Complexity in Financial Markets: Empirical evidence and models at the micro level

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Outline

• Introduction
• How an Order Book works
• Known results
• Present work
• Future developments
• Conclusions
Random walk hypothesis

\[ r(t) = \frac{p(t) - p(t - 1)}{p(t - 1)} = \frac{\delta p}{p} \approx \log p(t) - \log p(t - 1) \]
Stylized Facts

\[ C_r(\tau) = \langle r(t) r(t + \tau) \rangle \]

No simple correlations

\[ C_{|r|}(\tau) = \langle |r(t)||r(t + \tau)| \rangle \]

Volatility clustering

0 < \mu < 1

Returns are not correlated but not independent!

WELL OUT THE LEVY REGIME

fat tails

asymmetry
Lines of research

- *Find the stochastic model of stock price dynamics:*  
  Standard theory of Finance: price changes are independent and normal  
  Deviations from the simple RW: stochastic processes (directly on the price).  
  MACRO.

- *Agent Based Models:*  
  models of strategic interaction, collective behavior in financial market  
  Actions and interaction of agents ---> price  
  MESO.

- *Order book:*  
  double auction mechanism to match orders. If unmatched ---> limit order book  
  Direct investigation of price formation  
  MICRO.
How an Order Book works

Price is defined as the last transaction price or as the midprice \((a+b)/2\)

Volume

BID

spread

ASK

tick

b(t)

a(t)

gap

Price
How an Order Book works

Execution priority:

- Lower priced sell orders or higher priced buy orders have priority.
- First order placed has priority when multiple orders have same price.
Limit orders

Volume

tick

$\text{b}(t)$

$\text{a}(t)$

gap

Price
Limit orders

Volume

Price

tick

b(t) a(t)

gap
Patient traders place orders to buy or sell a given volume of stocks at a fixed price. Limit orders do not lead to an immediate transaction.
Market orders

tick

Volume

Price

gap
Market orders

tick

Volume

Price

b(t) a(t)

gap
Market orders

Impatient traders place orders to buy or sell a given volume, leading to an immediate transaction (and sometimes to an unfavorable market impact)
An unmatched limit order can be cancelled by the owner or can expire.
Returns and gaps

Price movements are equal to the first gap in most (>90%) cases.
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Orders with nonzero impact have volume equal to the best one in most (>80%) cases
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Orders with nonzero impact have volume equal to the best one in most (>80%) cases.
A striking result

Table 2. Summary table of the percentage of the time that non-zero changes in the best prices are equal to the first gap (left) and that the market order volume $\omega$ exactly matches the volume at the corresponding best price $V_{\text{best}}$ (right). Assuming a Bernoulli process the sample errors are of the order of 0.1–0.2%.

<table>
<thead>
<tr>
<th>Tick</th>
<th>% of non-zero returns equal to first gap</th>
<th>% of non-zero returns with $\omega = V_{\text{best}}$</th>
<th>Sell</th>
<th>Buy</th>
<th>Sell</th>
<th>Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZN</td>
<td>94.3</td>
<td>99.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>BAA</td>
<td>95.9</td>
<td>99.1</td>
<td></td>
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<tr>
<td>BLT</td>
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<td>99.2</td>
<td></td>
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<tr>
<td>BOOT</td>
<td>95.9</td>
<td>99.2</td>
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<tr>
<td>BSY</td>
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<td>99.7</td>
<td></td>
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<tr>
<td>DGE</td>
<td>96.3</td>
<td>99.7</td>
<td></td>
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<tr>
<td>GUS</td>
<td>95.8</td>
<td>99.5</td>
<td></td>
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<tr>
<td>HG.</td>
<td>95.9</td>
<td>99.5</td>
<td></td>
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<tr>
<td>LLOY</td>
<td>97.3</td>
<td>99.8</td>
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<tr>
<td>PRU</td>
<td>95.9</td>
<td>99.5</td>
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<tr>
<td>PSON</td>
<td>93.1</td>
<td>99.6</td>
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<tr>
<td>RIO</td>
<td>95.7</td>
<td>99.7</td>
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<tr>
<td>RTO</td>
<td>96.1</td>
<td>99.5</td>
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<tr>
<td>RTR</td>
<td>93.0</td>
<td>99.7</td>
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<tr>
<td>SBRY</td>
<td>95.6</td>
<td>99.4</td>
<td></td>
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<tr>
<td>SHEL</td>
<td>98.7</td>
<td>99.9</td>
<td></td>
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<tr>
<td>Average</td>
<td>95.6</td>
<td>99.5</td>
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</tbody>
</table>

A striking result

Liquidity

Liquidity is the degree to which an asset or security can be bought or sold in the market without affecting the asset's price. Liquidity is characterized by a high level of trading activity. Assets that can be easily bought or sold are known as liquid assets.

Liquidity is crucial to determine the price impact of a given order, that is the price variation caused by the arrival of the order.

The standard economic theory assumes infinite liquidity i.e. a zero price impact.
Liquid vs illiquid market

- Small price variations
- Behavior similar to a continuous system
Liquid vs illiquid market

- Large price variations
- The discreteness of the system is crucial

$p(t+1) - p(t) = \text{several ticks}$
Price Impact

The Price Impact Function (PIF) can be considered as the response function of a stock, that is ....

If an agent submit a “virtual” market order of volume $\omega$ at time $t$, what will be the average price change at time $t+\tau$?

$$\phi(\omega, \tau) = E[\Delta p(\tau)|\omega] \quad \Rightarrow \quad \phi(\omega, \tau) \approx \psi(\tau)\Phi(\omega)$$

The Price Impact Function of real market is a concave function with respect to the order volume

Markets are not in a linear response regime
\[ \Phi(\omega) \sim \omega^\beta \]

<table>
<thead>
<tr>
<th>Stock Exchange</th>
<th>( \beta \approx 0.2 - 0.4 )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYSE, LSE</td>
<td>small/medium order size</td>
<td>Farmer, Lillo, Mantegna</td>
</tr>
<tr>
<td>NYSE</td>
<td>( \beta \approx 0.5 )</td>
<td>Stanley, et al.</td>
</tr>
<tr>
<td>Citygroup</td>
<td>( \beta = 0.6 )</td>
<td>Almgren</td>
</tr>
<tr>
<td>( \Phi(\omega) \sim \ln(\omega) )</td>
<td>small/medium order size</td>
<td>Paris Bourse</td>
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<tr>
<td></td>
<td></td>
<td>Bouchaud, Potters, et al.</td>
</tr>
</tbody>
</table>
We want to study the role played by liquidity/granularity in price response but the normal PIF is calculated averaging on order book configurations with different liquidity/granularity.

We define the **Price Impact Surface** which is instead a function of volume and liquidity/granularity

\[
E\left[\Delta p(\Delta t) | \omega, g \right] = \phi(\omega, \Delta t \to 0, g)
\]
We define the **Price Impact Surface** which is instead a function of volume and liquidity/granularity

\[
E\left[ \Delta p(\Delta t) \mid \omega, g \right] = \phi(\omega, \Delta t \rightarrow 0, g)
\]

We can take the following expression as an operative definition of granularity:

\[
\langle g(t) \rangle = \frac{1}{N(t)} \sum_{j=1}^{N(t)} \frac{n_{s(t),j}}{s(t)}
\]

This is equivalent to the linear density of orders:

\[
\langle g(t) \rangle = \frac{\Omega}{L}
\]

Where \( \Omega \) is the total volume in that side of the book and \( L \) is his approximatively the depth of the book.
A microscopic version of Farmer's zero intelligence model

- A sell limit order can be placed in $[b(t), b(t) + k s(t)]$
- An incoming market order is placed outside $[b(t), a(t)]$
- An incoming limit order is placed inside $[b(t), a(t)]$
- A buy limit order can be placed in $[a(t) - k s(t), a(t)]$
Price Impact Surface
Model results for $T=400$ and $k=4$

\[
\phi(\omega, \Delta t \to 0, g | g) \sim \omega^{0.58} \quad \sim \omega^{0.5 \div 0.7}
\]

\[
\phi(\omega, \Delta t \to 0, g | \omega) \sim g^{-0.8 \div -1.0}
\]
Odd vs even spreads

Evidence: odd spreads are more frequent than even ones

Daily average

Conditional probability

Data: 20 NYSE stocks, 80 days from Oct. 2004 to Feb. 2005, tick by tick

Only a discreteness effect?
Consequently we adopt a non uniform deposition rule inside the spread.

\[ g(i | s) = \begin{cases} 
  g(1 | s) = \alpha \\
  g(i | s) = \frac{1 - \alpha}{s - 2} & i = 2, \ldots, s - 1 
\end{cases} \]
This simple change with respect to a zero-intelligence deposition model is enough to recover the empirical evidence.
Non uniform limit orders deposition causes intermittency

Uniform case
the OB is always quiet and compact, with small spreads and volatility clustering

Non uniform case
the OB displays different regimes of activity. Bursts of volatility and large spreads are clustered
Stylized Facts

Panel a: pdf(returns)
- Red: $\alpha=0.7$, $\pi=0.33$
- Blue: $\alpha=0.7$, $\pi=0.30$
- Orange: $\alpha=0.7$, $\pi=0.25$
- Green: $\text{unif}$, $\pi=0.33$
- Yellow: $\text{unif}$, $\pi=0.30$
- Light Yellow: $\text{unif}$, $\pi=0.25$

Panel b: pdf(spread)
- Red: $\langle s \rangle=2.77$, $\alpha=0.7$, $\pi=0.33$
- Blue: $\langle s \rangle=2.17$, $\alpha=0.7$, $\pi=0.30$
- Orange: $\langle s \rangle=1.71$, $\alpha=0.7$, $\pi=0.25$
- Green: $\langle s \rangle=15.8$, $\text{unif}$, $\pi=0.33$
- Yellow: $\langle s \rangle=2.07$, $\text{unif}$, $\pi=0.30$
- Light Yellow: $\langle s \rangle=1.80$, $\text{unif}$, $\pi=0.25$

Panel c: pdf($g$)
- Red: $\alpha=0.7$, $\pi=0.33$
- Blue: $\alpha=0.7$, $\pi=0.30$
- Orange: $\alpha=0.7$, $\pi=0.25$
- Green: $\text{unif}$, $\pi=0.33$
- Yellow: $\text{unif}$, $\pi=0.30$
- Light Yellow: $\text{unif}$, $\pi=0.25$

Panel d: Autocorrelation
- Red: $\alpha=0.7$, $\pi=0.33$
- Blue: $\alpha=0.7$, $\pi=0.30$
- Orange: $\alpha=0.7$, $\pi=0.25$
- Green: $\text{unif}$, $\pi=0.33$
- Yellow: $\text{unif}$, $\pi=0.30$
- Light Yellow: $\text{unif}$, $\pi=0.25$
Conclusions

• The order book offers a unique perspective on both price formation and agents' strategies
• The recent availability of a huge amount of data will permit a detailed study of empirical evidences and a quantitative test of theoretical models

• We have introduced a microscopic model to study the order book dynamics

• We investigated several features:
  • The price impact dependence on granularity
  • The asymmetry between odd and even spreads
  • The influence of strategic order deposition on both asymmetry and dynamics
Perspectives

• Analysis of the empirical Price Impact Surface, using various definitions of granularity: volume dependence? Tick size dependence? Whole order book?
• Analysis of Price Impact from real orders (hidden orders, time correlation, liquidity response...)

Final goal:

• To connect future volatility to present granularity, to use order book analysis for short term risk management
• M. Cristelli, V. Alfi, L. Pietronero, A. Z.
  *Liquidity Crisis, Granularity of the Order Book and Price Fluctuations*

• A. Z., M. Cristelli, V. Alfi, F. Ciulla, L. Pietroner
  *Asymmetric statistics of order books: The role of discreteness and evidence for strategic order placement*
Uniform and non uniform deposition

Conditional probabilities to have an even or an odd spread given s, the value of the previous spread

**Uniform case**

The only source of asymmetry is the OB discreteness

\[
P(e|o, s) = \frac{1}{2} \quad P(o|e, s) = \frac{1}{2} \frac{s}{s-1}
\]
\[
P(o|o, s) = \frac{1}{2} \quad P(e|e, s) = \frac{1}{2} \frac{s-2}{(s-1)}
\]

**Non uniform case**

Also the strategic order deposition contributes

\[
P(e|o,s) = \alpha + \frac{1 - \alpha}{2} \frac{s - 3}{s - 2}
\]
\[
P(o|e,s) = \alpha + \frac{1 - \alpha}{2}
\]
\[
P(o|o,s) = \frac{1 - \alpha}{2} \frac{s - 1}{s - 2}
\]
\[
P(e|e,s) = \frac{1 - \alpha}{2}
\]
Model parameters

• Tick size $\Delta q = 1$
• Orders volume $V = 1$
• Market orders probability $\pi = 1/3$
• Cancellation time $T = 400$
• Deposition length $k = 4$
• In the non uniform case, $\alpha$