\psi_i d\bar{\psi}_i dU_i O e^{-S}}{\prod_i \int d\psi d\bar{\psi} dU e^{-S}}$ numerically:

1. Generate $N$ configurations with weight $e^{-S}$ (Monte Carlo)
2. On each configuration measure $O_i$, $1 = 1 \ldots N$
3. Evaluate $O^{\text{ext}} = \frac{1}{N} \sum_i^N O_i$

**Main sources of systematic errors**

Extrapolation in:
- $a$ and in $L$
- physical light quark masses

Quenching (suppression of fermionic loop) not a big issue anymore

**Strong points**

- Method starting from first principles
- Errors can be arbitrarily lowered by using more computing power
Quark Masses

Relevance of Quark Mass values

\[ \mathcal{L}_{QCD} = \sum_q \bar{\psi}_q (\not\!\!p_q - m_q) \psi_q, \quad q = u, d, s, c, b, t \]

- \( m_q \): fundamental parameter NOT directly measurable (confinement)
- Values of \( m_q \) needed for many predictions in QCD

How we can calculate \( m_q \)?

- Choose quantity \( G^{phys} \) depending from \( m_q \) (\( M_{Hadr}, \Gamma_{Hadr\to X, \ldots} \))
- Calculate \( G (m_q) \) in QCD
- Find \( m_q \) solving: \( G (m_q) = G^{phys} \)

Which quantity use?

- High energy process \( \sim \) indipendent from \( m_{Up/Down/Strange} \)
- Mass of pseudoscalar mesons (pseudo-Goldstone bosons) highly sensible to quark masses value: \( m_\pi \to 0 \) if \( m_{u,d} \to 0 \)
How to calculate mass spectrum

- Choose operator $O$ with quantum number $\{n\}$ (parity, charge, etc)
- Calculate two point correlation functions

$$C(\tau) = \left\langle O^\dagger(\tau) O(0) \right\rangle = \frac{1}{Z} \int D[\psi] \left( \int d^3 x O^\dagger(\tau, x) O(0) \right) e^{-S[\psi]}$$

$$C(\tau) = \langle 0 | O_P^\dagger \sum_S |S\rangle e^{-m_S \tau} \langle S | \frac{1}{2m_s} O_P | 0 \rangle \rangle$$

$$\tau \rightarrow \infty : |\langle 0 | O_P | P \rangle|^2 e^{-m_P \tau} / 2m_P$$

$\rightarrow P$ lightest particle with quantum numbers $\{n\}$

Simplest operator for pseudoscalar meson: $O = \bar{\psi} \gamma_5 \psi$
Calculation of di $m_l = (m_u + m_d) / 2$ on lattice

- Assume isospin symmetry ($m_u = m_d$)
- Fix lattice spacing $a$, volume $L$
- Choose $m_l$ and calculate $\langle O^\dagger(\tau)O \rangle$, from which extract $m_{PS}(m_l)$
- Variate $m_l$ and calculate $m_{PS}$ up to reproducing physical pion $m_{\pi}^{phys}$
- Repeat lowering $a$ and enlarging $L$, extrapolating to $a \rightarrow 0$, $L \rightarrow \infty$

On lattice one has always $m_l \gg m_{\pi}^{phys}$:
- one need to invert a matrix that get more and more singular with the decrease of $m_l$
- volume constraint $L \gg 1/m_{\pi}$

Finite volume effects treatable analitically
- constraint: $L \gtrsim 2fm$, $M_{\pi}L \gg 1$
- Colangelo-Durr-Haefeli formulas [hep-lat/0503014]
Which Lattice?

Lattice setup

- Improved Regularization, $\hat{\mathcal{L}} \xrightarrow{a \rightarrow 0} \mathcal{L}$ faster
- $O(a)$ effects removed from results: easier extrapolation to the continuum
- $N_f = 2$ dynamical flavour, lights degenerate
- In future, $N_f - 2 + 1 + 1$: 2 lights + strange + charm

To perform continuum limit, 4 lattice spacings, ranging from 0.05 – 0.10 fm
Some technical detail

- Various sea masses $10 - 50\,\text{MeV}$ for the “chiral limit”
- Two volumes (for some setups) to check volume effects

Typical statistics: $300 - 500$ uncorrelated configurations
Statistical is not the main source of errors
$M_{\pi}^2$ fit in $SU(2)\chi PT$

$M_{\pi}^2 = 2B_0 m_l [1 + m_l \log(m_l/\Lambda^3) + k a^2]$}

SU2-$\chi$PT

$M_{\pi}^2 = \text{Discret.} NLO$

$M_{\pi}^2 \text{ fisico} = (135.0 \text{ MeV})^2$

$m_{l \text{MS 2GeV}}$ (MeV)

$m_1$ (MeV)

0 10 20 30 40 50 60

0 50000 1e+05 1.5e+05 2e+05 2.5e+05 3e+05

$a=0.100 \text{ fm}$

$a=0.085 \text{ fm}$

$a=0.067 \text{ fm}$

$a=0.054 \text{ fm}$

Continuo
Discretization effects for $M^2_\pi$

![Graph showing $M^2_\pi$ vs $a^2$](image)

- $a^2$ in fm$^2$
- $M^2_\pi$ in GeV$^2$
- Points for different $\beta$ values:
  - $\beta=3.80$
  - $\beta=3.90$
  - $\beta=4.05$
  - $\beta=4.20$
- Fit curve
- Physical $\pi$
- $\Delta M^2_\pi / M^2_\pi \sim 10\%$
Quark Masses (preliminary results)

Light \( \frac{m_u + m_d}{2} \)

Strange

Charm

Coming soon: \( b \) quark mass
Decay constant

Leptonic decays of pseudo-scalar meson

\[ \Gamma(P \rightarrow l\nu) \propto \frac{G_F^2}{8\pi} |V_{q_1q_2}| f_P^2 m_l^2 M_P^2 \left(1 - \frac{m_l^2}{M_P^2}\right)^2 \]

General Importance

- CKM matrix elements can be determined from leptonic decays
- Decay constant needed

The case of \( f_B \)

Quite large deviation between theoretical prevision and data for \( \Gamma(B \rightarrow \tau\nu) \)

- Large discretization effects on lattice
- Extrapolation in \( m_B \) needed
- Static point (HM-\( \chi \)PT needed)

Extrapolation of $f_B$
Future works

Near future
- Calculation of $b$ quark mass
- Calculation of $f_K, f_D$
- New method for extraction of $f_B$

Other flavour topics
- Semi-leptonic decays (important for $V_{ub}$)
- Studies with $N_f = 2 + 1 + 1$

X space renormalization
- New method for nonperturbative renormalization of operators, alternative to $RI - MOM$ and Schroedinger Functional
- Method based upon stochastic approximation applied on the configuration space [hep-lat/0406019]