Exploiting path-polarization hyperentangled photons for multiqubit quantum information protocols

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March 30, 2015
Introduction

A bit is the minimal amount of classical information which can assume a value of either 0 or 1. A *qubit* is the quantum equivalent to the bit: it is described by a vector in a two-dimensional Hilbert space spanned by the computational basis $\{ |0\rangle, |1\rangle \}$ and can be represented as

$$ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{1} $$

where $\alpha$ and $\beta$ are complex coefficients such that $\alpha^2 + \beta^2 = 1$. A multiqubit system is *separable* if it can be written as a product tensor of single qubit systems and it is *entangled* if it is not possible.

An entangled system is a useful resource in quantum information as it shows peculiar properties that don’t have an equivalent in the classical world. Indeed such states are often prerequisites for many protocols used both in quantum computation and in quantum cryptography which provide both computational speedup over classical algorithms and secure information exchange between communicating parties.

Furthermore entanglement underlines deep non-local properties of the quantum world, and thus entangled states provide an interesting benchmark for testing physics on a fundamental level. The efficient generation, manipulation and detection of quantum states is still an openly debated problem.

Photons indeed are excellent candidates to represent qubits. Within the quantum optics framework, the quantum states of photons can be easily manipulated using both linear and non linear optical elements. In particular single photon *generation* can be attained using spontaneous parametric down conversion (SPDC) regarding information encoded in the path and polarization degrees of freedom of photons, photon state *manipulation* may be obtained using optical basic elements such as beam splitters, phase shifters, polarizing beam splitters and wave plates, and very efficient avalanche photo diodes may be used as single photon *detectors*. Furthermore light radiation can be easily transferred using either free space or waveguides and it has been shown that entangled state may be preserved for very long distances.

One of the most challenging tasks when dealing with the study of quantum systems is to increase the dimension of the Hilbert space so to investigate more complex structures.

In order to increase the number of qubits of a quantum system one can simply increase the number of entangled carriers so that each particle carries one qubit. It is worth noting that creating and controlling entanglement between an increasing number of particles becomes exponentially difficult due to decoherence effects. Exploiting different degrees of freedom of a single particle enables the encoding of more than one qubit in it, effectively reducing the number of particles needed to perform a quantum algorithm, and thus achieving higher generation rates and simpler experimental setup compared to the one photon=one qubit approach.

The entanglement of two particles over different degrees of freedom is called *hyper-entanglement*.

Once again photons are an optimal support to quantum information as it has been shown that path, polarization, orbital angular momentum, number and even time-energy may be exploited as degrees of freedom.
Project of PhD research

Figure 1: Two photons/four qubits path- polarization- hyperentangled source. A type I BBO crystal is injected with an UV pump (P=100mW, \(\lambda = 355\text{nm}\), violet line in figure) from right to left and generates correlated photons of H polarization by spontaneous parametric down conversion process (SPDC). The HH cone passes through a quarter wave plate (QWP) at 45° and becomes VV, then it is reflected by a spherical mirror \(M_s\) through the output on the right. Meanwhile the pump which is not affected by the BBO and passes through it passes trough a small hole in the center of the QWP, and it passes again through the BBO from left to right after being reflected by the spherical mirror. The BBO produces an HH cone which is superposed with the previous VV one. Changing the distance between the spherical mirror and the crystal determines the phase between the two cones. A four hole brass mask centered on the two cones selects the two indistinguishable couples \(l_A, r_B\) and \(r_A, l_B\).

During my PhD I will work using a two photons hyperentangled source in which a type I SPDC crystal produces a state of the form:

\[
|\psi_{he}\rangle = \frac{1}{\sqrt{2}}(|HH\rangle_{12} + |VV\rangle_{12}) \otimes \frac{1}{\sqrt{2}}(|rl\rangle_{12} + |lr\rangle_{12})
\]  

(2)

where 1, 2 identify the two photons, H and V are respectively the horizontal and vertical polarization of the photons and r, l denote the path the photons take exiting from the source. This state is a tensor product of a Bell state of two photons in both path and polarization. In fig. 1 is shown the schematic representation of the source.

The main goal of my thesis will be the study of quantum protocols and algorithms in which the qubits are represented by different degrees of freedom of photons. In particular my research will proceed twofold: my main interest will be the study of these kind of hyperentangled states using a platform exploiting the novel technique of integrated photonics, while other systems that would currently be much more difficult to implement with integrated photonic will be implemented in bulk optics.

Simultaneous control of two degrees of freedom of photon pairs on a chip

The realization of complex optical schemes consisting of a large number of elements requires the use of the waveguide technology to achieve the desired scalability, stability and miniaturization of the devices. For the last two decades this technique have met with important experimental success with the introduction of miniaturized photonic devices and circuits built on different material platforms.
Inherently stable interferometer networks, composed of waveguides, beam splitters and phase shifters, built in two dimensions on different material platforms, such as silicon, silicon nitride and others, are realized by lithography, a well established technique already developed for telecom wavelengths. It makes possible to fabricate a virtually infinite number of replica of the same circuit by using a single mask and represents at the moment the strongest candidate for a large-scale production of IQCs. While it has been demonstrated that the operation complexity performed by such systems may be very high, it is worth to remember that only path-encoded qubits are allowed in such systems, since polarization qubits are degraded by the intrinsic large birefringence of the material substrate or of the waveguide itself. On the other hand, several applications in the quantum domain, such as quantum computation and quantum communications, may greatly benefit from the possibility of manipulating and controlling polarization qubits.

Femtosecond laser writing, recently introduced for IQC applications [5, 6], allows to write in three dimensions circular transverse waveguide profiles able to support the propagation of nearly Gaussian modes with any polarization state, while keeping highly stable the phase of path-encoded qubits [7]. Besides, this technique makes possible to perform arbitrary transformations of the polarization state by suitable integrated devices, such as polarization beam splitters [5] and waveguide-based optical waveplates [8].

This line of research will be one of the main focuses of my PhD thesis and it will deal with an important novelty in the quantum information scenario: injecting a path-polarization hyperentangled state in a chip is the first attempt to exploit contextually different degrees of freedom in the same device, in order to compress the information over a smaller number of particles over an integrated network.

The main goal is to obtain first a proof of principle demonstration that in an integrated photonic circuit path-polarization entanglement of the input state is preserved. The same scheme will then be adopted to analyze both hyperentangled and cluster states.

The relevance of this research lies on the fact that, once the proof of principle is complete, it will be possibile to go to more complex circuits exploiting the two degrees of freedom at the same time. A few examples can be 3D quantum walks or multimode interferometers which would greatly benefit from the injection of hyperentangled states. Additionally the experimental simultaneous control of two degrees of freedom of photon pairs on a chip will permit the performing of one-way quantum computation protocols.

The main achievement of the scheduled experiment will be to demonstrate the presence of simultaneous entanglement in both path and polarization after injecting the four output modes of the hyperentangled source in an integrated device. In particular the photonic chip will present two integrated beam splitters which will be used to detect path entanglement of the state. The main technical achievement of the experimental setup consists of addressing each of the four modes of the source simultaneously so to inject them inside the device. As long optical fiber does not guarantee phase stability, we will address each mode in free space and we will inject them inside a 7 cm long fiber array coupled with the chip. Polarization analysis will be again performed in free space. Phase stability and independent polarization compensation will be the main concern of
Figure 2: Schematic representation of the integrated device used for the experiment. It consists in two integrated beam splitters which are injected respectively with the two modes of photon A and the two modes of photon B. A third BS present an integrated half eave plate in one of the inputs, and will be used to engineer cluster states.

the apparatus, and two qubits tomographic reconstruction and bosonic interference like effects will be the main tools to observe simultaneous path-polarization entanglement.

Furthermore will explore genuine multipartite entanglement by engineering a 4 qubit cluster state encoded in path and polarization of two photons starting from eq. 2:

$$|C_4⟩ = \frac{1}{2}(|H_Ar_AH_Bl_B⟩ + |V_Ar_AV_BL_B⟩ + |H_AL_AH_BR_B⟩ - |V_AL_AV_BR_B⟩) = \frac{1}{\sqrt{2}}(|Φ^+⟩|r_Al_B⟩ + |Φ^-⟩|l_Ar_B⟩).$$

A multipartite entanglement witness will be measured after the interaction with the integrated device to show that the multiqubits correlations are preserved. We will use the cluster state to implement various quantum algorithms.

Multiqubit protocols

While integrated photonics is indeed a promising technique that will allow to study increasingly complex systems in future, for a number of purposes it is still more practical to perform more intricated experiments using bulk optics. Because of this I’m pursuing a second line of research exploiting the multiple degree of freedom entangled photon states in more traditional ways. Here we briefly summarize a few examples of the experiments that are planned along this direction.

Entanglement activation from local noisy channel

Entanglement is the most precious resource in quantum information processing [9, 10]. However, what is most precious is often also most fragile. Indeed, a profound opponent of entanglement is noise: decoherence and dissipation both typically decrease entanglement, unless they are tailored specifically in a collective way, such as, for example, in correlated noisy channels [11, 12], or in engineered dissipation of coupled systems [13] or in the presence of tunable noise [14]. Hence, in general, except for peculiar experimental conditions, noise represents a strong obstacle for entanglement.

In this experiment, recently concluded and reported in [15], I will operate to introduce a scenario where, with fixed given resources of input states and gates, no entanglement can be produced, unless one switches on any local non-unital noise (such as dissipation). In this setting entanglement is zero without noise, and the degree of
the produced entanglement grows with the amount of the introduced local noise. An important aspect of this quantum effect is the choice of the dimension of the underlying Hilbert space: as long as only two qubits are present, we show that an adversary is always able to prevent the production of entanglement by applying a suitable local rotation.

I will also show that, however, by embedding the protocol into a higher-dimensional Hilbert space, concretely by using four qubits, the creation of entanglement via local noise can be made robust against any possible unitary action of an adversary. Interestingly, in order to achieve this robustness it is sufficient to have input states that exhibit just classical correlations. Experimentally, the different dimensional settings of two and four qubits are implemented using the path and polarisation degrees of freedom of photons and the fragility of the effect in low dimensions is demonstrated.

![Figure 3: Schematic representation of the experiment: a) Two-qubit scheme: the entangled output state $\rho_{\text{out}}$ is generated by starting from the initial state $I_2 \otimes \langle 0 | 0 \rangle$, and using local unitaries $U_A$ and $U_C$ and a CNOT gate. A local noisy element can be switched on by turning the knob. b) The same protocol in the presence of an adversary player using a local unitary $V_A$ to try and prevent the generation of entanglement. c) Four-qubit scheme: here $\rho_{AB}$ represents a bipartite state diagonal in the computational basis, with the same block of gates defined in the two-qubit scenario. The aim is to switch on entanglement of $\rho_{\text{out}}$ with respect to the cut $AB|CD$ by exploiting local noise.](image)

The reason behind the effect described above is an intricate relationship between the concepts of separability, quantum correlations and entanglement: a necessary ingredient for the production of entanglement in our scenario is the generation of nonvanishing off-diagonal terms (“quantum coherences”) in the initial density matrix, which may arise via a local non-unital channel. However, the presence of quantum coherences is not sufficient for robustness of the protocol, in the sense explained above. As mentioned above, a sufficient ingredient for robustness is the presence of classical correlations within the initial state. These classical correlations are turned into correlations of quantum nature via a local non-unital channel, which still keeps the quantum state separable. Finally, the quantum correlations are activated into entanglement.

The experiment has been performed with bulk optics by using the two photons generated by the hyperentanglement source above described. In the two qubit version of the protocol we manipulate path and polarization degrees of freedom of a single photon. In particular the noisy channel is implemented by realizing an amplitude damping channel in the path degree of freedom, inside a two-level Sagnac interferometer and we measured a proper entanglement witness varying the noise of the system. In the four qubit protocol we use path and polarization of two photons and we demonstrate the protocol robustness for an adversary player manipulation of the system. In Fig. 3 we show a schematic representation of the two protocols.
Revealing three-qubit entanglement using thermodynamics

Entanglement has always been a crucial element for quantum information protocols and different approaches to investigate the presence of entanglement in quantum systems have been proposed. In a two qubit system Bell’s inequalities represent the standard tool to demonstrate the presence of non classical correlations, as they are built in order to prove that it is not possible that Quantum Mechanics properties cannot be explained with the presence of an hidden variable theory.

Another approach which is used to detect entanglement in a two qubit system is the measurement of extractable thermodynamical work from a system. This derives from the first relation of the uncertainty principle was formulated by Heisenberg for the case of momentum and position. The subsequent work of Robertson showed that

\[ \Delta R \Delta S \geq \frac{1}{2} |\langle [R, S] \rangle| \] (4)

where \( \Delta R \) and \( \Delta S \) (for two observables R and S) are characterized in terms of standard deviations. An improved version of this inequality is quantified by the entropy:

\[ H(R) + H(S) \geq \log_2 \frac{1}{c} \] (5)

where \( H(R) \) denotes the Shannon entropy of the probability distribution of the outcomes when R is measured and the term \( \frac{1}{c} \) quantifies the complementarity of the two observables.

Mathematically these relations don’t apply to the case of an observer holding quantum information about the system: in the extreme case of an observer that holds a particle maximally entangled with one-another (a quantum memory), it is possible to correctly predict the outcome of whichever measurement is chosen. Bob’s task is to use entanglement to minimize his uncertainty about Alice’s measurement outcome.

A quantification of this effect is described by the uncertainty relation shown in (6):

\[ H(R|B) + H(S|B) \geq \log_2 \frac{1}{c} + H(A|B) \] (6)

where the Shannon entropy is now replaced by the von Neumann entropy; \( H(A|B) \) is an extra term appearing on the right-side that quantifies the amount of entanglement between the particle and the quantum memory: this quantity is mathematically negative only for entangled states [17]. If the particle A and the quantum memory B are maximally entangled, then \( H(A|B) = -\log_2 d \), where \( d \) is the dimension of the Hilbert space in which the state of the particle A lives.

The maximal complementarity, \( \max(c) \), between R and S is \( \frac{1}{2} \), giving \( \log_2 \frac{1}{c} = 1 \); as \( \log_2 \frac{1}{c} \) can’t be larger than \( \log_2 d \), the right-hand-side term of equation (6) cannot be greater than zero for \( \rho_{AB} \) maximally entangled.

In this case the outcome can be correctly predicted if Alice chooses R or S and communicates classically the measurement choice to Bob. Entanglement allows us to reduce uncertainties about both observables than would be possible with only classical information. This has been experimentally done in [18].
The aim of this project is to provide an alternative method to distinguish between different kinds of tripartite entangled states, such as the entangled three-qubit Greenberger Horne Zeilinger (GHZ) state ($|GHZ⟩ = \frac{1}{\sqrt{2}}(|000⟩ + |111⟩)$) and from a Werner state ($|W⟩ = \frac{1}{\sqrt{3}}(|001⟩ + |010⟩ + |100⟩)$) based on extracting thermodynamical work from a quantum optical system. We will firstly reproduce the results obtained in [18] using a two qubit protocol, and then we will use the hyperentangled source to produce mixtures of GHZ and W states and measure the extractable work of the three qubit protocol proposed in [19]. We will confront our results with other entanglement witnesses for three dimensional systems, like Mermin inequalities (see [20] for details) for different input states.

0.1 Single qubit protocols

In this section I will present another planned experiment that doesn’t regard technically a multiqubit protocol, but it will nevertheless be performed by using the photons produced by the hyperentanglement source.

**Weak measurement in complementarity principle**

In this experiment I’m planning to investigate the wave-particle duality relation $D^2 + V^2 \leq 1$ in a quantum system subject to weak measurements and decoherence. In the inequality D is known as distinguishability and V is the visibility of the interference fringes. In a Mach-Zehnder interferometer injected with classical light we observe interference fringes at the output of the second beam splitter showing that light behaves like a wave. If we inject single photons it shows a similar behaviour, so that we can observe peaks and dips in the number of output photons by changing the phase of one of the arms. If we remove the second beam splitter we can observe particle behaviour of photons which singularly bunch either in one arm of the interferometer or the other.

We propose to investigate this scenario by using weak measurements, as it has been demonstrated that weak measurements help preserving the entanglement on a system [21], but their effect on the wave-particle duality relation is still unknown. In our scheme we will use a single qubit input state encoded in photon polarization injected in a Mach-Zehnder interferometer. If the photon passes through one of the arms of the interferometer, it is subject to a weak measurement, performed by using multiple glass plates that rotate around their brewster angle, and an amplitude damping channel in order to introduce controlled decoherence between the two arms. Measurements of visibility and distinguishability are then performed as a function of the damping parameter and of the weak measurement parameter in order to analyze the behaviour of the duality relation.
Bibliography


