Electroweak nuclear response in the quasi elastic sector

Ph.D. Research Project

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1 Motivations

Neutrino Physics is entering the age of precision measurements. Several experiments have detected neutrino oscillations, providing unambiguous evidence that neutrinos, assumed to be massless in the Standard Model of Particle Physics, have in fact non vanishing masses.

Reactor neutrino experiments carried out in the last five years (Double Chooz [1], Daya Bay [2] and RENO [3]) recently reported high quality measurements of the $\theta_{13}$ mixing angle, the value of which turned out to be $\sim 10$ deg. The large $\theta_{13}$ mixing angle will enable future experiments, such as the Long- Baseline Neutrino Experiment (LBNE) in the United States [4], to search for leptonic CP violation in appearance mode, thus addressing one of the outstanding problems in Particle Physics. However, these searches will involve high precision determinations of the oscillation parameters, which in turn require a deep understanding of neutrino interactions with matter. In view of the achieved and expected experimental accuracies, the treatment of nuclear effects is indeed regarded as one of the main sources of systematic uncertainty. The authors of Ref. [5] argued that the uncertainty on neutrino energy reconstruction [6], arising from the limitations of the nuclear models employed in data analysis, may hamper the extraction of the CP violating phase from an oscillation result.

Over the past decade, it has become more and more evident that the Relativistic Fermi Gas Model (RFGM), routinely employed in simulation codes, is not adequate to account for the complexity of strong interaction dynamics and the variety of reaction mechanisms contributing to the detected signals.

A striking manifestation of the above problem is the large discrepancy between the results of Monte Carlo simulations and the double differential cross section of charged current quasi elastic (CCQE) interactions in carbon, recently measured by the Mini-BooNE collaboration [7].

A strong effort is being made to develop a fully quantitative description of the neutrino-nucleus cross section, based on information provided by the large body of accurate electron-nucleus scattering data.

The main difficulty involved in the generalisation of the approaches successfully employed to analyse electron scattering data to the case of neutrino interactions stems from the fact that, while in electron scattering the beam energy is fixed, in neutrino scattering the measured cross section is obtained by averaging over different energies, distributed according to the neutrino flux. As a consequence, the observed energy of the outgoing charged lepton does not determine the amount of energy transferred to the nuclear target, which in turn determines the dominant reaction mechanism.

In this Thesis we want to analyse how the contribution of processes leading to the production of $2p2h$ final states affects the nuclear response. As a first step we take into account the electromagnetic channel as a large set of data are available to check the validity of our results.
2 Neutrino- nucleus Charged Current Quasi Elastic (CCQE) interactions

The double differential inclusive cross section of the neutrino-nucleus scattering process \( \nu_\ell (k) + A(p_0) \rightarrow \ell^- (k') + X(p_X) \), where A and X denote the target nucleus, in its ground state, and the undetected final state, respectively, can be written in the form [8]

\[
\frac{d^2 \sigma}{d \Omega_{k'} dk'_0} = \frac{G_F^2 V_{ud}^2}{16 \pi^2} \frac{|k'|}{|k|} L_{\mu \nu} W_{A}^{\mu \nu}.
\]  

(1)

The lepton tensor is completely determined by the lepton kinematics, whereas the nuclear tensor \( W_{A}^{\mu \nu} \), containing all the information on strong interaction dynamics, describes the response of the target nucleus. Its definition

\[
W_{A}^{\mu \nu} = \sum_X \langle 0| J_A^{\mu \dagger} |X \rangle \langle X| J_A^{\nu} |0 \rangle \delta^{(4)}(p_0 + q - p_X),
\]

(2)

with \( q = k - k' \), involves the target initial and final states \( |0 \rangle \) and \( |X \rangle \), carrying four momenta \( p_0 \) and \( p_X \), respectively, as well as the nuclear current operator.

The nuclear current expression is determined by requiring that its electromagnetic part satisfies the continuity equation: \( q \cdot j(q) = [H, \rho(q)] \) where the Hamiltonian includes two- and possibly three-nucleon interactions. As the isospin and momentum dependence of the two-nucleon interaction leads to non vanishing commutators with the one-body charge operator, a consistent treatment of the nuclear matter, based on the use of a realistic Hamiltonian requires the introduction of the two-body currents. The nuclear electroweak current operator is expanded into a sum of one- and two-body terms which operate on the nucleon degrees of freedom:

\[
J_A^{\mu} = \sum_i J_i^{\mu} + \sum_{j>i} J_{ij}^{\mu},
\]

(3)

where \( J_{ij}^{\mu} \) denotes the two-nucleon contribution arising from meson-exchange processes.

In the QE regime, the composite structure of the hadrons in the interaction vertices can be parametrized in terms of form factors. Exploiting the CVC and PCAC hypothesis, the_CCQE neutrino-nucleus process is described in terms of the electromagnetic form factors, \( F_1 \) and \( F_2 \) and the axial form factor \( F_A \). For the electromagnetic case, gauge invariance actually puts constraints on the form factors by linking the divergence of the two-body currents to the commutator of the charge operator with the nucleon-nucleon interaction. The latter contains form factors too, but these are obtained phenomenologically by fitting nucleon-nucleon data. Thus the continuity equation reduces the model dependence of the longitudinal part of the corresponding two-body current by relating it to the form of the interaction [9].

For \( A \leq 12 \), accurate calculations of the tensor \( W_{A}^{\mu \nu} \) can be carried out within non relativistic nuclear many-body theory [10, 11]. However, the event analysis of accelerator-based neutrino experiments requires theoretical approaches that can be applied in the relativistic regime. The importance of relativistic effects can be easily grasped considering that the mean values of momentum transfer of QE processes obtained by averaging over the MiniBooNE [7] and Minerva [12] neutrino fluxes turn out to be \( \sim 640 \) and \( \sim 880 \) MeV, respectively. While non relativistic approaches can always be used for the description of the initial state of the target nucleus, which is independent of momentum transfer, at large \( |q| \) the treatment of both the nuclear current and the hadronic final state unavoidably requires approximations.
The impulse approximation (IA) scheme rests on the assumptions that at momentum transfer $\mathbf{q}$ such that $q^{-1} \ll d$, $d$ being the average separation distance between nucleons in the target nucleus, the contribution of the two-nucleon current can be disregarded and the final state $|X\rangle$ can be written in the factorized form

$$|X\rangle = |p\rangle \otimes |n_{A-1}, p_n\rangle,$$  \hspace{1cm} (4)

where the state $|p\rangle$ describes a non interacting nucleon carrying momentum $p$, while $|n_{A-1}, p_n\rangle$ describes the $(A-1)$-particle spectator system in the state $n$, with momentum $p_n$.

Within the IA, the nuclear cross section – including only the contributions of the one-body current - can be written in terms of the cross section of the elementary scattering process, involving an individual nucleon, and the target spectral function

$$d\sigma_{IA} = \int d^3k \ dE P(k, E) d\sigma_{elem},$$  \hspace{1cm} (5)

where

$$P(k, E) = \sum_n \langle 0 \left\{ |n_{A-1}, p_n\rangle \otimes |k\rangle \right\} |^2 \delta(E + E_0 - E_n),$$  \hspace{1cm} (6)

yields the probability distribution of removing a nucleon with momentum $k$ leaving the residual system with an excitation energy $E$ [13].

Exploiting the Källén Lehman representation of the Green’s function, the spectral function is most conveniently separated into two parts, according to its energy dependence. The single particle, or pole, term $P_{s.p.}(k, E)$, corresponds to the contribution from the one hole $(1h)$ intermediate states and turns out to be sharply peaked at $E = -e(k)$, $e(k)$ being the binding energy of the $1h$ state. The width of the peak provides a measure of the lifetime of the hole state and goes to zero as it approaches to the Fermi surface.

The integral of $P_{s.p.}(k, E)$ over the energy gives the strength $Z(k)$ of the hole state which is quenched with respect to unity, due to NN correlations. The other part, denoted as correlated spectral function $P_{corr}(k, E)$, correspond to the contribution from $n$-hole-$(n-1)$-particle states. It would be strictly zero in the absence of NN correlations and its leading contribution comes from $2h1p$ states. This part has a completely different energy dependence as compared to $P_{s.p.}(k, E)$, showing a widespread background extending up to large values of both $k$ and $E$ with a maximum at $E \sim k^2/2m$.

It has been suggested [14] that the excess CCQE cross section observed by the MiniBooNE collaboration may be ascribed to the occurrence of events with $2p2h$ states. It is very important to point out that, in scattering processes involving interacting many-body systems, $2p2h$ final states can be produced through the action of both one- and two-nucleon currents.

In order for the matrix element of a one-body operator between the target ground state and a $2p2h$ final state to be non vanishing, the effects of dynamical nucleon-nucleon (NN) correlations, must be included in the definition of the nuclear wave functions.

Thus, a consistent description of these processes within a realistic model of nuclear dynamics, requires that all mechanisms leading to their appearance – Initial State Correlations (ISC) among nucleons in the target ground state, Final State Interactions (FSI) between the struck nucleon and the spectator particles, and interactions involving two-nucleon meson exchange currents (MEC) – be included. In existing calculations [15, 16], however, the initial and final states are described within the Independent Particle Model (IPM), in which the interaction effects are expressed in terms of a mean field, $2p2h$ final states can only be excited by two-body operators.
The ISC contribution to the $2p2h$ amplitude arises from processes in which the beam particle couples to one of these high-momentum nucleon. Within the IA, the effects of ISC on the nuclear cross section at large momentum transfer can be taken into account using a realistic spectral function.

Figure 1: Inclusive electron-carbon cross section at beam energy $E_e = 730$ MeV and electron scattering angle $\theta_e = 37^\circ$, plotted as a function of the energy loss. Dashed line has been obtained by retaining only the correlation part of the spectral function. The data points are taken from Ref. [13].

Figure 1 illustrates the $2p2h$ contribution to the electron-carbon cross section, at beam energy $E_e = 730$ MeV and electron scattering angle $\theta_e = 37^\circ$, arising from ISC. The solid line corresponds to the result of the full calculation, carried out within the IA using the spectral function of Ref. [13], while the dashed line has been obtained including only the amplitudes involving $2p2h$ final states.

The starting point of our work consists on the observation that the presence of strongly correlated $np$ pairs leads to important interference effects between the amplitudes associated with one- and two- nucleon currents and that this term may play a crucial role.

In Ref. [17] we reported the results of the calculation of the sum rule of the electromagnetic response of the carbon in the transverse channel, carried out within the Green Function Monte Carlo (GFMC) computational scheme. Exploiting the completeness of the set of final state entering the definition of the nuclear inclusive cross section, these sum rules can be easily related to the energy loss integrals of the transverse components of the tensor describing the target response to electromagnetic interactions. Chosing the $z$-axis along the direction of the momentum transfer, $\mathbf{q}$, the transverse sum rule can be written in the form

$$ S_T(q) = \int d\omega S_T(q,\omega) $$

where

$$ S_T(q,\omega) = S^{xx}(q,\omega) + S^{yy}(q,\omega) $$

$$ S^{\alpha\beta} = \sum_N \langle 0|J^\alpha_A|N\rangle\langle N|J^\beta_A|0\rangle\delta(E_0 + \omega - E_N) $$
The results of Fig. 2 clearly show that interference terms provide a sizeable fraction of the sum rule.

While providing valuable and previously unavailable information, the study of Ref. [17] has been performed using non relativistic initial and final states, and a non relativistic reduction of the nuclear current. Moreover, the results only include the energy-integrated response, and cannot be directly compared to experimental data.

We plan to carry out a full calculation of the double differential neutrino-carbon cross section, using the relativistic expressions of the one- and two-body currents, and consistently taking into account the terms arising from their interference.

Our work will be based on a generalisation of the factorization ansatz of Eq. (4), allowing one to write the nuclear matrix element of the two-nucleon current in the form

\[
\langle X | j^\mu_{ij} | 0 \rangle = \int d^3k d^3k' M_m(k, k') \langle pp' | j^\mu_{ij} | kk' \rangle . 
\]

The nuclear amplitude \( M_m(k, k') \) given by

\[
M_m(k, k') = \{ \langle n_{(A-2)}, p_m | \otimes \langle kk' | \} | 0 \rangle ,
\]

turns out to be independent of \( q \), it can be readily obtained within non relativistic many body theory.

The connection with the spectral function formalism becomes apparent noting that the two-nucleon spectral function \( P(k, k', E) \), yielding the probability of removing two nucleons from the nuclear ground state leaving the residual system with excitation energy \( E \) (compare to Eq. (6)), is defined as

\[
P(k, k', E) = \sum_m |M_m(k, k')|^2 \delta(E + E_0 - E_m) ,
\]

where \( M_m(k, k') \) is the nuclear amplitude and \( E_0 \) is the ground state energy.

It is worth noting that both Eq. (4) and (10), have been obtained assuming that all FSI between the nucleon interacting with the beam particles and the spectators can be neglected. In inclusive processes, FSI lead to a shift of the energy loss spectrum, arising from interactions between the knocked out nucleon and the mean field of the recoiling
nucleus, and a redistribution of the strength from the quasi free bump to the tails, resulting from rescattering processes. Theoretical studies of electron-nucleus scattering suggest that in the kinematical region relevant to the MiniBooNE analysis the former mechanism, which does not involve the appearance of two particle-two hole final states, dominates, thus this contribution is not expected to be critical. The inclusion of FSI within the IA scheme has been recently discussed in Ref. [19]. A different approach, based on a Monte Carlo simulation, is described in Ref. [20].

3 Two-nucleon electromagnetic currents

Following the discussion of Refs. [21, 22], as a first step, two different contributions to the two-body electromagnetic current have been analysed. One is a direct consequence of the requirement of current conservation. This requirement gives rise to \( \pi^-\) meson exchange currents. Due to the constraints from current conservation, there is little ambiguity in the construction of these currents.

\[
\text{Figure 3: Feynman diagram representation of the isovector two-body currents associated with pion exchange. Solid lines: nucleons; dashed lines: pions; wavy lines: photons.}
\]

The second contribution to the two-body current we consider are the \( \Delta^-\) isobar currents. These are much more model dependent since they are not constrained by model conservation.

We start by deriving the lowest-order pionic MECs. As our starting point we take the pseudovector \( \pi N\) Lagrangian:

\[
L_{\pi N} = \bar{\psi}_N(i\partial - M)\psi_N + \frac{1}{2}(\partial_\mu \phi \cdot \partial^\mu \phi - m_\pi^2 \phi \cdot \phi) + f_{\pi NN} \bar{\psi}_N i\gamma_\mu (\partial_\mu \phi) \cdot \tau \gamma^5 \psi_N ,
\]

where \( \psi_N\) is the nucleon field, \( \phi\) is the isovector pion field and \( \tau\) denotes the Pauli isospin matrices. Minimal substitution \( \partial_\mu \rightarrow \partial_\mu + ieA_\mu\) for the charged components of the pion field leads to the interaction vertices with which the currents of Fig. 3 are constructed.

Using the labelling in the figure and defining the four-momenta \( q_1 = p_1 - k_1 \) and \( q_2 = p_2 - k_2\) [the four-momentum carried by the virtual photon is then \( q = -(q_1 + q_2)\)] their relativistic expression is:

\[
J^\mu_{in-flight}(q_1, q_2) = -i \frac{1}{\sqrt{2}} \frac{f_{\pi NN}^2 f_{\pi\pi}}{m_\pi^2} (\tau^{(1)} \times \tau^{(2)})_3 \Pi(q_1)_{(1)} \Pi(q_2)_{(2)} (q_2 - q_1)^\mu \]

6
for the pion-in-flight (diagram (c)) and
\[
J_{\text{seagull}}^\mu(q_1, q_2) = -i \frac{1}{\sqrt{2}} \frac{f_{\pi NN} f_{\gamma NN}}{m_\pi^2} (\mathcal{T}^{(1)} \times \mathcal{T}^{(2)})_3 [\Pi(q_2)(\gamma^\mu \gamma^5)^{(1)}(q_1)(\gamma^\mu \gamma^5)^{(2)}] \tag{15}
\]
for the seagull current (diagrams (a) and (b)).

In the above:
\[
\Pi(q_i)(i) = \frac{(q_i \gamma^5)(i)}{q_i^2 - m_p^2} \tag{16}
\]

\(m_\pi\) and \(M\) are the pion and nucleon masses, \(f_{\pi NN} (f_{\pi NN}^2/4\pi = 0.08)\) the pseudo-vector pion-nucleon coupling constant. The other coupling constants are \(f_{\gamma NN} = 1\) and \(f_{\gamma NN} = f_{\pi NN}\). The index \((i)\) attached to the vertex operators distinguishes between the two interacting nucleons.

In the approach of Ref. [23], the correlations are introduced by adding an “ad hoc” contribution to the two-body current operator acting on the uncorrelated initial and final states. This leads to the appearance of four diagrams – in addition to the ones of Fig. 3 – which are not treated as genuine MEC, but as correlation corrections to the nuclear wave function.

In order to derive the contribution to the two-body current due to the excitation of a \(\Delta\) isobar in the intermediate state (see the four diagrams in Fig.4), we have to specify an effective Lagrangian that involves interacting pion, nucleon, and \(\Delta\) fields. In this picture the \(\Delta\) isobar is treated as a separate degree of freedom not as a \(\pi N\) resonance. We therefore have to extend the Lagrangian of Eq.(13) with a spin- and isospin- \(\frac{3}{2}\) field representing the \(\Delta\) isobar. We use the Lagrangian of Peccei, which is based on the Rarita-Schwinger (RS) formalism. The main reason for this choice is that the \(\Delta\) part of this Lagrangian is well suited for our relativistic analysis, since special care is taken of the off-shell extrapolation of the interaction vertices. This guarantees that, also off-shell, only the spin-\(\frac{3}{2}\) component of the RS \(\Delta\) field contributes to the amplitudes. A second important ingredient is the chiral symmetry from which the \(\pi N\) sector of the Lagrangian is constructed. The resulting linear \(\pi N\) coupling is of pseudovector type. Since we neglect the non linear terms (in the pion field) that arise, the \(\pi N\) sector is already displayed in Eq. (13). We now discuss the most important features of the incorporation of the \(\Delta\) field.

Although there is a general freedom in the expression for the free \(\Delta\) Lagrangian, due to invariance under transformations that do not have an effect on the spin-\(\frac{3}{2}\) of the field, we will choose the simplest form possible:

\[
\mathcal{L}_\Delta = -\bar{\psi}_\Delta^\mu [\gamma_\mu \gamma_\nu \bar{\psi}_\Delta^\nu - \gamma_\nu \gamma_\mu \bar{\psi}_\Delta^\nu - \gamma_\mu i \partial_\nu \bar{\psi}_\Delta^\nu + \gamma_\nu i \partial_\mu \bar{\psi}_\Delta^\nu] - M_\Delta (g_{\mu \nu} - \gamma_\mu \gamma_\nu) \bar{\psi}_\Delta^\mu \tag{17}
\]
This choice can be considered as choosing a specific gauge for the transformations mentioned above.

Next we discuss the interactions. The interaction vertices $O^{\mu\nu}$ are restrained by the condition
\[ \gamma_\mu O^{\mu\nu} = 0 \] (18)
This is a generalization of the property
\[ \gamma_\mu \psi^\mu_\Delta = 0 \] (19)
of the on-shell free $\Delta$ field, which assures that there is no direct coupling to the spin-$\frac{1}{2}$ component possible. This leads to the $\pi N \Delta$-interaction term:
\[ L_{\pi N \Delta} = \frac{i f_{\pi N \Delta}}{m_\pi} \bar{\psi}_\Delta (4g_{\mu\nu} - \gamma_\mu \gamma_\nu) T^\dagger \psi_N \partial^\nu \phi + \text{h.c.} \] (20)
where $T^\dagger$ represents the $2 \times 4$ isospin coupling matrices, which obey the relation
\[ T^i (T^\dagger)^j = \delta^{ij} - \frac{1}{3} \tau^i \tau^j. \] (21)

A shorthand notation for the isobar current is then given by:
\[ J_\mu^{\Delta}(q_1, q_2) = -\frac{1}{V^2} \frac{f_{\pi N N} f_{\pi N \Delta} f_{N \Delta}}{2M^{2}_{\pi}} \left\{ \left( \frac{2}{3} \tau^3 (2) - \frac{i}{3} (\tau^1 (1) \times \tau^2 (2))_3 \right) \left( j_\mu^{(a)}(k_a, q_2, q) \gamma_5 \right) \right\} \] (22)
where $k_a \equiv k_1 - q$, $k_b \equiv p_1 - q$ and $f_{\pi N \Delta} = 0.54, f_{\gamma N \Delta} = 5$ yield the strength of the coupling of the $\Delta$ to the EM and pionic fields, respectively.

We introduce the following definitions
\[ j_\mu^{(a)}(k_a, q_2, q) = (4k_\beta - \hat{q}_2 \gamma_\beta) S^{\beta\gamma}(k_a, M_\Delta) \frac{1}{2}(\gamma_\mu \gamma_\gamma + q_\mu \gamma_\gamma) \] (23)
and
\[ j_\mu^{(b)}(p_b, q_2, q) = \frac{1}{2}(\gamma_\beta \gamma_\mu + q_\mu \gamma_\beta) S^{\beta\gamma}(p_b, M_\Delta)(4q_2 - \gamma_\gamma \hat{q}_2); \] (24)

the analogous expressions associated with the diagrams (c) and (d) are simply obtained through the interchange $(1 \leftrightarrow 2)$, which also implies $k_a \leftrightarrow p_c$ and $p_b \leftrightarrow k_d$.

The Rarita-Schwinger (RS) $\Delta$ propagator is given by
\[ S^{\beta\gamma}(k, M_\Delta) = \frac{k + M_\Delta}{k^2 - M_\Delta^2} \left( g^{\beta\gamma} - \frac{\gamma_\beta \gamma_\gamma}{3} - \frac{2k^\beta k^\gamma}{3M^2_\Delta} - \frac{\gamma^\beta k^\gamma - \gamma^\gamma k^\beta}{3M_\Delta} \right) \] (25)

In order to include the effects of two-body currents in electron-nucleus interactions, the full two-body current operator, given by the sum of the pion and $\Delta$-isobar contribution:
\[ J_\mu^{\text{full}} = J_\mu^{\text{in-flight}} + J_\mu^{\text{seagull}} + J_\mu^{\Delta} \] (26)
has to be replaced in the matrix element of Eq. (10).
4 Summary & Perspectives

In this Thesis we want to consistently calculate the total neutrino-carbon CCQE cross section, including the contribution given by the relativistic one- and two-body electroweak current.

Over the past few years, the availability of the double-differential CCQE cross section measured by the MiniBooNE collaboration and the results of a new generation of theoretical studies have led to a better understanding of neutrino-nucleus interactions in a broad kinematical range, as well as to the identification of a number of outstanding unresolved issues. The excess of CCQE events in carbon reported by the MiniBooNE collaboration [7], is likely to be ascribable to the occurrence of processes other than single nucleon knock out, as advocated in Ref. [14].

In the presence of NN correlations final states with two nucleons in the continuum can be produced through three different mechanisms–ISC, FSI, MEC– involving both one- and two-nucleon currents. Models in which the processes associated to MEC are treated within the framework of the IPM, such as those of Refs. [15, 16], appear to be conceptually inconsistent, although the impact of this issue on the numerical results needs to be carefully investigated. A fully consistent analysis of the role of two particle-two hole final state within a realistic model of nuclear structure obviously requires that all mechanisms leading to the appearance of these final states be included, using a quantum-mechanical approach properly taking into account interference between the transition amplitudes of the one- and two-nucleon currents.

The results we obtained so far only provide evidence of the importance of interference terms in the electromagnetic response [17]. However, in our view this finding is a strong indication that they play a similar role in the weak response, as confirmed by Ref. [24]. The authors of this paper find that the effects of interference between one and two nucleon weak currents is in fact critical in determining the integrated neutral current response of carbon at momentum transfer larger than $1 \sim \text{fm}^{-1}$.

The extension of the factorization scheme underlying the IA appears to be a viable option for the development of a unified treatment of processes involving one- and two- nucleon currents in the region of large momentum transfer, which is known to be relevant for neutrino experiments. As the ultimate aim of our work is the calculation of the neutrino-nucleus cross section, in addition to the electromagnetic two-body currents, reported in the preceding section, even the fully relativistic expression of the axial contribution has to be included.

In our study, we will employ expressions of the one- and two- nucleon currents that reduce to those reported in Ref. [25] – extensively employed in the analysis of electron scattering data – in the non relativistic limit. The calculation of the nuclear amplitudes will be carried out within the Correlated Basis Function (CBF) approach, developed in Refs. [26, 27, 28].

References