Out of equilibrium statistical mechanics is a wide research field that deals with all those phenomena that require a statistical description, focusing on their behaviour while they are going to thermodynamical equilibrium or when an equilibrium state cannot be reached at all. Such systems are a common part of everyday life, e.g. in weather changes, chemical reactions, biological activity. My thesis will concentrate mostly on two kinds of systems: granular matter and active matter. Both are intrinsically out of equilibrium, presenting symmetric properties: granular matter ceaselessly dissipates energy by mean of inelastic collisions, while active matter needs to produce energy to keep its state of continuous motion. Generally, such systems are made by an amount of particles so large that statistical mechanics is necessary to represent their dynamics, but not so large that fluctuations can be easily neglected as in the usual thermodynamic limit. The lack of an established general method has led to the formulation of new mathematical methods such as fluctuating hydrodynamics and macroscopic fluctuation theory [1], which rely on the existence of a scale separation in time and space for some observables. Furthermore, granular and active matter models are driving intense research and applications in biophysics, material science, nanotechnologies [2].

Our aim is to study the behaviour of both kinds of models, deriving from microscopical dynamics the evolution of macroscopic quantities described above; a common way to perform this study is to find the large deviation function of thermodynamic currents, in order to derive stochastic evolution equations for the hydrodynamic fields. Such computations can apply both to lattice models [3, 4, 5, 6] or to molecular models [7, 4, 8, 9].

My PhD thesis’ work is now focusing on the analytical and numerical study of a granular lattice model: starting from the Kipnis-Marchioro-Presutti model [3], we introduced a similar system of $N$ particles on a unidimensional lattice with microscopical velocities, evolving by mean of inelastic collisions with different choices for the transition rates (e.g. Maxwell molecules or hard spheres), where total momentum is conserved and energy is dissipated; in such system, we derived hydrodynamics equations and described the homogeneous cooling state (in agreement with expected theoretical results [10]). A linear analysis of its stability has shown the presence of a critical value of the dissipation coefficient separating two cooling regimes (homogeneous or not). Furthermore, we simulated the system evolution finding a good agreement with the theory developed under the local equilibrium assumption, but also some small discrepancies even in the homogeneous regime. A further analysis has shown that also below the critical dissipation rate, local equilibrium approximation is violated and correlations between particles cannot be neglected. We then found an exact result for the 2-points correlation function and shown that its behaviour explains the observed discrepancies in cooling.

In the future, we will look at the characteristic nonequilibrium properties of the model, such as energy fluctuations, fluctuating currents and transport coefficients, both in homogeneous and nonhomogeneous states. We will study fluctuation-dissipation relations and try to deduce some general features that can be observed also in other models. The
inclusion of conserved momentum is a new feature with respect to usual fluctuating hydrodynamics models; indeed, in active matter systems velocity field is important because we are dealing with particles carrying momentum and kinetic energy. However, a kinetic microscopic approach to understand macroscopic features is still missing. With these prescriptions, we aim to introduce an active matter model on lattice and to describe its hydrodynamics.

*References*


